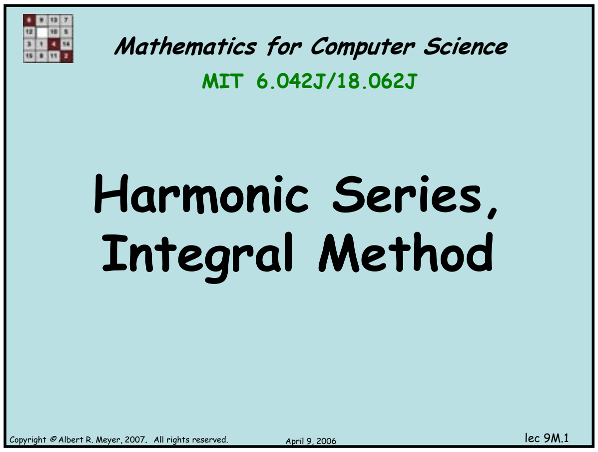
Mathematics for Computer Science
MIT 6.042J/18.062J
Harmonic Series,
Integral Method
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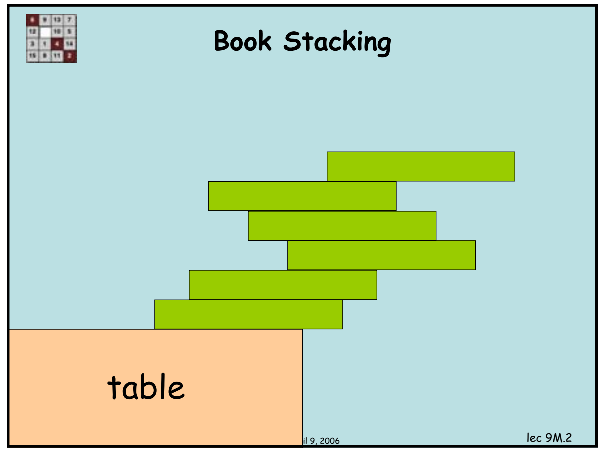
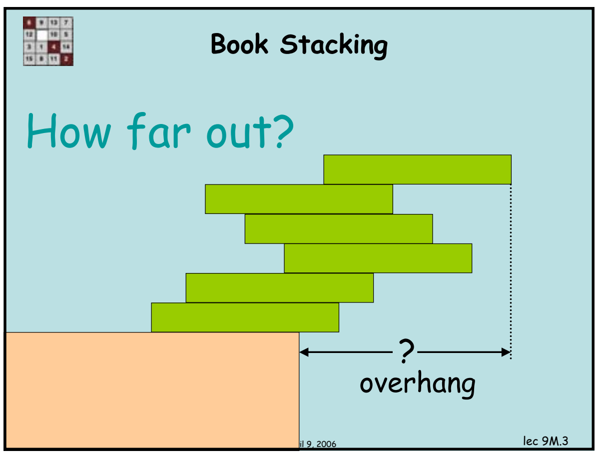
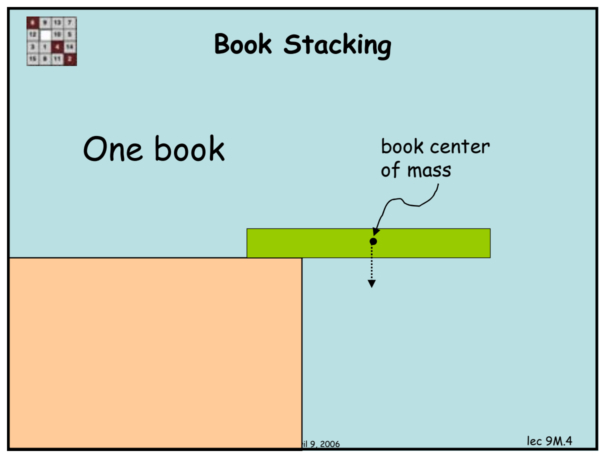
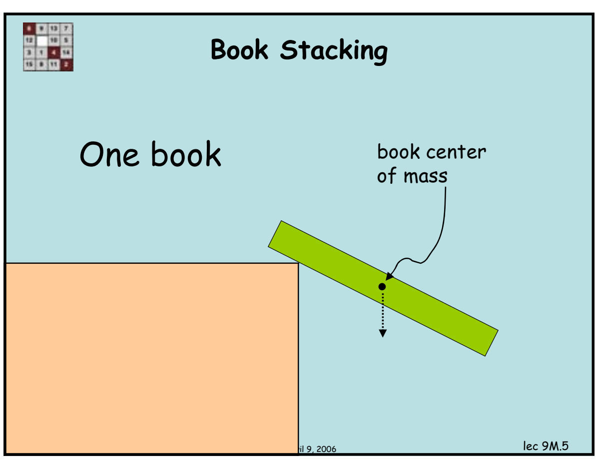
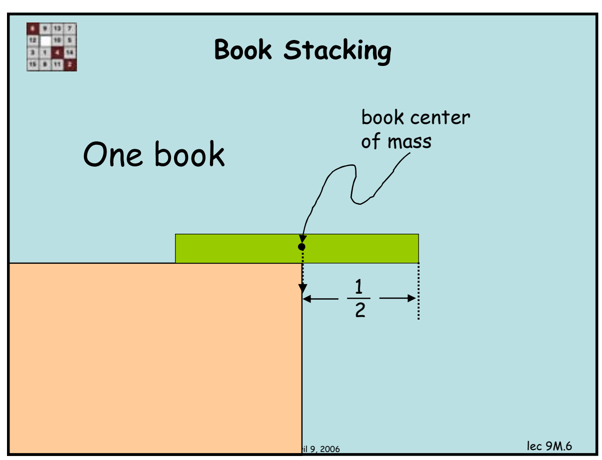
April 9, 2006
lec 9M.1

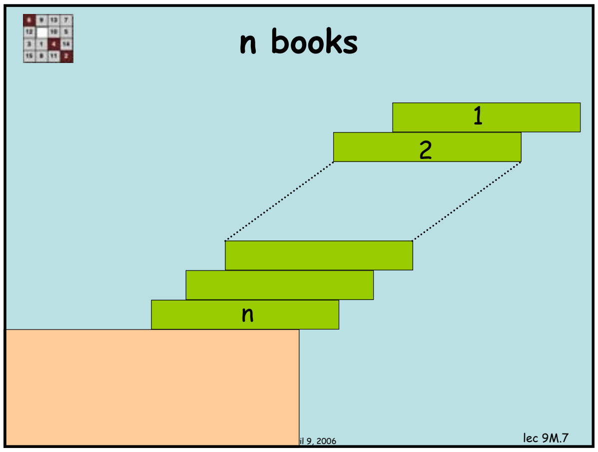

Book Stacking
table
lec 9M.2

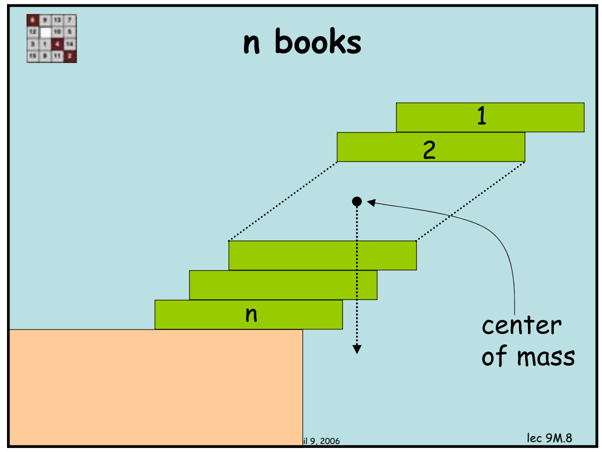

Book Stacking
How far out?
?
overhang
lec 9M.3

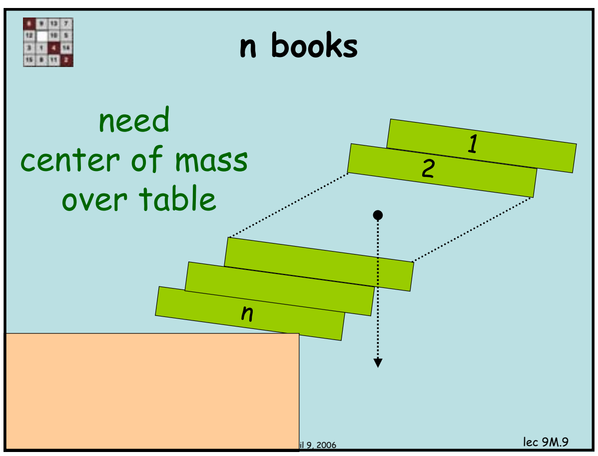

Book Stacking
One book
book center
of mass
lec 9M.4

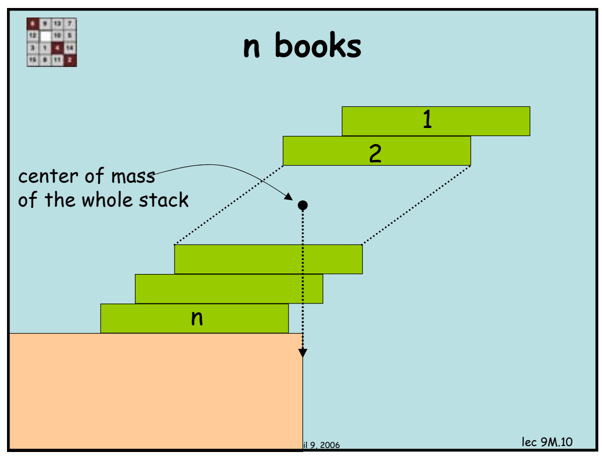

Book Stacking
One book
book center
of mass
lec 9M.5

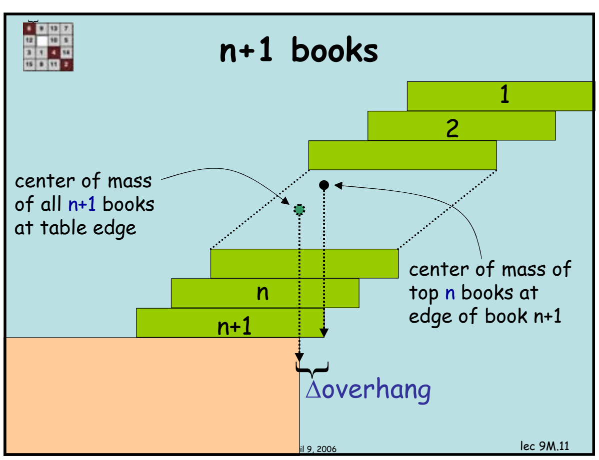

Book Stacking
One book
book center
of mass
1/2
lec 9M.6











Δoverhang ::=

horizontal distance from n-book to (n+1)-book centers of mass

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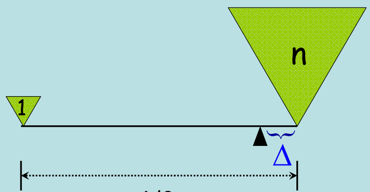
Δ overhang



$$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$$

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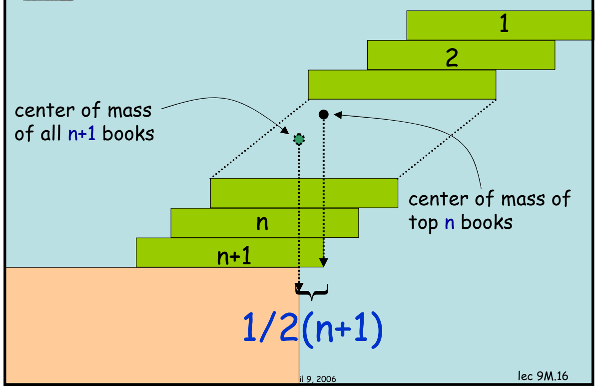
n+1 books



lec 9M.16

Book stacking summary

B_n ::= overhang of n books

$B_1 = 1/2$

$$B_{n+1} = B_n + \frac{1}{2(n+1)}$$

$$B_n = \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

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$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

n^{th} Harmonic number

$$B_n = H_n/2$$

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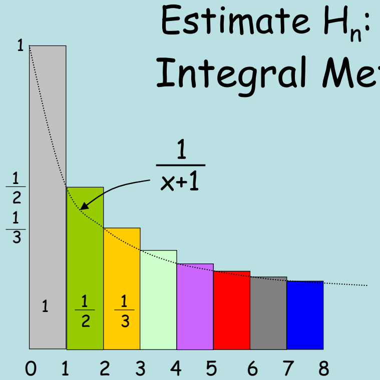
Estimate H_n: Integral Method



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$$\int_0^n \frac{1}{x+1}\,dx \le 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$

$$\int_1^{n+1} \frac{1}{x}\,dx \le H_n$$

$$\ln(n+1) \le H_n$$

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Book stacking

Now $H_n \to \infty$ as $n \to \infty$, so

overhang can be as big desired

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Book stacking

for overhang 3, need $B_n \geq 3$

$H_n \geq 6$

integral bound: $\ln(n+1) \geq 6$

so can do with $n \geq \lceil e^6 - 1 \rceil$ = 403 books

actually calculate H_n:

227 books are enough.

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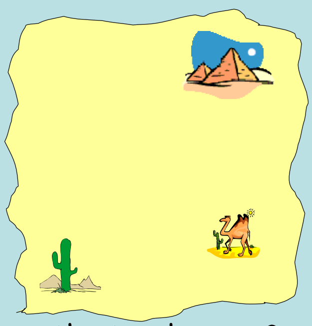
Crossing a Desert

Gas depot

truck



How big a desert can the truck cross?

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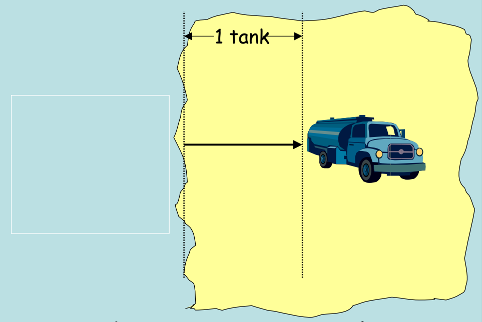
1 Tank of Gas



D_1::= max distance on 1 tank = 1

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D_n ::=

max distance into the desert using n tanks of gas from the depot

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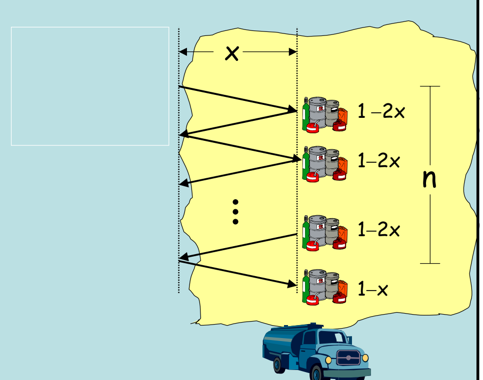
n+1 Tanks of Gas



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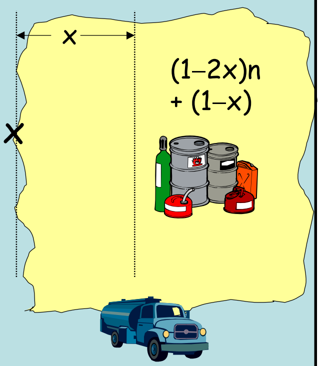
n+1 Tanks of Gas

So have:

grow depot at x to be n tanks; continue from x with n tank method.



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$$\text{Set } \overbrace{(1-2x)n + (1-x)}^{\text{depot at } x} = n.$$

Then using n tank strategy from position x, gives

$$D_{n+1} = D_n + x$$

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$$(1-2x)n + (1-x) = n$$

$$x = \frac{1}{2n+1}$$

$$\boxed{D_{n+1} = D_n + \frac{1}{2n+1}}$$

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$$D_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$$

$$\int_0^n \frac{1}{2(x+1)-1}\,dx \le D_n$$

$$\frac{\ln(2n+1)}{2} \le D_n$$

Can cross any desert!

 April 9, 2006 lec 9M.31

Team Problems

Problems 1—3

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