MO417 – Complexidade de Algoritmos
Segundo Semestre de 2008
Nona Lista de Exercícios

Caminhos Mínimos

1. Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers.

2. We are given a directed graph G = (V, E) on which each edge (u, v) $\in$ E has an associated value r(u, v), which is a real number in the range 0 $\leq$ r(u, v) $\leq$ 1 that represents the reliability of a communication channel from vertex u to vertex v. We interpret r(u, v) as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

3. Suppose we change the line:

   "while $Q \neq \emptyset$"

   of Dijkstra's algorithm to the following:

   "while $|Q| > 1$"

   This change causes the while loop to execute $|V| - 1$ times instead of $|V|$ times. Is this proposed algorithm correct?

4. Let G = (V, E) be a weighted, directed graph with weight function $w : E \rightarrow \{0, 1, \ldots, W\}$ for some nonnegative integer W. Modify Dijkstra's algorithm to compute the shortest paths from a given source vertex s in O$(WV + E)$ time.

5. Modify your algorithm from previous exercise to run in O$((V + E) \lg W)$ time.

6. Modify the Bellman-Ford algorithm so that it sets d[v] to $-\infty$ for all vertices v for which there is a negative-weight cycle on some path from the source to v.

7. Given a weighted, directed graph G = (V, E) with no negative-weight cycles, let m be the maximum over all pairs of vertices u, v $\in$ V of the minimum number of edges in a shortest path from u to v. Here, the shortest path is by weight, not the number of edges. Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m + 1$ passes.

8. Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm.