

4	9	13	7
12		10	5
3	1	6	14
15	8	11	2

Counting

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4	9	13	7
12		10	5
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15	8	11	2

Counting in Gambling

A pair of Jacks is



what *fraction* of poker hands?
(*probability* of a pair of Jacks)

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4	9	13	7
12		10	5
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Counting in Algorithms

- How many comparisons are needed to *sort* n numbers?
- How many multiplications to compute d^n ?

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4	9	13	7
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Counting in Games



- How many different configurations for a Rubik's cube?



- How many different chess positions after n moves?



- How many weighings to find the one counterfeit among 12 coins?

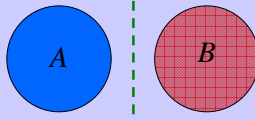
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4	9	13	7
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Sum Rule



If sets A and B are **disjoint**, then

$$|A \cup B| = |A| + |B|$$

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4	9	13	7
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The Sum Rule

- Class has 43 women, 54 men so total enrollment = $43 + 54 = 97$
- 26 lower case letters, 26 upper case letters, and 10 digits, so total characters = $26+26+10 = 62$

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4	9	13	7
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The Product Rule

If there are 4 boys and 3 girls, there are possible
 $4 \times 3 = 12$
 married couples.

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4	9	13	7
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Product Rule

If $|A| = m$ and $|T| = n$, then
 $|A \times T| = mn.$

$$A = \{a, b, c, d\}, \quad T = \{1, 2, 3\}$$

$$A \times T = \{ (a,1), (a,2), (a,3), \\ (b,1), (b,2), (b,3), \\ (c,1), (c,2), (c,3), \\ (d,1), (d,2), (d,3) \}$$

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4	9	13	7
12	10	6	
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Product Rule: Counting Strings

The number of length-4 strings
 from alphabet $B ::= \{0,1\}$

$$= |B \times B \times B \times B| \\ = 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

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4	9	13	7
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3	1	8	14
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Product Rule: Counting Strings

The number of length- n strings
 from an alphabet of size m is
 $m^n.$

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4	9	13	7
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Example: Counting Passwords

- between 6 & 8 characters long
- starts with a letter
- case sensitive
- other characters: digits or letters

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4	9	13	7
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Counting Passwords

$$L ::= \{a, b, A, z, A, B, A, Z\}$$

$$D ::= \{0, 1, A, 9\}$$

$$P_n ::= \text{length } n \text{ passwords}$$

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4	9	13	7
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Counting Passwords

$$L ::= \{a, b, A, z, A, B, A, Z\}$$

$$D ::= \{0, 1, A, 9\}$$

$$P_6 =$$

$$L \times (L \cup D) \times (L \cup D) \times (L \cup D) \times (L \cup D) \times (L \cup D) \\ = L \times (L \cup D)^5$$

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4	9	13	7
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Counting Passwords

$$L ::= \{a, b, A, z, A, B, A, Z\}$$

$$D ::= \{0, 1, A, 9\}$$

$$P_n ::= \text{length } n \text{ passwords}$$

$$= L \times (L \cup D)^{n-1}$$

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4	9	13	7
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Counting Passwords

$$\begin{aligned} |L \times (L \cup D)^{n-1}| &= |L| \cdot |L \cup D|^{n-1} \\ &= |L| \cdot (|L| + |D|)^{n-1} \\ &= 52 \cdot 62^{n-1} \end{aligned}$$

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4	9	13	7
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Counting Passwords

The set of Passwords:

$$P = P_6 \cup P_7 \cup P_8$$

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4	9	13	7
12		10	5
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Counting Passwords

$$\begin{aligned} |P| &= |P_6| + |P_7| + |P_8| \\ &= 52 \cdot 62^5 + 52 \cdot 62^6 + 52 \cdot 62^7 \\ &= 186125210680448 \\ &\approx 19 \cdot 10^{14} \end{aligned}$$

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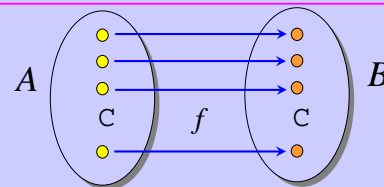
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4	9	13	7
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Mapping Rule: Bijections

If f is a **bijection** from A to B ,

then $|A| = |B|$



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Size of the **Power Set**

How many subsets of finite set A ?

$H(A)$ = the **power set** of A

= the set of all subsets of A

for $A = \{a, b, c\}$,

$H(A) = \{\emptyset, \{a\}, \{b\}, \{c\},$

$\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

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Bijection: $H(A)$ and Binary Strings

$A: \{a_1, a_2, a_3, a_4, a_5, \dots, a_n\}$

subset: $\{a_1, a_3, a_4, \dots, a_n\}$

string: $1 \ 0 \ 1 \ 1 \ 0 \ \dots \ 1$

a bijection, so

$$|\text{\textit{n}-bit binary strings}| = |H(A)|$$

$$2^n$$

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Counting Doughnut Selections

five kinds of doughnuts

select a dozen:

 chocolate
  (none) lemon-filled
  sugar
  glazed
  plain

$A ::=$ all selections of a dozen doughnuts

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Bit Strings with four 1's

$B ::=$ 16-bit words with four 1's, e.g.

0011000000100100
 00 1 1 000000 1 00 1 00
 chocolate
  lemon-filled
  sugar
  glazed
  plain

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Bijection from A to B

c chocolate, l lemon, s sugar, g glazed, p plain

maps to

$0^c 10^l 10^s 10^g 10^p$

$$|A| = |B|$$

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Team Problems

Problems
1. 3

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