

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem; Benés Network

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lec 7F.1

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

Planar Graphs are 5-Colorable

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lec 7F.2

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

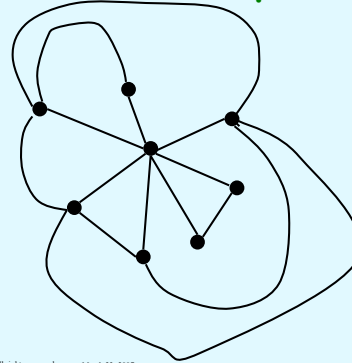
- $\deg(v) \leq 5$ for some v
- K_5 is **not** planar
- subgraphs are planar
- two new facts:

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lec 7F.3

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

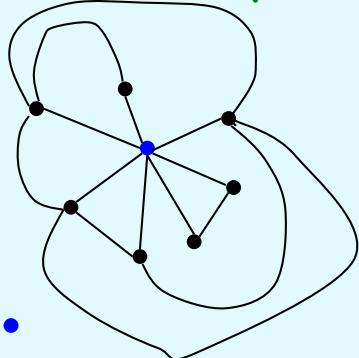


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lec 7F.4

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs



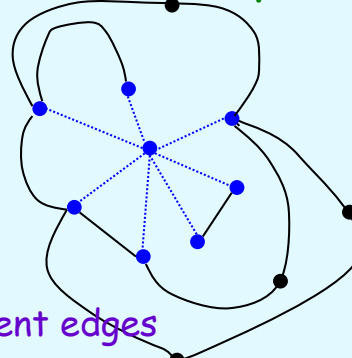
delete •

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lec 7F.5

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs



& incident edges

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lec 7F.6

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

& incident edges

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

- then can connect any two of its adjacent vertices, ●, and stay planar:

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

- merging adjacent vertices in a planar graph leaves a planar graph

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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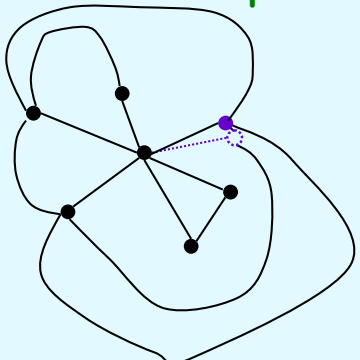
6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

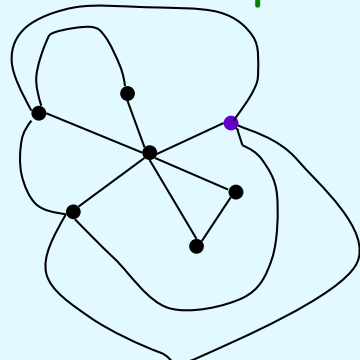
Planar Graphs



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Planar Graphs



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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

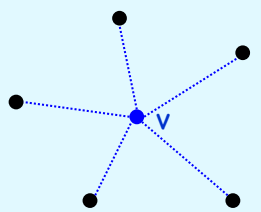
By induction on # vertices:
 case 1: vertex of $\deg \leq 4$.
 remove vertex, v ,
 5-color remainder, then
 enough colors left
 to color v . OK

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

case 2: vertex v of $\deg = 5$.

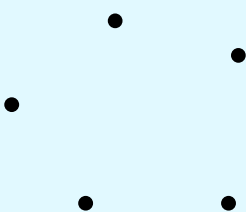


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

remove v

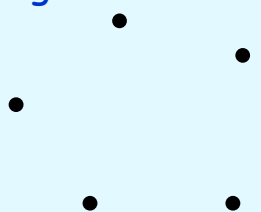


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

remaining 5 not all adjacent



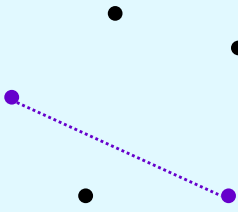
else would have K_5

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

pick 2 not adjacent



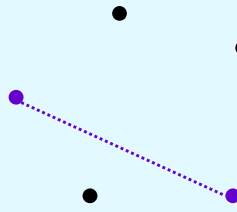
add edge (still planar)

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

merge (still planar)

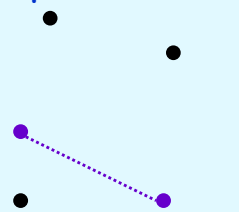


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

merge (still planar)

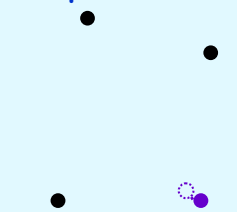


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

merge (still planar)

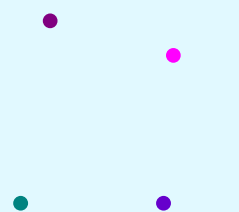


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

now 5-color

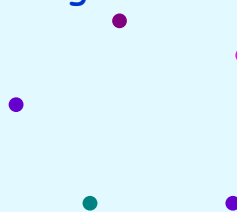


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

now unmerge

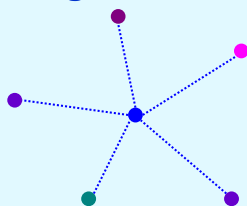


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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

5-Color Theorem

now unmerge, restore v



only 4 colors adjacent to v , so **OK**

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lec 7F.32

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Benés Network

see

<http://theory.csail.mit.edu/classes/6.042/spring07/slides6f.pdf>

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lec 7F.48

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Team Problems

Problems 1-3

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