The Problem of Sorting Permutations by Prefix and Suffix Rearrangements

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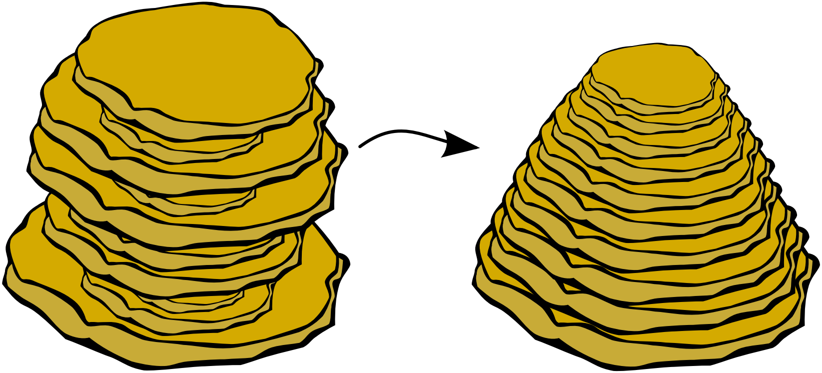
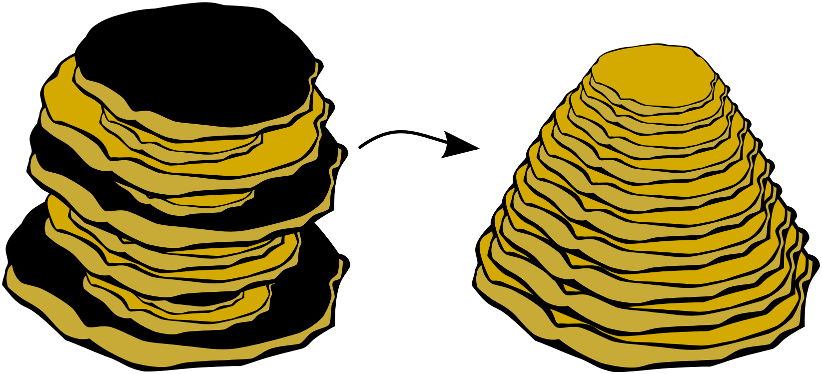
Pancake Flipping

The Pancake Flipping Problem (Dweighter, 1975 [1])



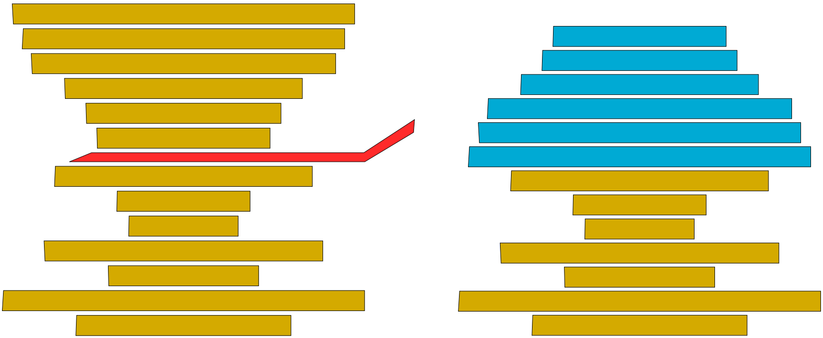
Pancake Flipping

The Burnt Pancake Flipping Problem (Gates and Papadimitriou, 1979 [2])



Pancake Flipping

Allowed moves: prefix reversals



Reinterpreted as a genome rearrangements problem (1995) [3]

Genome Rearrangements

- A type of large scale mutation that can occur in a genome
- Reversal: inverts a segment of a genome
- Transposition: exchanges the position of two consecutive segments of a genome
- Sorting by Genome Rearrangements: a form of comparing two genomes and inferring their evolutionary distance
- Our work: prefix and suffix versions of reversals and transpositions

Pancake Flipping

Sorting by Prefix Reversals

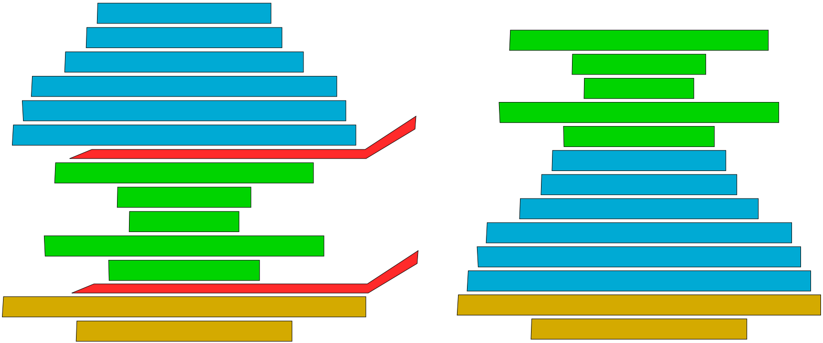
- NP-hard (2012) [4]
- Best-known approximation algorithm has factor 2 (2005) [5]

Sorting by Signed Prefix Reversals

- Unknown complexity
- Best-known approximation algorithm has factor 2 (1995) [6]

Other Problems

Sorting by Prefix Transpositions (Dias and Meidanis, 2002 [7])
Unknown complexity, best-known approximation factor 2



Pancake Flipping with Two Spatulas (Sharmin *et al.*, 2010 [8])
Unknown complexity, best-known approximation factor $2 + 4/b_{upr}(\pi)$

Definitions and Notations

- Permutation: $\pi = (\pi_1 \ \pi_2 \ \ldots \ \pi_n)$ where $\pi_i = \pi(i)$
- Unsigned permutation: $\pi_i \in \{1, 2, \ldots, n\}$ and $\pi_i \neq \pi_j$ for all $i \neq j$
- Signed permutation: $\pi_i \in \{-n, -(n-1), \ldots, -1, 1, 2, \ldots, n\}$ and $|\pi_i| \neq |\pi_j|$ for all $i \neq j$
- Extended: $(\pi_0 = 0 \ \pi_1 \ \pi_2 \ \ldots \ \pi_n \ \pi_{n+1} = n + 1)$
- Permutations also represent rearrangements
- Composition: $\pi \cdot \sigma = (\pi_{\sigma_1} \ \pi_{\sigma_2} \ \ldots \ \pi_{\sigma_n})$
- Identity permutation: $\iota_n = (1 \ 2 \ \ldots \ n)$

Definitions and Notations

- Reversal: $\rho(i,j)$ with $1 \leq i < j \leq n$

$$\pi = (\pi_1 \ \ldots \ \pi_{i-1} \ \underline{\pi_i \ \pi_{i+1} \ \ldots \ \pi_{j-1} \ \pi_j} \ \pi_{j+1} \ \ldots \ \pi_n)$$
$$\pi \cdot \rho(i,j) = (\pi_1 \ \ldots \ \pi_{i-1} \ \underline{\pi_j \ \pi_{j-1} \ \ldots \ \pi_{i+1} \ \pi_i} \ \pi_{j+1} \ \ldots \ \pi_n)$$

Example:

$$\pi = (3 \ \underline{1 \ 5 \ 2 \ 7} \ 4 \ 6)$$
$$\pi \cdot \rho(2,5) = (3 \ \underline{7 \ 2 \ 5 \ 1} \ 4 \ 6)$$

- Prefix reversal: $\rho_p(j) \equiv \rho(1,j)$
- Suffix reversal: $\rho_s(i) \equiv \rho(i,n)$

Definitions and Notations

- Signed reversal: $\bar{\rho}(i,j)$ with $1 \le i \le j \le n$

$$\begin{array}{rl} \pi = & (\pi_1 \ \dots \ \pi_{i-1} \ \underline{\pi_i \ \ \pi_{i+1} \ \dots \ \pi_{j-1} \ \ \pi_j} \ \pi_{j+1} \ \dots \ \pi_n) \\ \pi \cdot \bar{\rho}(i,j) = & (\pi_1 \ \dots \ \pi_{i-1} \ \underline{-\pi_j \ -\pi_{j-1} \ \dots \ -\pi_{i+1} \ -\pi_i} \ \pi_{j+1} \ \dots \ \pi_n) \end{array}$$

Example:

$$\begin{array}{rl} \pi = & (-3 \ \underline{+1 \ -5 \ +2 \ +7} \ -4 \ -6) \\ \pi \cdot \bar{\rho}(2,5) = & (-3 \ \underline{-7 \ -2 \ +5 \ -1} \ -4 \ -6) \end{array}$$

- Signed prefix reversal: $\bar{\rho}_p(j) \equiv \bar{\rho}(1,j)$
- Signed suffix reversal: $\bar{\rho}_s(i) \equiv \bar{\rho}(i,n)$

Definitions and Notations

- Transposition: $\tau(i, j, k)$ with $1 \leq i < j < k \leq n + 1$

$$\pi = (\pi_1 \ldots \pi_{i-1} \ \underline{\pi_i \ \pi_{i+1} \ldots \pi_{j-1}} \ \underline{\pi_j \ \pi_{j+1} \ldots \pi_{k-1}} \ \pi_k \ldots \pi_n)$$
$$\pi \cdot \tau(i, j, k) = (\pi_1 \ldots \pi_{i-1} \ \underline{\pi_j \ \pi_{j+1} \ldots \pi_{k-1}} \ \underline{\pi_i \ \pi_{i+1} \ldots \pi_{j-1}} \ \pi_k \ldots \pi_n)$$

  Example:

$$\pi = (3 \ \underline{1 \ 5} \ \underline{2 \ 7 \ 4} \ 6)$$
$$\pi \cdot \tau(2, 4, 7) = (3 \ \underline{2 \ 7 \ 4} \ \underline{1 \ 5} \ 6)$$

- Prefix transposition: $\tau_p(j, k) \equiv \tau(1, j, k)$
- Suffix transposition: $\tau_s(i, j) \equiv \tau(i, j, n + 1)$

Definitions and Notations

Sorting sequence: Sequence of rearrangements that, when applied to π, transform it into ι_n

$$
\begin{array}{rcl}
\pi & = & (\underline{2 \quad 4 \quad 5} \quad 3 \quad 1) \\
\cdot\rho_p(3) & \rightarrow & (\underline{5 \quad 4 \quad 2 \quad 3 \quad 1}) \\
\cdot\rho_p(5) & \rightarrow & (\underline{1 \quad 3} \quad 2 \quad 4 \quad 5) \\
\cdot\rho_p(2) & \rightarrow & (\underline{3 \quad 1 \quad 2} \quad 4 \quad 5) \\
\cdot\rho_p(3) & \rightarrow & (\underline{2 \quad 1} \quad 3 \quad 4 \quad 5) \\
\cdot\rho_p(2) & \rightarrow & (1 \quad 2 \quad 3 \quad 4 \quad 5)
\end{array}
$$

Definitions and Notations

- Length of $\rho(i, j)$: $\ell = j - i + 1$
- Length of $\tau(i, j, k)$: $\ell = k - i$
- Cost of a rearrangement of length ℓ: $f(\ell)$
  - Normally, $f(\ell) = \ell^{\alpha}$, for $\alpha \geq 0$
- Sorting sequence with k rearrangements of length ℓ_1, ℓ_2, ..., ℓ_k has cost $f(\ell_1) + f(\ell_2) + \ldots + f(\ell_k)$
- Rearrangement model: defines the allowed rearrangements in a sorting problem

Goals

Given a permutation π, a rearrangement model β, and an α, find the **distance** $d^{\alpha}_{\beta}(\pi)$

$$\begin{array}{rcl} \pi & = & (\underline{2 \quad 4 \quad 5 \quad 3} \quad 1) \\ \cdot\rho_p(4) & \rightarrow & (\underline{3 \quad 5 \quad 4} \quad 2 \quad 1) \\ \cdot\rho_p(3) & \rightarrow & (\underline{4 \quad 5} \quad 3 \quad 2 \quad 1) \\ \cdot\rho_p(2) & \rightarrow & (\underline{5 \quad 4 \quad 3 \quad 2 \quad 1}) \\ \cdot\rho_p(5) & \rightarrow & (1 \quad 2 \quad 3 \quad 4 \quad 5) \end{array}$$

$$d^{0}_{pr}(\pi) = 4$$

$$d^{1}_{pr}(\pi) = 14$$

Goals

Given all permutations of a size n, a rearrangement model β, and an α, find the **diameter** $D^{\alpha}_{\beta}(n)$

$d^0_{pr}(1\ 2\ 3\ 4) = 0$	$d^0_{pr}(1\ 2\ 4\ 3) = 3$	$d^0_{pr}(1\ 3\ 2\ 4) = 3$
$d^0_{pr}(1\ 3\ 4\ 2) = 3$	$d^0_{pr}(1\ 4\ 2\ 3) = 3$	$d^0_{pr}(1\ 4\ 3\ 2) = 3$
$d^0_{pr}(2\ 1\ 3\ 4) = 1$	$d^0_{pr}(2\ 1\ 4\ 3) = 3$	$d^0_{pr}(2\ 3\ 1\ 4) = 2$
$d^0_{pr}(2\ 3\ 4\ 1) = 2$	$d^0_{pr}(2\ 4\ 1\ 3) = 4$	$d^0_{pr}(2\ 4\ 3\ 1) = 3$
$d^0_{pr}(3\ 1\ 2\ 4) = 2$	$d^0_{pr}(3\ 1\ 4\ 2) = 4$	$d^0_{pr}(3\ 2\ 1\ 4) = 1$
$d^0_{pr}(3\ 2\ 4\ 1) = 3$	$d^0_{pr}(3\ 4\ 1\ 2) = 3$	$d^0_{pr}(3\ 4\ 2\ 1) = 2$
$d^0_{pr}(4\ 1\ 2\ 3) = 2$	$d^0_{pr}(4\ 1\ 3\ 2) = 3$	$d^0_{pr}(4\ 2\ 1\ 3) = 3$
$d^0_{pr}(4\ 2\ 3\ 1) = 4$	$d^0_{pr}(4\ 3\ 1\ 2) = 2$	$d^0_{pr}(4\ 3\ 2\ 1) = 1$

$$D^0_{pr}(4) = 4$$

Traditional Approach

- The cost to sort a permutation is given by the amount of rearrangements that were used to do it
- $f(\ell) = \ell^{\alpha} = 1$ because $\alpha = 0$

Known Results

Rearrangements	Best Approx. Factor	Complexity
Reversals	1.375 [9]	NP-hard [10]
Sig. Reversals	1 [11]	P [11]
Transpositions	1.375 [12]	NP-hard [13]
Reversals and Transpositions	2.8334 [14, 15]	Unknown
Sig. Reversals and Transpositions	2 [16]	Unknown
Pref. Reversals	2 [5]	NP-hard [4]
Sig. Pref. Reversals	2 [6]	Unknown
Pref. Transpositions	2 [7]	Unknown
Pref. Reversals and Transpositions	$2 + 4/b_{upr}(\pi)$ [17]	Unknown

Known Results

Rearrangements	Diameter	
	Lower Bound	Upper Bound
Reversals	$n-1$ [18]	
Sig. Reversals	$n+1$ [19]	
Transpositions	$\lfloor \frac{n+1}{2} \rfloor + 1$ [12]	$\lfloor \frac{2n-2}{3} \rfloor$ [20]
Reversals and Transpositions	-	-
Sig. Reversals and Transpositions	-	-
Pref. Reversals	$\frac{15n}{14}$ [21]	$\frac{18n}{11} + O(1)$ [22]
Sig. Pref. Reversals	$\frac{3n+3}{2}$ [21]	$2n-6$ [23]
Pref. Transpositions	$\lfloor \frac{3n+1}{4} \rfloor$ [24]	$n - \log_{7/2} n$ [25]
Pref. Reversals and Transpositions	-	-

Obtained Results

Rearrangements	Approx. Factor
Pref. and Suf. Reversals	2
Pref. and Suf. Transpositions	2
Pref. and Suf. Reversals and Transpositions	$2 + 4/b_{upsrt}(\pi)$
Sig. Pref. and Suf. Reversals	2
Sig. Pref. Reversals and Transpositions	$2 + 4/b_p(\pi)$
Sig. Pref. and Suf. Reversals and Transpositions	$2 + 4/b_{upr}(\pi)$

Obtained Results

Rearrangements	Diameter	
	Lower Bound	**Upper Bound**
Pref. and Suf. Reversals	$n-1$	$\frac{18n}{11}+O(1)$
Pref. and Suf. Transpositions	$\lceil \frac{n-1}{2} \rceil + 1$	$n - \log_{7/2} n$
Pref. Reversals and Transpositions	$\lceil \frac{n}{2} \rceil$	$n - \log_{7/2} n$
Pref. and Suf. Reversals and Transpositions	$\lceil \frac{n-1}{2} \rceil$	$n - \log_{7/2} n$
Sig. Pref. and Suf. Reversals	n	$2n-6$
Sig. Pref. and Suf. Transpositions	$\lceil \frac{n}{2} \rceil + 1$	$2n-6$
Sig. Pref. and Suf. Reversals and Transpositions	$\lceil \frac{n-1}{2} \rceil$	$2n-6$

Obtained Results

- Pref. and Suf. Reversals
- Pref. and Suf. Transpositions
- Pref. and Suf. Reversals and Transpositions
- Sig. Pref. and Suf. Reversals
- Sig. Pref. Reversals and Transpositions
- Sig. Pref. and Suf. Reversals and Transpositions

Sorting by Signed Prefix and Suffix Reversals (SbPS$\bar{\text{R}}$)

- Breakpoint: occur between two consecutive elements in π that should not be consecutive

$$(0 \quad 3 \;\textbf{.}\; -2 \;\textbf{.}\; 4 \quad 5 \;\textbf{.}\; -1 \quad 6)$$

- Strip: maximal sequence of elements of π without breakpoints

- Lower bound:

$$d_{p\bar{r}}(\pi) \geq b(\pi)$$

SbPS$\bar{\text{R}}$

Main idea of algorithm 2−PS$\bar{\text{R}}$:

- While π is not sorted:
  - If it is possible to remove 1 breakpoint with one prefix/suffix reversal, then do it;
    - $(-k \;\ldots\ldots\; k+1 \;\ldots\ldots)$
    - $(\ldots\ldots\; k-1 \;\ldots\ldots\; -k)$
  - Otherwise, if it is possible to remove 1 breakpoint with two prefix/suffix reversals, then do it;
    - $(\ldots\ldots\; k \;\ldots\ldots\; -(k+1) \;\ldots\ldots)$
    - $(\ldots\ldots\; k \;\ldots\ldots\; k+1 \;\ldots\ldots)$
    - $(\ldots\ldots\; k \;\ldots\ldots\; -(k-1) \;\ldots\ldots)$
  - Otherwise the permutation has a special form.

SbPS$\bar{R}$

Lemma

If π is an signed permutation for which is not possible to remove one breakpoint with one or two prefix/suffix reversals, then π is of one of the three following forms:

1. $\bar{\eta_n} = (-n \;\; -(n-1) \;\; \dots \;\; -1)$;
2. $\lambda^{sa}_{b+1} = (\underbrace{p_b+1 \; p_b+2 \; \dots \; n}_{\ell_{b+1}} \underbrace{p_{b-1}+1 \; p_{b-1}+2 \; \dots \; p_b}_{\ell_b} \dots\dots \underbrace{1 \; 2 \; \dots \; p_1}_{\ell_1})$;
3. $\lambda^{sd}_{b+1} = (\underbrace{-p_1 \; -(p_1-1) \; \dots \; -1}_{\ell_1} \underbrace{-p_2 \; -(p_2-1) \dots \; -(p_1+1)}_{\ell_2} \dots\dots \underbrace{-n \; -(n-1) \; \dots \; -(p_b+1)}_{\ell_{b+1}})$,

where $b = b(\pi) \geq 1$ and $\ell_i \geq 1$ for all $1 \leq i \leq b(\pi) + 1$.

SbPS$\bar{R}$

Lemma

Let π be one of the signed permutations described in the previous lemma. If $\pi = \bar{\eta_n}$, then one signed prefix reversal $\bar{\rho}_p(n)$ sorts it. Otherwise, at most $b(\pi) + 2$ prefix and suffix reversals sort it.

Lemma

For any signed permutation π, 2-PS$\bar{R}(\pi) \leq 2b(\pi) + 1$.

Theorem

$\mathrm{S}\mathrm{B}\mathrm{PS}\bar{\mathrm{R}}$ *is 2-approximable.*

Length-Weighted Approach – Polynomial Cost Function

- The cost to sort a permutation is the sum of the cost of the rearrangements that were used to do it
- $f(\ell) = \ell^{\alpha}$ and $\alpha > 0$

Known Results

Rearrangements	Parameter	Best Approx. Factor
Reversals [26]	$0 < \alpha < 1$	-
	$\alpha = 1$	$O(\lg n)$
	$1 < \alpha < 2$	$O(\lg n)$
	$2 \leq \alpha < 3$	2
	$\alpha \geq 3$	1
Sig. Reversals [27]	$0 < \alpha < 1$	-
	$\alpha = 1$	$O(\lg n)$
	$1 < \alpha < 2$	$O(\lg n)$
	$\alpha \geq 2$	$O(1)$
Reversals of length at most k [28]	$k = \Omega(n)$	$O(\log n)$
	$k = o(n)$	$2 \lg^2 n + \lg n$

Known Results

Rearrangements	Parameter	Diameter: Lower Bound	Diameter: Upper Bound
Reversals [26]	$0 < \alpha < 1$	$\Omega(n)$	$O(n \lg n)$
	$\alpha = 1$	$\Omega(n \lg n)$	$O(n \lg^2 n)$
	$1 < \alpha < 2$	$\theta(n^\alpha)$	$\theta(n^\alpha)$
	$\alpha \geq 2$	$\theta(n^2)$	$\theta(n^2)$
Sig. Reversals [27]	$0 < \alpha < 1$	$\Omega(n)$	$O(n \lg n)$
	$\alpha = 1$	$\Omega(n \lg n)$	$O(n \lg^2 n)$
	$1 < \alpha < 2$	$\theta(n^\alpha)$	$\theta(n^\alpha)$
	$\alpha \geq 2$	$\theta(n^2)$	$\theta(n^2)$
Reversals of length at most k [28]	$0 < \alpha < 1$	$\Omega(n + n^2 k^{\alpha-2})$	$O(n \log n + n^2 k^{\alpha-2})$
	$\alpha = 1$	$\Omega(n \log n + \frac{n^2}{k})$	$O(n \log n \log k + \frac{n^2}{k})$
	$1 < \alpha < 2$	$\Omega(n^2 k^{\alpha-2})$	$\Omega(n^2 k^{\alpha-2})$
	$\alpha \geq 2$	$\theta(n^2)$	$\theta(n^2)$

Obtained Results

Rearrangements	α	Approximation Factor
Pref. Reversals, Pref. Transpositions, and Pref. Reversals and Transpositions	$0 < \alpha < 1$	$O(n^\alpha)$
	$\alpha = 1$	$O(\lg^2 n)$
	$1 < \alpha < 2$	$O(1)$
	$2 \leq \alpha < 3$	10
	$\alpha \geq 3$	5
	$\alpha \to \infty$	$\frac{2^\alpha}{2^\alpha - 2} + \frac{2^{2\alpha+1}}{(2^\alpha - 2)^2}$
Pref. and Suf. Reversals, Pref. and Suf. Transpositions, and Pref. and Suf. Reversals and Transpositions	$\alpha = 1$	$O(\lg^2 n)$
Reversals	$0 < \alpha < 1$	$O(n^\alpha)$
Transpositions, Reversals and Transpositions	$0 < \alpha < 1$	$O(n^\alpha)$
	$\alpha = 1$	$O(\lg^2 n)$
	$1 < \alpha < 2$	$O(\lg n)$
	$2 \leq \alpha < 3$	2
	$\alpha \geq 3$	1

Obtained Results

Rearrangements	α	Approximation Factor
Sig. Pref. Reversals and Sig. Pref. Reversals and Transpositions	$0 < \alpha < 1$	$O(n^{\alpha})$
	$\alpha = 1$	$O(\lg^2 n)$
	$\alpha > 1$	$O(1)$
Sig. Pref. and Suf. Reversals and Sig. Pref. and Suf. Reversals and Transpositions	$\alpha = 1$	$O(\lg^2 n)$

Obtained Results

Rearrangements	α	Diameter	
		Lower Bound	**Upper Bound**
Pref. Reversals, Pref. Transpositions, and Pref. Reversals and Transpositions	$0 < \alpha < 1$	$\Omega(n)$	$O(n \lg n)$
	$\alpha = 1$	$\Omega(n \lg n)$	$O(n \lg^2 n)$
	$\alpha > 1$	$\Theta(n^\alpha)$	
Transpositions, and Reversals and Transpositions	$0 < \alpha < 1$	$\Omega(n)$	$O(n \lg n)$
	$\alpha = 1$	$\Omega(n \lg n)$	$O(n \lg^2 n)$
	$1 < \alpha < 2$	$\Theta(n^\alpha)$	
	$\alpha \geq 2$	$\Theta(n^2)$	
Sig. Pref. Reversals and Sig. Pref. Reversals and Transpositions	$0 < \alpha < 1$	$\Omega(n)$	$O(n \lg n)$
	$\alpha = 1$	$\Omega(n \lg n)$	$O(n \lg^2 n)$
	$\alpha > 1$	$\Theta(n^\alpha)$	

Obtained Results

- Pref. Reversals
- Pref. Transpositions
- Pref. Reversals and Transpositions
- Pref. and Suf. Reversals
- Pref. and Suf. Transpositions
- Pref. and Suf. Reversals and Transpositions
- Reversals
- Transpositions
- Reversals and Transpositions
- Sig. Pref. Reversals
- Sig. Pref. Reversals and Transpositions
- Sig. Pref. and Suf. Reversals
- Sig. Pref. and Suf. Reversals and Transpositions

Sorting by Length-Weighted Prefix Reversals (SbWPR)

Main idea of algorithm `WPR`: divide-and-conquer strategy

- If no base case applies:
  - Let m be the median of the interval // let $\pi = (4\ 7\ 1\ 8\ 3\ 9\ 5\ 6\ 2)$, $m = 5$
  - Partition the interval // $(9\ 6\ 8\ 7\ 5 \mid 1\ 4\ 3\ 2)$
  - Recursively sort first part // $(9\ 8\ 7\ 6\ 5 \mid 1\ 4\ 3\ 2)$
  - Reverse whole interval // $(2\ 3\ 4\ 1 \mid 5\ 6\ 7\ 8\ 9)$
  - Recursively sort second part // $(1\ 2\ 3\ 4 \mid 5\ 6\ 7\ 8\ 9)$

SbWPR

Lower bound:

$$d^1_{pr}(\pi) \geq n$$

Lemma

For any unsigned permutation π with n valid elements and $\alpha = 1$, `partitionWPR`(π, n) is in $O(n \lg n)$ and `WPR`(π, n) is in $O(n \lg^2 n)$.

Theorem

For $\alpha = 1$, $\textsc{SbWPR}$ is $O(\lg^2 n)$-approximable.

Length-Weighted Approach – Exponential Cost Function

- The cost to sort a permutation is the sum of the cost of the rearrangements that were used to do it
- $f(\ell) = 2^{\ell}$

Obtained Results

Rearrangements	Approximation Factor	Diameter
Pref. Reversals	3	$\Theta(2^n)$
Pref. Transpositions and Pref. Reversals and Transpositions	2	$\Theta(2^n)$
Reversals	1.5	$\Theta(n^2)$
Transpositions and Reversals and Transpositions	1.125	$\Theta(n^2)$
Sig. Pref. Reversals and Sig. Pref. Reversals and Transpositions	$4 + n/2^{n-1}$	$\Theta(2^n)$

Other Obtained Results

- Binary string: 001011010110
- Sorted binary string: 000000111111

Other Obtained Results

Sorting Binary Strings by Length-Weighted Operations ($f(\ell) = \ell^\alpha$)

Rearrangements	α	Approximation Factor	Diameter
Pref. Reversals, Pref. Transpositions, and Pref. Reversals and Transpositions	$0 < \alpha < 1$	$O(\lg n)$	$\Theta(n)$
	$\alpha = 1$	$O(\lg n)$	$\Theta(n \lg n)$
	$1 < \alpha < 2$	$O(1)$	$\Theta(n^\alpha)$
	$2 \leq \alpha < 3$	4	
	$\alpha \geq 3$	3	
	$\alpha \to \infty$	$(2^{\alpha+1})(2^\alpha - 2)$	
Pref. and Suf. Reversals, Pref. and Suf. Transpositions, and Pref. and Suf. Reversals and Transpositions	$\alpha = 1$	$O(\lg n)$	-
Transpositions, Reversals and Transpositions	$0 < \alpha < 1$	$O(1)$	$\Theta(n)$
	$\alpha = 1$	$O(\lg n)$	$\Theta(n \lg n)$
	$1 < \alpha < 2$	$O(1)$	$\Theta(n^\alpha)$
	$\alpha \geq 2$	1	$\Theta(n^2)$
Sig. Pref. Reversals and Sig. Pref. Reversals and Transpositions	$0 < \alpha < 1$	$O(\lg n)$	$\Theta(n)$
	$\alpha = 1$	$O(\lg n)$	$\Theta(n \lg n)$
	$\alpha > 1$	$O(1)$	$\Theta(n^\alpha)$
Sig. Pref. and Suf. Reversals and Sig Pref. and Suf. Reversals and Transpositions	$\alpha = 1$	$O(\lg n)$	-

Other Obtained Results

Sorting Binary Strings by Length-Weighted Operations ($f(\ell) = 2^\ell$)

Rearrangements	**Approximation Factor**	**Diameter**
Pref. Reversals	3	$\Theta(2^n)$
Pref. Transpositions and Pref. Reversals and Transpositions	2	$\Theta(2^n)$
Reversals	1.125	$\Theta(n^2)$
Transpositions and Reversals and Transpositions	1.125	$\Theta(n^2)$
Sig. Pref. Reversals and Sig. Pref. Reversals and Transpositions	$4 + n/2^{n-1}$	$\Theta(2^n)$

Published Contributions

2014	"Sorting Permutations by Prefix and Suffix Versions of Reversals and Transpositions" [29] Latin American Theoretical Informatics Symposium
2014	"On Sorting of Signed Permutations by Prefix and Suffix Reversals and Transpositions" [30] International Conference on Algorithms for Computational Biology
2014	"On the Diameter of Rearrangement Problems" [31] International Conference on Algorithms for Computational Biology
2014	"A General Heuristic for Genome Rearrangement Problems" [32] Journal of Bioinformatics and Computational Biology
2015	"Approximation Algorithms for Sorting by Length-Weighted Prefix and Suffix Operations" [33] Theoretical Computer Science

Contributions Under Review

2015	"Sorting Permutations and Bit Sequences by Length-Weighted Rearrangements" Theoretical Computer Science
2016	"Sorting Permutations by Prefix and Suffix Rearrangements" (minor) Journal of Bioinformatics and Computational Biology

Thank you!

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