Mathematics for Computer Science

MIT 6.042J/18.062J

Counting

Counting in Gambling

A pair of Jacks is

what *fraction* of poker hands?

(probability of a pair of Jacks)

Counting in Algorithms

- How many comparisons are needed to *sort* n numbers?
- How many multiplications to compute d^n ?

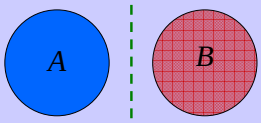
Counting in Games

- How many different configurations for a Rubik's cube?

- How many different chess positions after n moves?

- How many weighings to find the one counterfeit among 12 coins?

Sum Rule



If sets A and B are disjoint, then

$$|A \cup B| = |A| + |B|$$

The Sum Rule

- Class has 43 women, 54 men so total enrollment = 43 + 54 = 97
- 26 lower case letters, 26 upper case letters, and 10 digits, so total characters = 26+26+10 = 62

The Product Rule

If there are 4 boys and 3 girls, there are possible

$$4 \times 3 = 12$$

married couples.

Product Rule

If $|A| = m$ and $|T| = n$, then $|A \times T| = mn$.

A = {a, b, c, d}, T = {1, 2, 3}

$A \times T$ = { (a,1), (a,2), (a,3),
(b,1), (b,2), (b,3),
(c,1), (c,2), (c,3),
(d,1), (d,2), (d,3) }

Product Rule: Counting Strings

The number of length-4 strings from alphabet $B ::= \{0,1\}$

$$= |B \times B \times B \times B|$$

$$= 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

Product Rule: Counting Strings

The number of length-n *strings* from an *alphabet* of size m is

$$m^n.$$

Example: Counting Passwords

- between 6 & 8 characters long
- starts with a letter
- case sensitive
- other characters: digits or letters

Counting Passwords

$$L ::= \{a, b, \texttt{A}\ , z, A, B, \texttt{A}\ , Z\}$$

$$D ::= \{0, 1, \texttt{A}\ , 9\}$$

$P_n ::=$ length n passwords

Counting Passwords

$$L ::= \{a, b, \texttt{A}, z, A, B, \texttt{A}, Z\}$$

$$D ::= \{0, 1, \texttt{A}, 9\}$$

$$P_6 = L \times (L \cup D) \times (L \cup D) \times (L \cup D) \times (L \cup D) \times (L \cup D)$$

$$= L \times (L \cup D)^5$$

Counting Passwords

$$L ::= \{a, b, \texttt{A}, z, A, B, \texttt{A}, Z\}$$

$$D ::= \{0, 1, \texttt{A}, 9\}$$

$P_n ::=$ length n passwords

$$= L \times (L \cup D)^{n-1}$$

Counting Passwords

$$\left| L \times (L \cup D)^{n-1} \right| = |L| \cdot |L \cup D|^{n-1}$$

$$= |L| \cdot (|L| + |D|)^{n-1}$$

$$= 52 \cdot 62^{n-1}$$

Counting Passwords

The set of Passwords:

$$P = P_6 \cup P_7 \cup P_8$$

Counting Passwords

$$|P| = |P_6| + |P_7| + |P_8|$$

$$= 52 \cdot 62^5 + 52 \cdot 62^6 + 52 \cdot 62^7$$

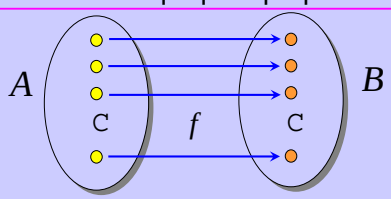
$$= 186125210680448$$

$$\approx 19 \cdot 10^{14}$$

Mapping Rule: Bijections

If f is a bijection from A to B,

then $|A| = |B|$



Size of the Power Set

How many subsets of finite set A?

$\mathtt{H}(A)$ = the power set of A

= the set of all subsets of A

for A = {a, b, c},

$\mathtt{H}(A)$ = {∅, {a}, {b}, {c},

{a,b}, {a,c}, {b,c}, {a,b,c} }

Bijection: $\mathtt{H}(A)$ and Binary Strings

A: $\{a_1, a_2, a_3, a_4, a_5, \ldots, a_n\}$

subset: $\{a_1, a_3, a_4, \ldots, a_n\}$

string: 1 0 1 1 0 … 1

a bijection, so

$|n\text{-bit binary strings}| = |\mathtt{H}(A)|$

2^n

Counting Doughnut Selections

five kinds of doughnuts

select a dozen:

chocolate, (none) lemon-filled, sugar, glazed, plain

A ::= all selections of a dozen doughnuts

Bit Strings with four 1's

B::= 16-bit words with four 1's, e.g.

0011000000100100

00 1 1 000000 1 00 1 00

chocolate, lemon-filled, sugar, glazed, plain

Bijection from A to B

c chocolate, l lemon, s sugar, g glazed, p plain

maps to

$0^c10^l10^s10^g10^p$

$|A| = |B|$

Team Problems

Problems 1 . 3