Multiscale Methods in Visual Computing

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Multiscale Methods in Visual Computing

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Multiscale method to solve a problem with many variables:

1. Scale down the number of variables by a constant factor;
2. Solve (recursively) the reduced problem;
3. Expand the solution to the original scale;
4. Adjust the solution iteratively.

Typically reduces the asymptotic exponent and the actual time.

Example: the one-dimensional heat equilibrium problem

A metal bar is heated or cooled along its length.

Constant-temperture heat sinks at the ends.

Constant heat power added $(+)$ or removed $(-)$ at position x is $P(x)$.

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What is the limiting temperature distribution $T(x)$?

Poisson problem in one dimension 4

Poisson equation:

$$\partial_{xx} T(x) = -\kappa P(x)$$

Exact solution known: double integral of $-\kappa P$.

Discretized version of problem

$T_0, T_1, \ldots, T_n$: temperatures at equally-spaced points.

$P_1, P_2, \ldots, P_{n-1}$: input-output heat power at those points.

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Poisson system:

$$\begin{cases} T_0 & = \; 0 \\ T_i - \dfrac{1}{2}(T_{i-1} + T_{i+1}) & = \; \dfrac{\kappa}{2} P_i \\ T_n & = \; 0 \end{cases}$$

Example problem

Input-output power:

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True solution:

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Gauss-Seidel with random guess

Initial guess and error:

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Gauss-Seidel with random guess

After 1 iteration

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Gauss-Seidel with random guess

After 2 iterations

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Gauss-Seidel with random guess

After 3 iterations

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Gauss-Seidel with random guess

After 4 iterations

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Gauss-Seidel with random guess

After 5 iterations

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Gauss-Seidel with random guess

After 6 iterations

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Gauss-Seidel with random guess

After 7 iterations

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Gauss-Seidel with random guess

After 8 iterations

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Gauss-Seidel with random guess

After 9 iterations

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Gauss-Seidel with random guess

After 10 iterations

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Gauss-Seidel with random guess

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Gauss-Seidel with random guess

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Gauss-Seidel with random guess

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Gauss-Seidel with random guess

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Gauss-Seidel with random guess

After 100 iterations

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Gauss-Seidel with random guess

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Gauss-Seidel with random guess

After 400 iterations

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Gauss-Seidel with random guess

After 800 iterations

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Gauss-Seidel with random guess

After 1600 iterations

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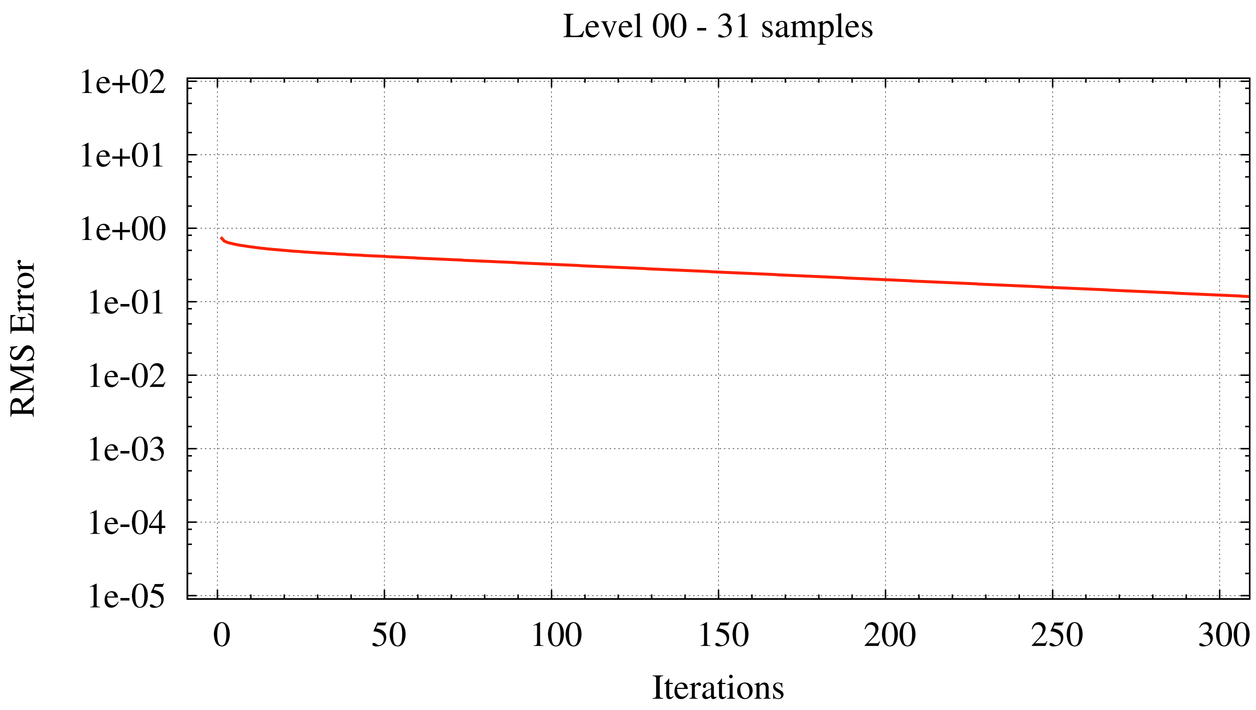
Very low convergence rate!



Convergence rate depends on smoothness of error

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Guess with low-frequency error

Initial guess and error:

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Guess with low-frequency error

After 1 iteration

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

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Guess with low-frequency error

After 100 iterations

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Guess with low-frequency error

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Guess with low-frequency error

After 400 iterations

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Guess with low-frequency error

After 800 iterations

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Guess with low-frequency error

After 1600 iterations

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Guess with medium-frequency error

Initial guess and error:

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Guess with medium-frequency error

After 1 iteration

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Guess with medium-frequency error

After 2 iterations

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Guess with medium-frequency error

After 3 iterations

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Guess with medium-frequency error

After 4 iterations

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

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Guess with medium-frequency error

After 100 iterations

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Guess with medium-frequency error

After 200 iterations

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Guess with high-frequency error

Initial guess and error:

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Guess with high-frequency error

After 1 iteration

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Guess with high-frequency error

After 2 iterations

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Guess with high-frequency error

After 3 iterations

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Guess with high-frequency error

After 4 iterations

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Guess with high-frequency error

After 5 iterations

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Guess with high-frequency error

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Guess with high-frequency error

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Guess with high-frequency error

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Guess with high-frequency error

After 9 iterations

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Guess with high-frequency error

After 10 iterations

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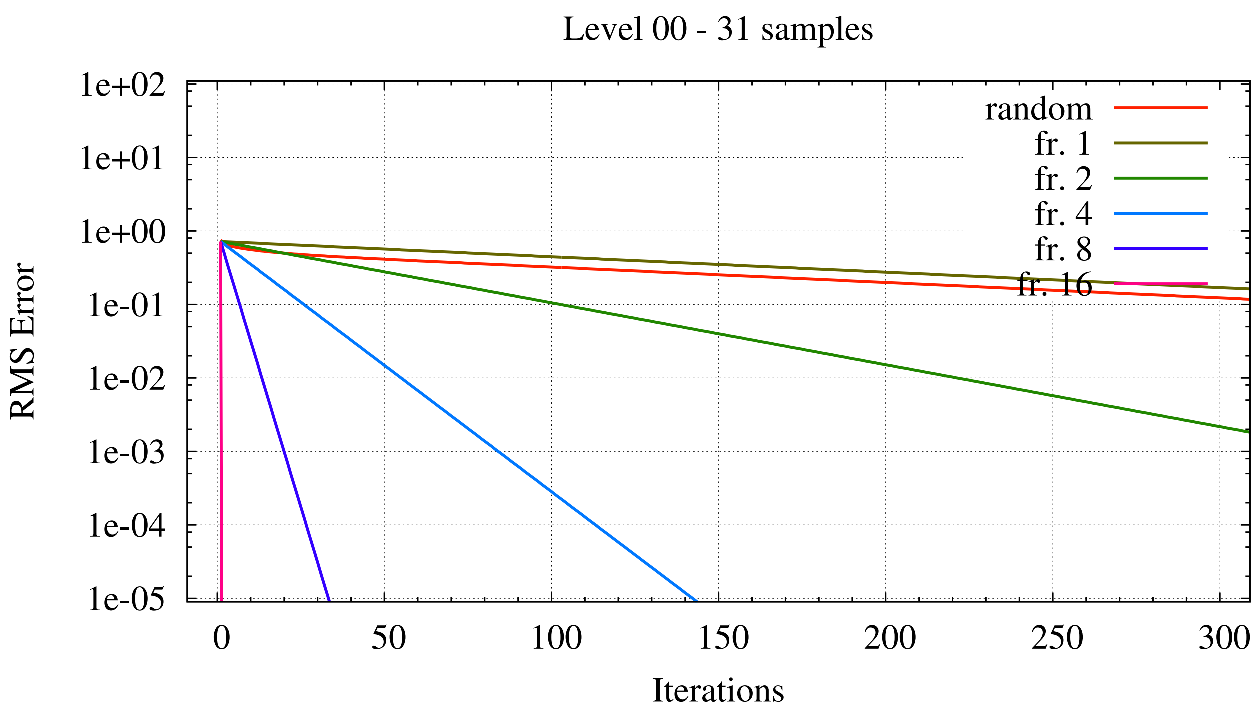
Convergence rate is inversely proportional to $\lambda^2 = (n/f)^2$



How do we get a guess with only high-frequency error?

Reduce the number of samples in half

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Solve the smaller version

Initial guess and error:

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Solve the smaller version

After 1 iteration

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Solve the smaller version

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Solve the smaller version

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Solve the smaller version

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Solve the smaller version

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Solve the smaller version

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Solve the smaller version

After 100 iterations

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Solve the smaller version

After 200 iterations

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Now expand the solution

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Iterate from expanded solution

Initial guess and error:

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Iterate from expanded solution

After 1 iteration

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Iterate from expanded solution

After 2 iterations

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Iterate from expanded solution

After 3 iterations

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Iterate from expanded solution

After 4 iterations

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Iterate from expanded solution

After 5 iterations

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Iterate from expanded solution

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Iterate from expanded solution

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Iterate from expanded solution

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Iterate from expanded solution

After 9 iterations

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Iterate from expanded solution

After 10 iterations

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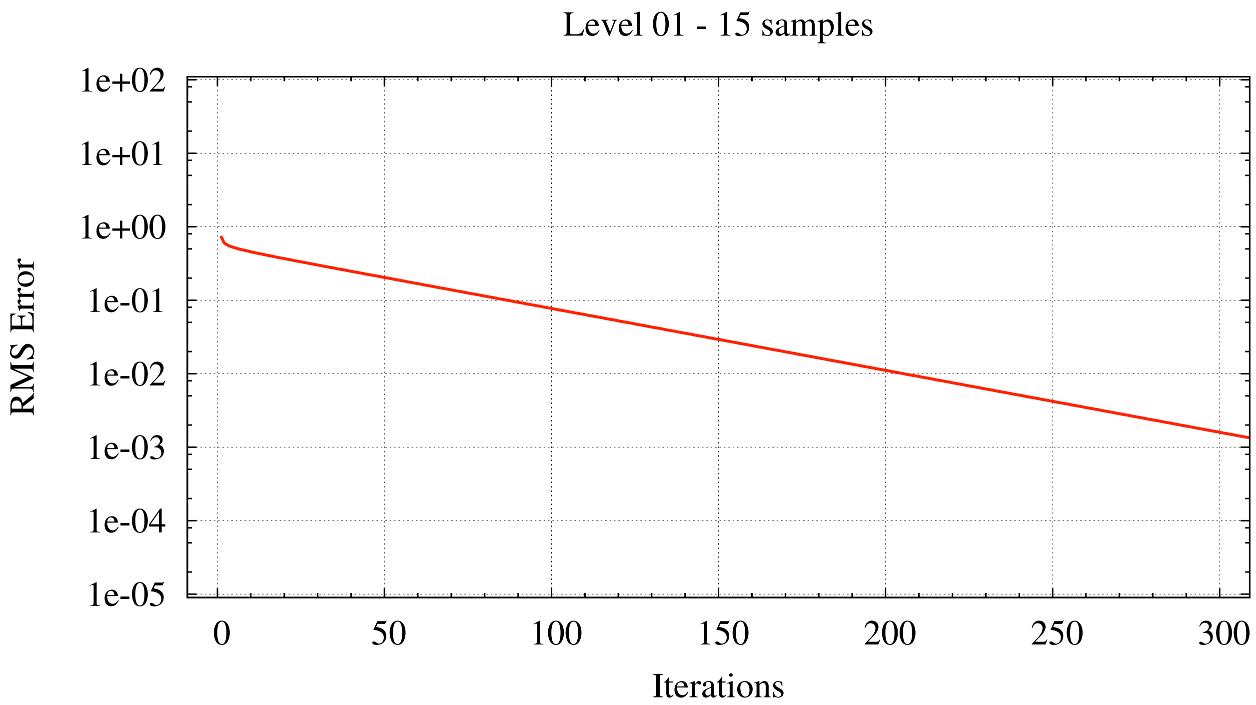
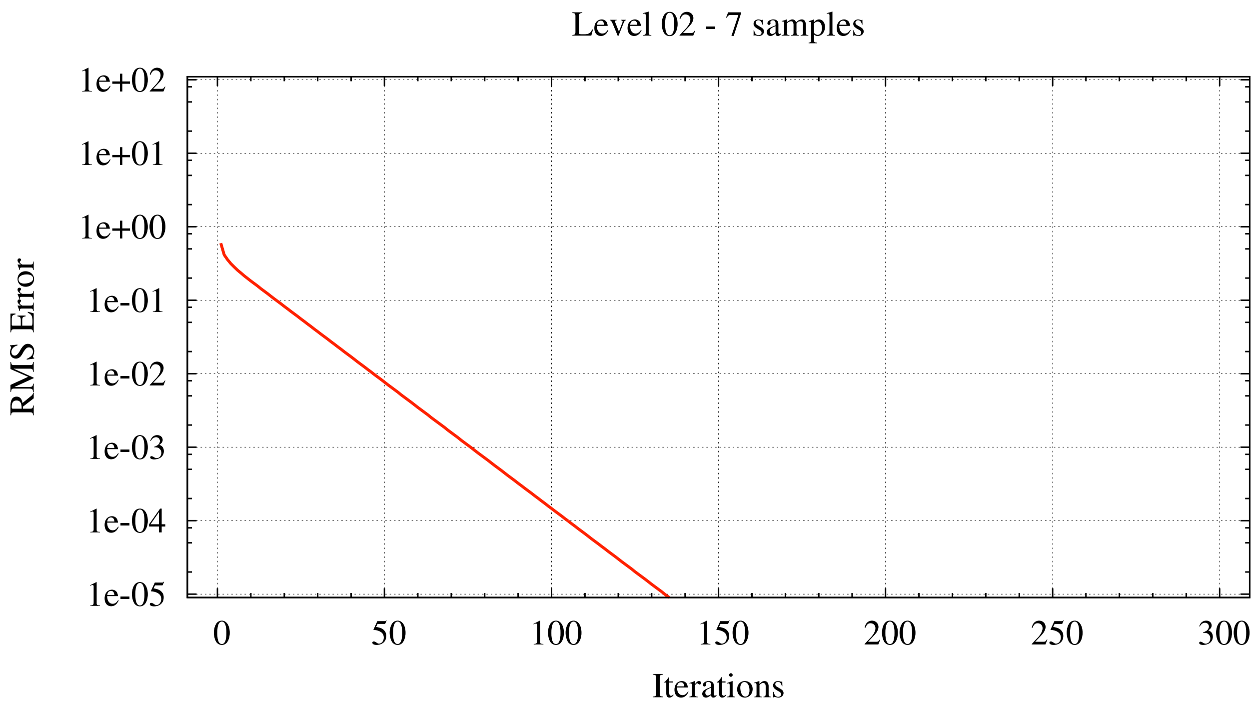
The 1/2-size problem converges 4 times faster



The 1/4-size problem converges 16 times faster



The mathematical Gradient Integration Problem (GIP):

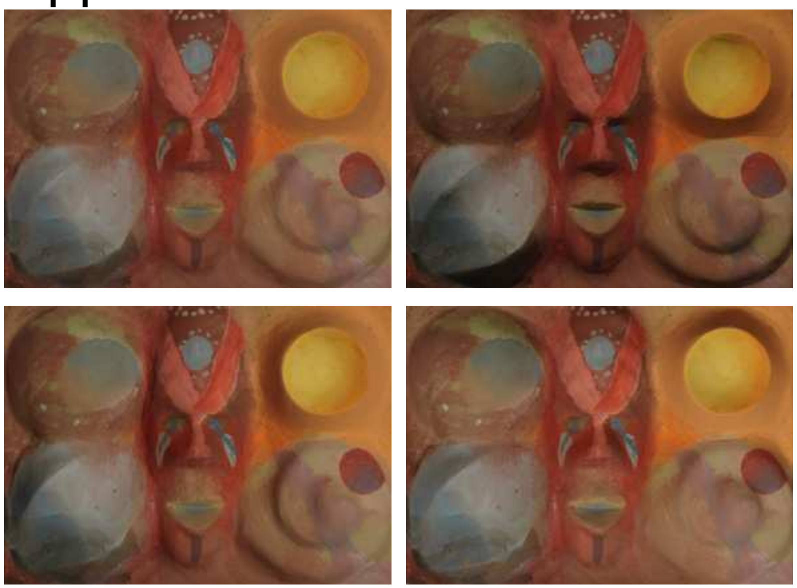
Input: A gradient map $(F, G) : \mathbb{R}^2 \mapsto \mathbb{R}^2$

Output: A height map $Z : \mathbb{R}^2 \mapsto \mathbb{R}$ such that

$$\partial_x Z = F \qquad \partial_y Z = G$$

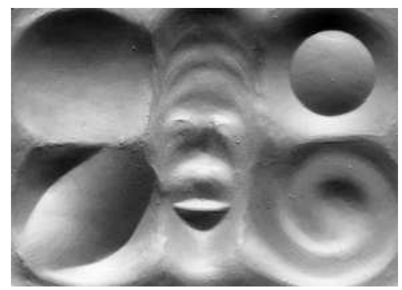
The Gradient Integration Problem 111

Application: Photometric Stereo



$F =$

$G =$



The *discrete* Gradient Integration Problem:

Input: A discrete, gradient map $f, g \in \mathbb{R}^{m \times n}$
and a weight map $w \in \mathbb{R}^{m \times n}$

Output: A discrete height map $z \in \mathbb{R}^{(m+1)\times(n+1)}$ such that

$$(\Delta_x z)[x,y] \approx f[x,y] \qquad (\Delta_y z)[x,y] \approx g[x,y]$$

with confidence proportional to $w[x,y]$. Data F, G is discretely sampled

Data contains noise, errors, and holes (where $w[x,y] = 0$)

Cliffs (step discontinuities) in the height Z

Direct Poisson

Convert the mathematical GIP to a Poisson differential equation

$$(\partial_{xx} Z + \partial_{yy} Z)(x, y) = \partial_x F(x, y) + \partial_y G(x, y)$$

Discrete version is a system of N linear equations
Can take weights $w[x, y]$ into account
System's matrix is sparse, $\Theta(N)$ entries
Solve the system by Gauss LU or similar
Robust but expensive: $\Theta(N^{1.15})$ space, $\Theta(N^{1.5})$ time

Iterative Poisson

Convert GIP to a system of N linear equations as in Direct Poisson
Solve the system by Gauss-Seidel iteration

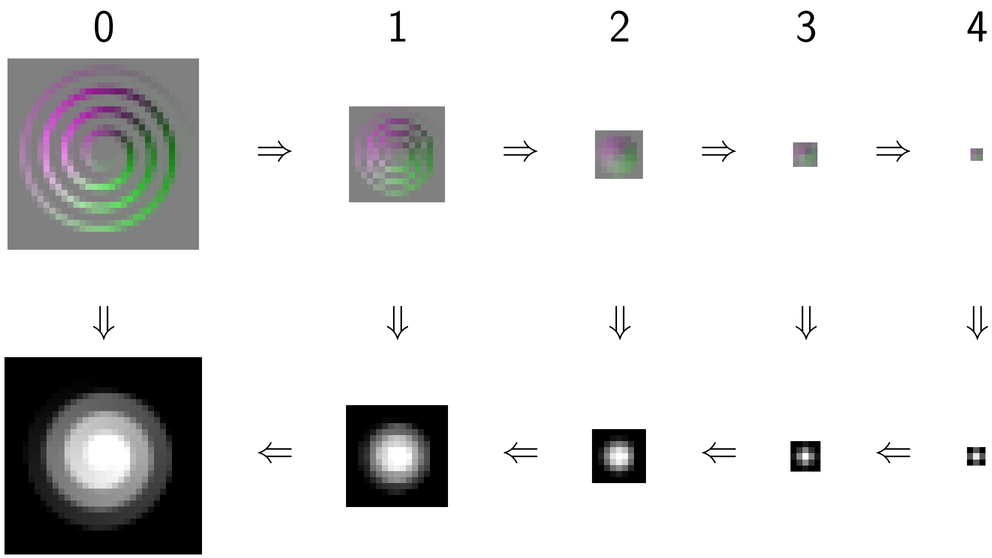
Only $\Theta(N)$ space, $\Theta(N)$ time per iteration
BUT requires $\Omega(N)$ iterations to converge

Total time is $\Omega(N^2)$

Multiscale Iterative Poisson
[Saracchini, Stolfi et al. 2009]

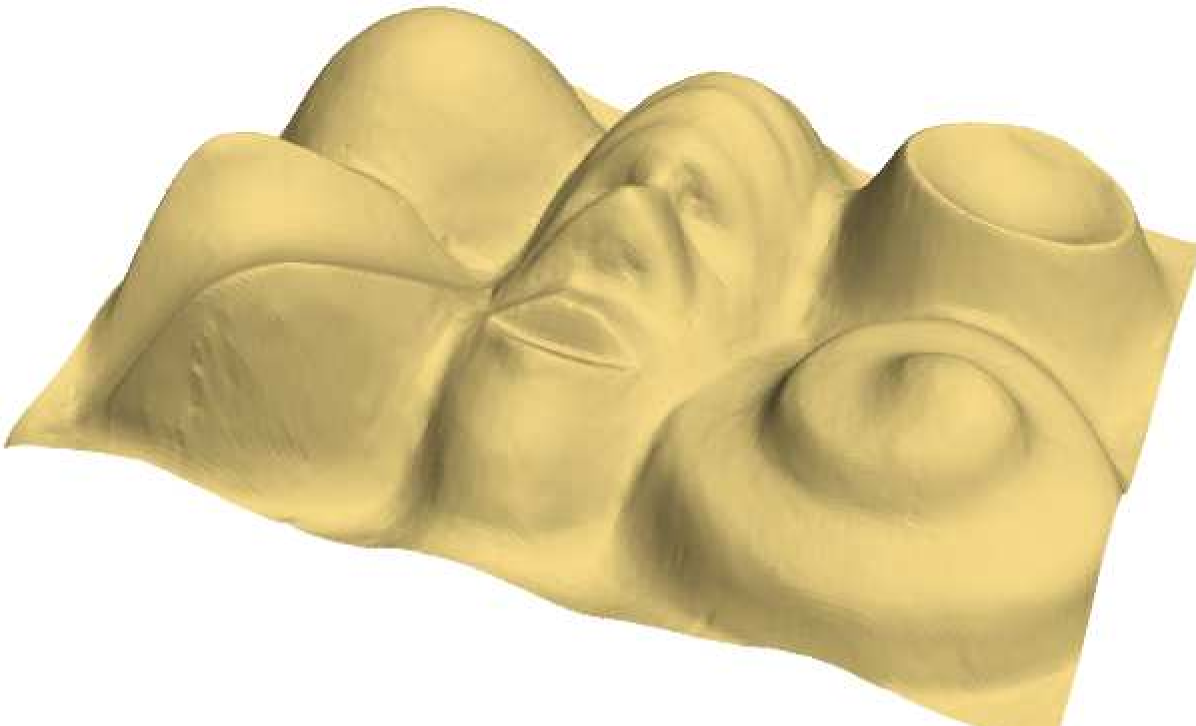
- Convert GIP to a system of N linear equations as in Direct Poisson
- Reduce the input maps f, g, w by 1/2 to f', g', w'
- Recursively integrate f', g', w' obtaining z'
- Expand the solution z' to a full size solution z
- Improve the solution z by Gauss-Seidel iteration.

Multiscale method

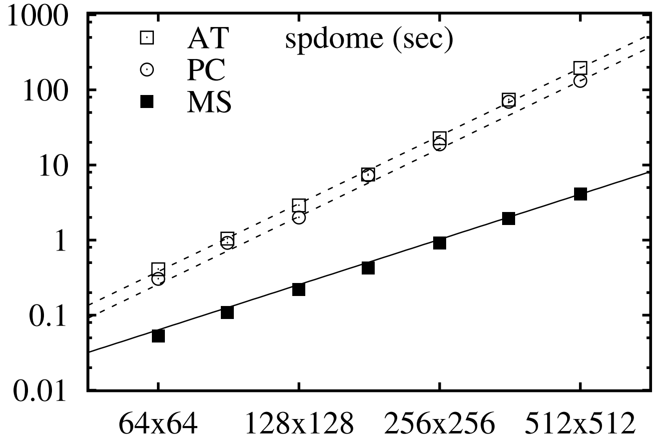


Only $\Theta(N)$ space, $\Theta(N)$ time per iteration
Converges in $O(1)$ iterations at each scale

Total time and space are $\Theta(N)$.



Time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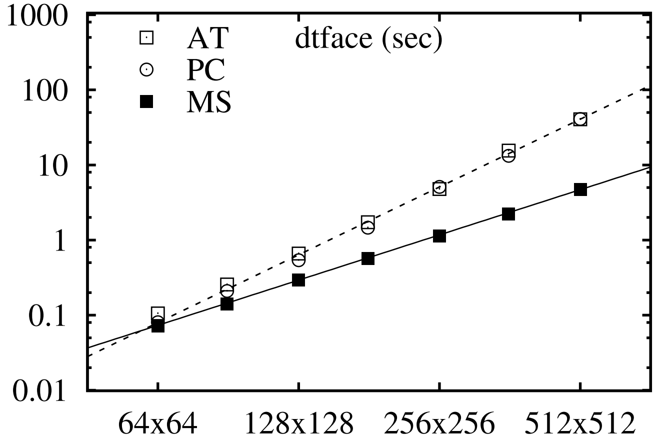
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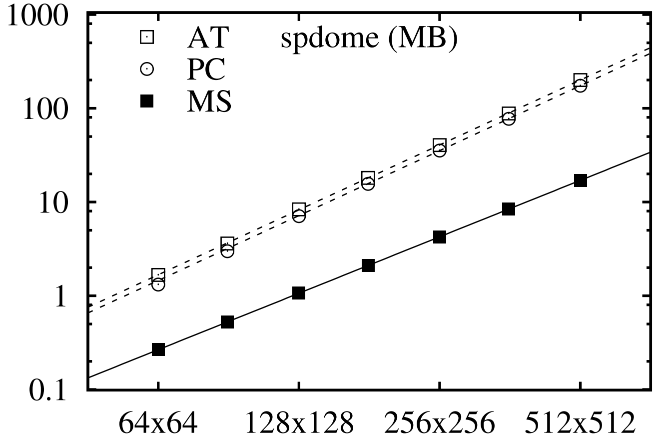
512x512

Space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10
1
0.1
AT
PC
MS
spdome (MB)
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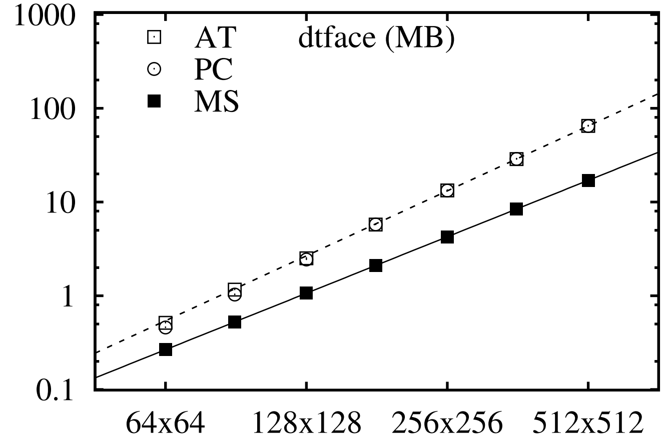
1000
100
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dtface (MB)
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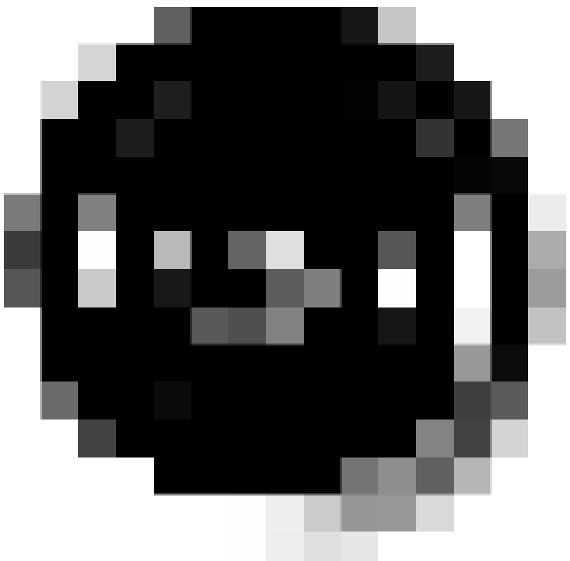
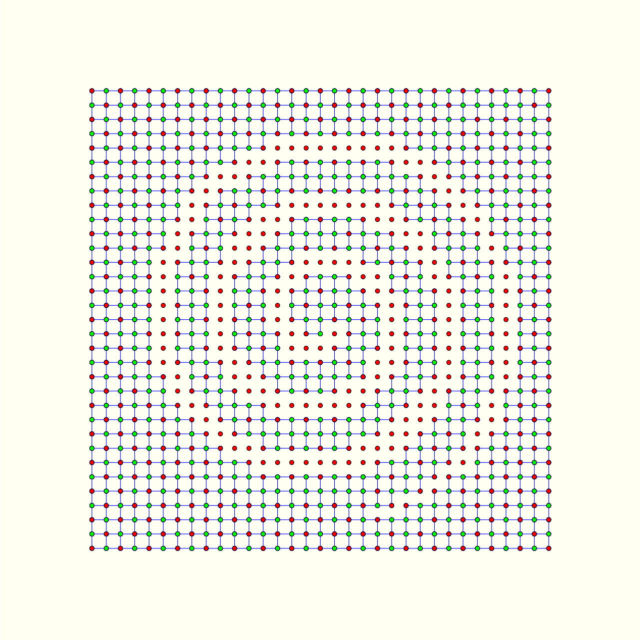
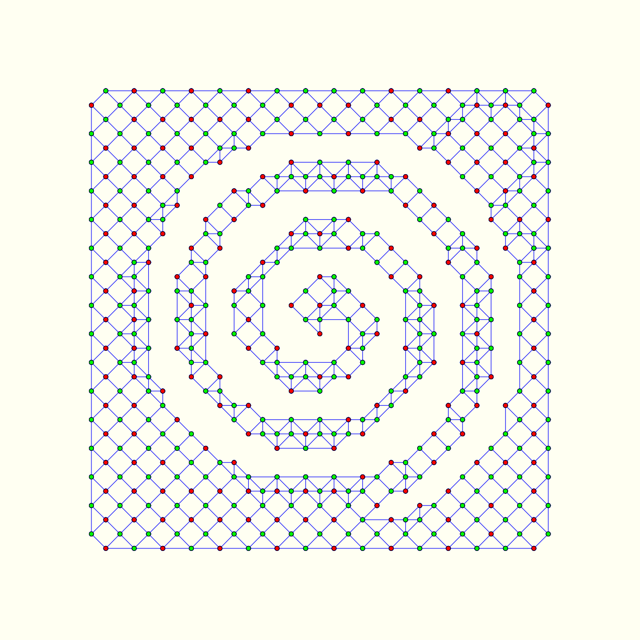
Main limitation:
loss of connectivity at coarser scales



Graph-based multiscale:





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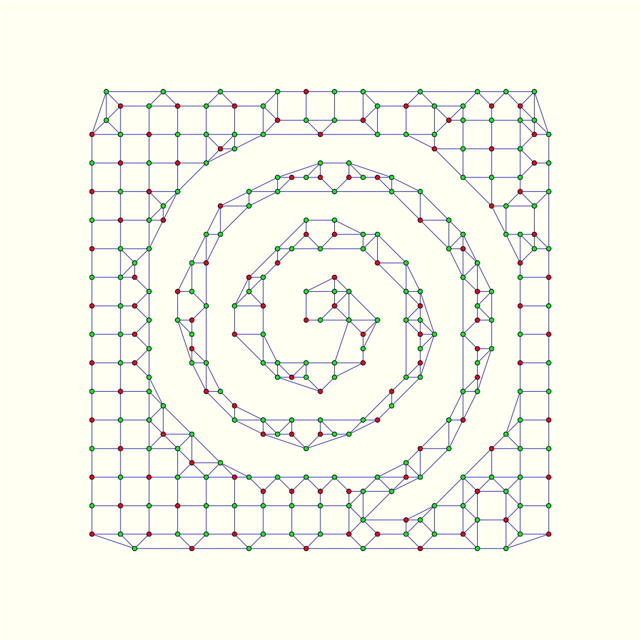

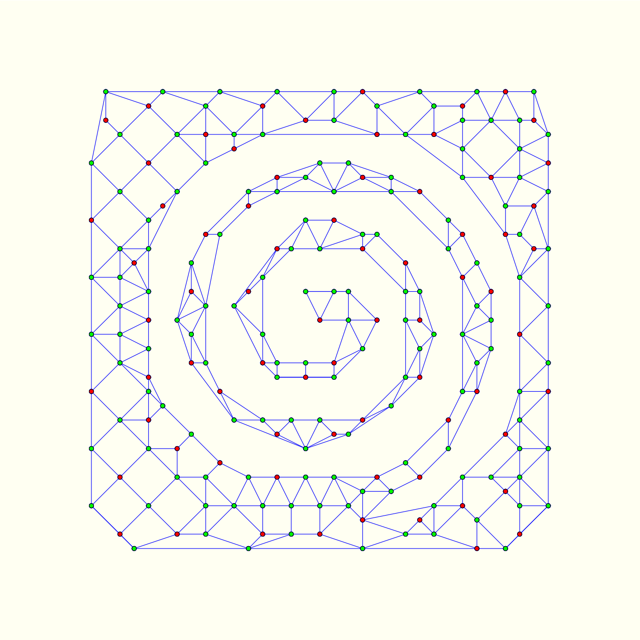

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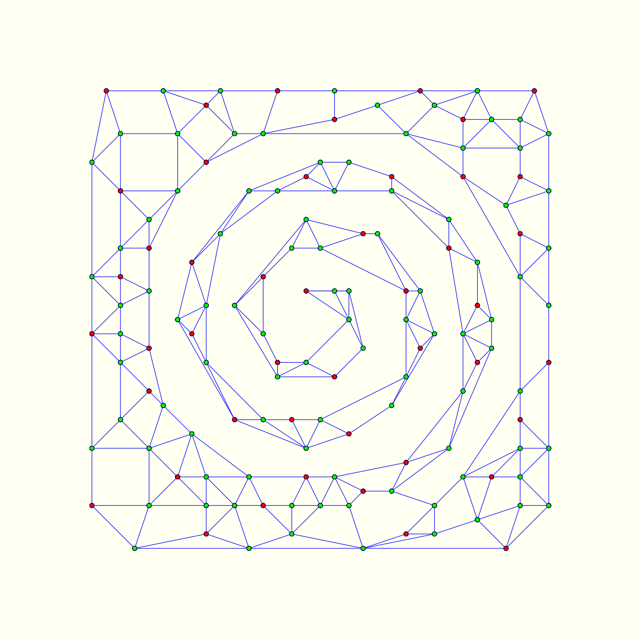

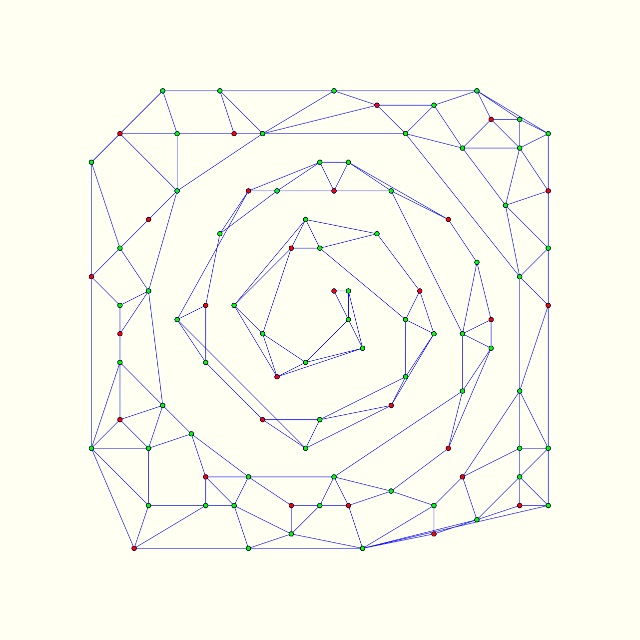

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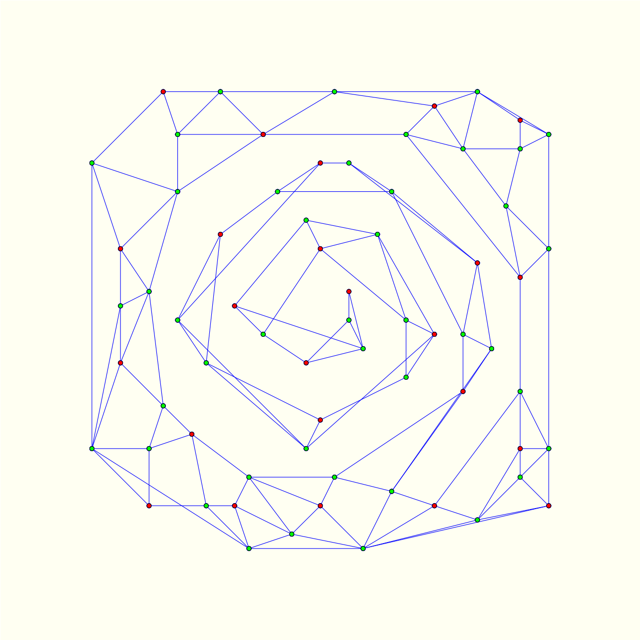

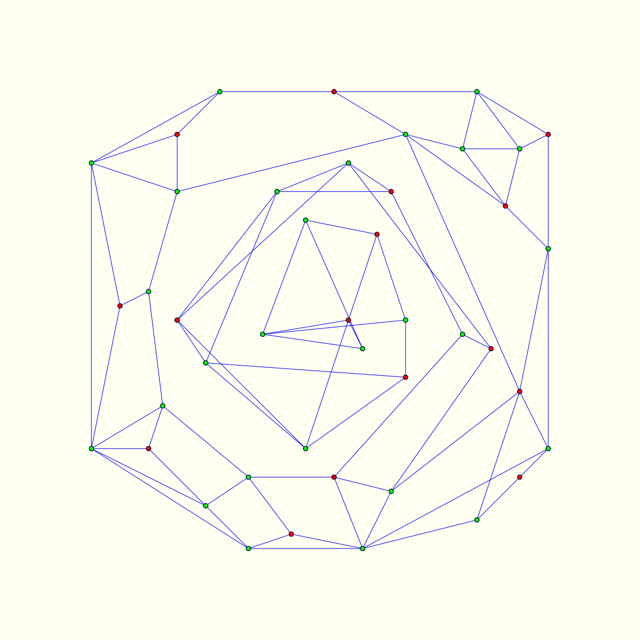

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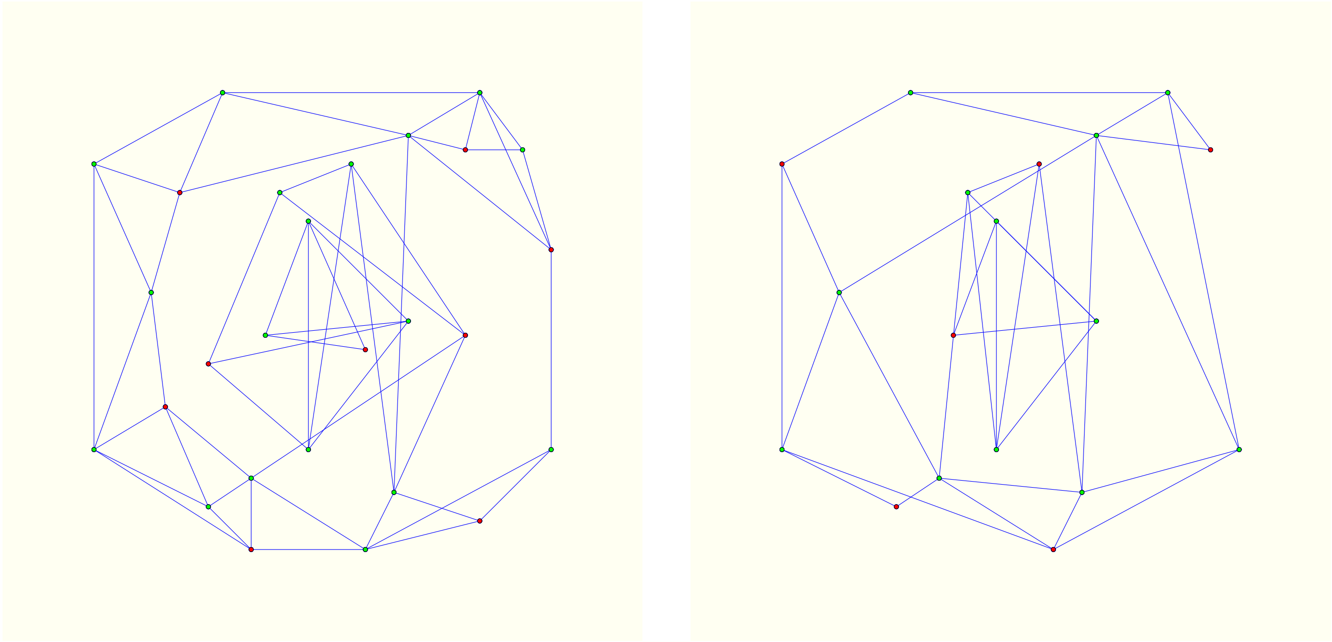

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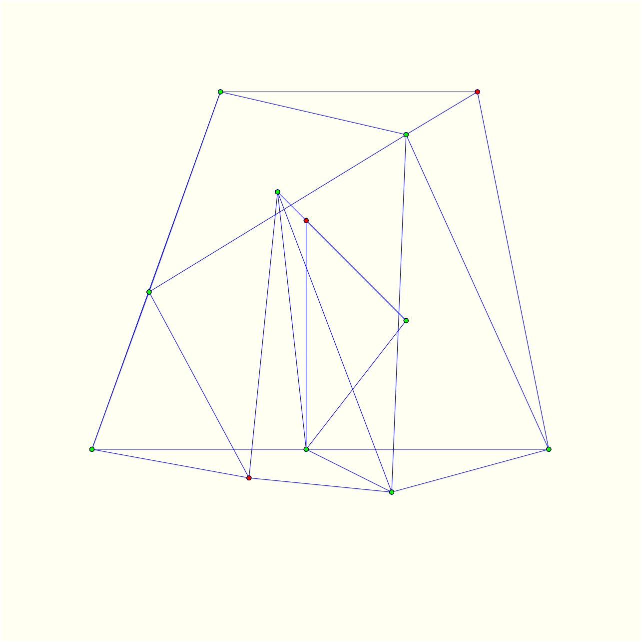

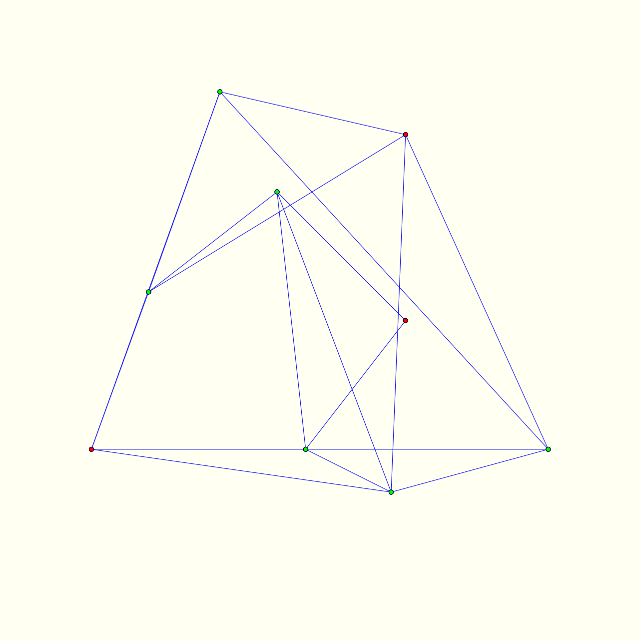

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