Mathematics for Computer Science
MIT 6.042J/18.062J

5-Color Theorem; Benés Network

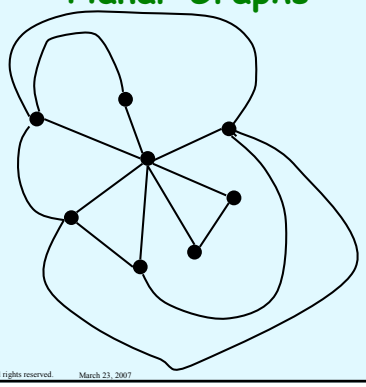
5-Color Theorem

Planar Graphs are 5-Colorable

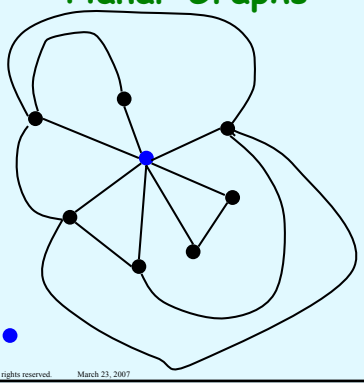
Planar Graphs

- deg(v) ≤ 5 for some v
- K_5 is not planar
- subgraphs are planar
- two new facts:

Planar Graphs

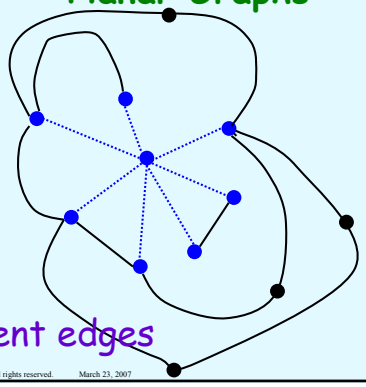


Planar Graphs



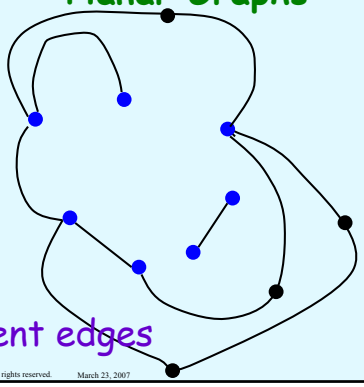
delete •

Planar Graphs



& incident edges

Planar Graphs

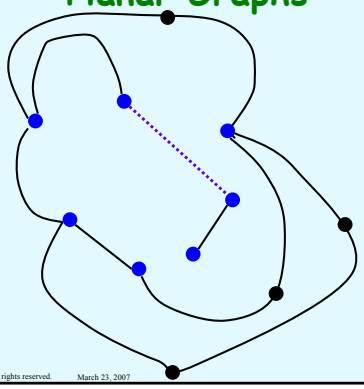


& incident edges

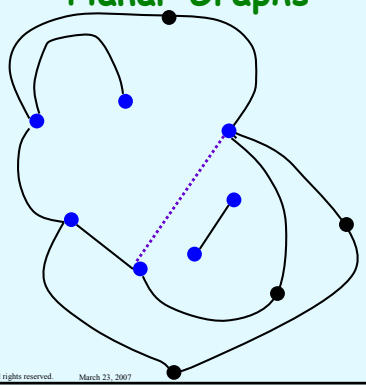
Planar Graphs

- then can connect any two of its adjacent vertices, ●, and stay planar:

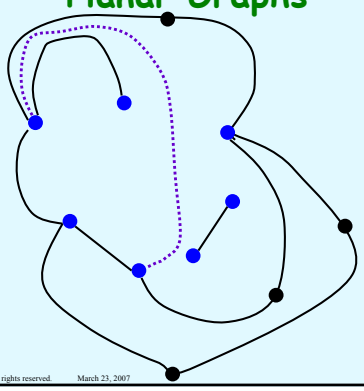
Planar Graphs



Planar Graphs

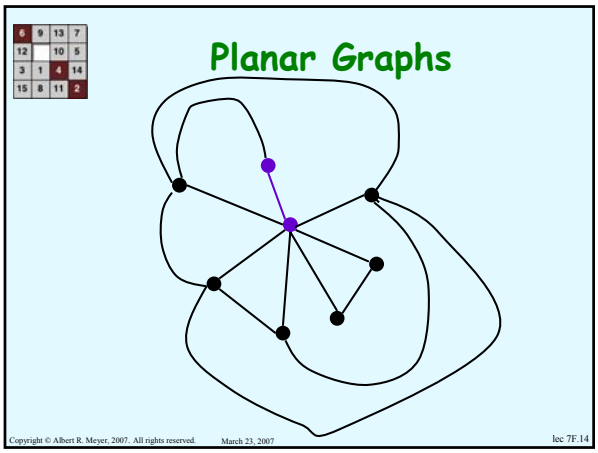


Planar Graphs



Planar Graphs

- merging adjacent vertices in a planar graph leaves a planar graph


Planar Graphs
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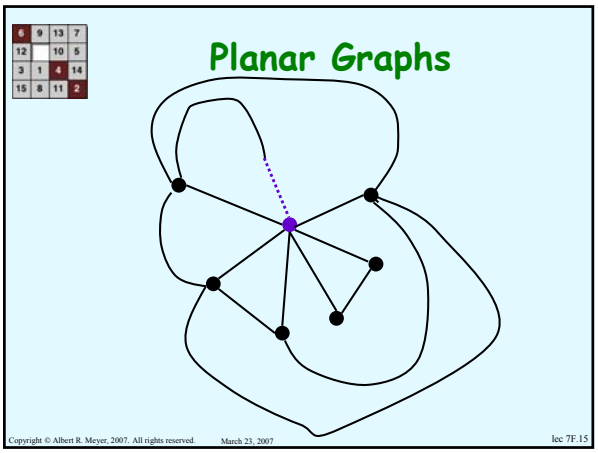
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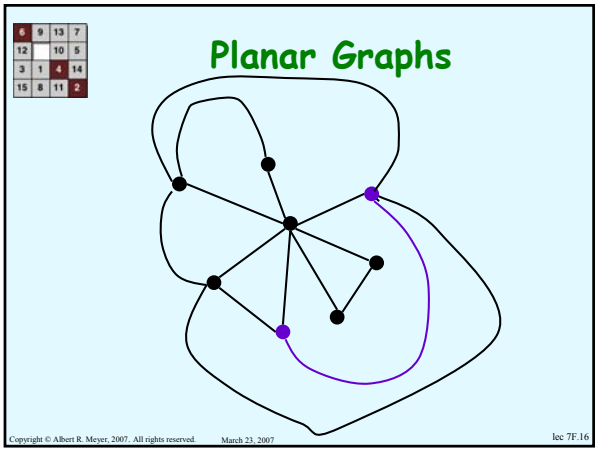
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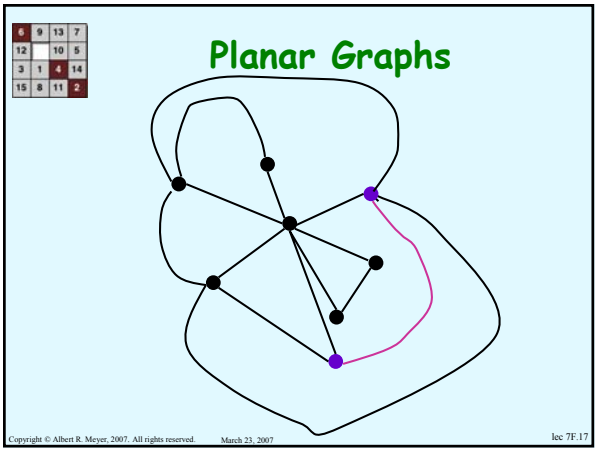
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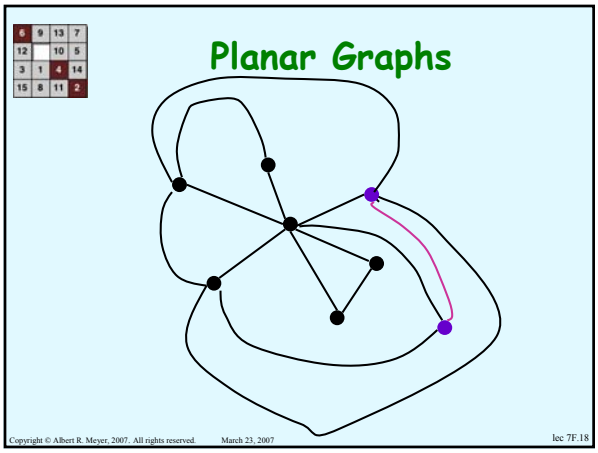
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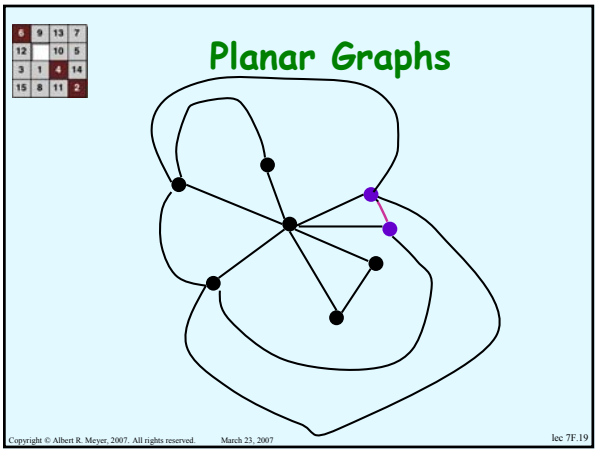
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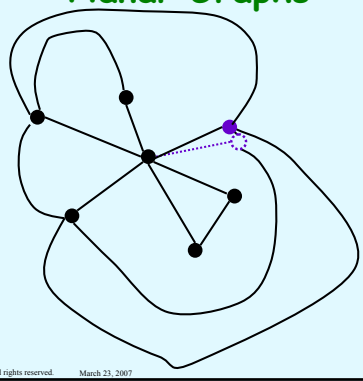
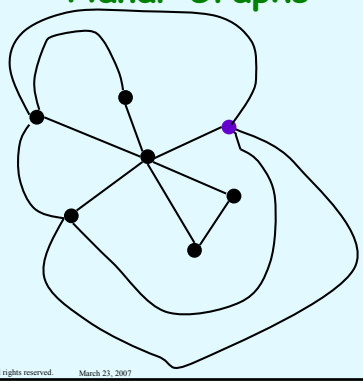
Planar Graphs



Planar Graphs



5-Color Theorem

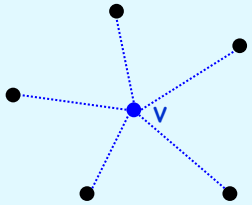
By induction on # vertices:

case 1: vertex of deg ≤ 4.

remove vertex, v,
5-color remainder, then
enough colors left
to color v. OK

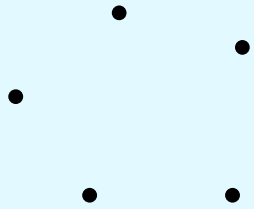
5-Color Theorem

case 2: vertex v of deg = 5.



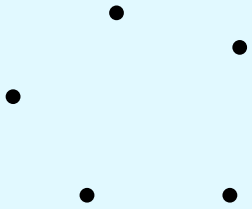
5-Color Theorem

remove v



5-Color Theorem

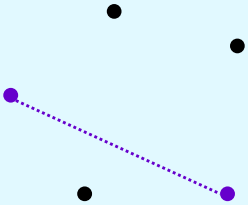
remaining 5 not all adjacent



else would have K_5

5-Color Theorem

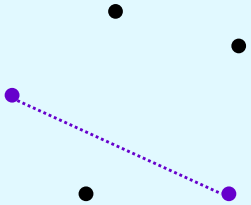
pick 2 not adjacent



add edge (still planar)

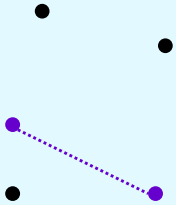
5-Color Theorem

merge (still planar)



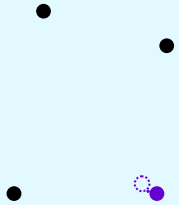
5-Color Theorem

merge (still planar)



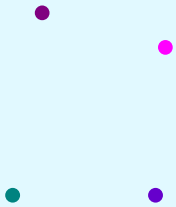
5-Color Theorem

merge (still planar)



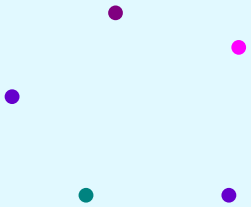
5-Color Theorem

now 5-color



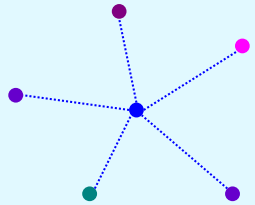
5-Color Theorem

now **un**merge



5-Color Theorem

now unmerge, restore v



only 4 colors adjacent to v, so **OK**

Benés Network

see
http://theory.csail.mit.edu/classes/6.042/spring07/slides6f.pdf

Team Problems

Problems 1–3