Mathematics for Computer Science

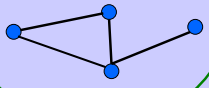
MIT 6.042J/18.062J

Simple Graphs: Degrees, Isomorphism, Paths

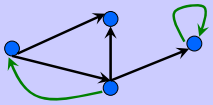
 March 7, 2007 lec 5W.1

Types of Graphs

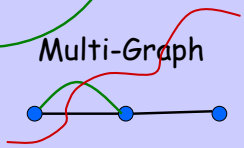
Simple Graph



Directed Graph



Multi-Graph



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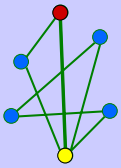
A Simple Graph

edge

vertices, *V*

undirected edges, *E*

::= { ●, ○ }



"*adjacent*"

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Vertex Degree

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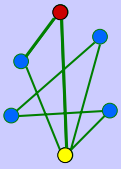
Vertex degree

degree of a vertex is # of *incident* edges



deg(●) = 2

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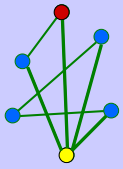
Vertex degree

degree of a vertex is # of *incident* edges



deg(○) = 4

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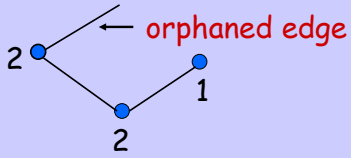
Impossible Graph

Is there a graph with vertex degrees 2,2,1?

NO!



Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

2+2+1 = odd, so impossible

Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

Proof: Each edge contributes 2 to the sum on the right

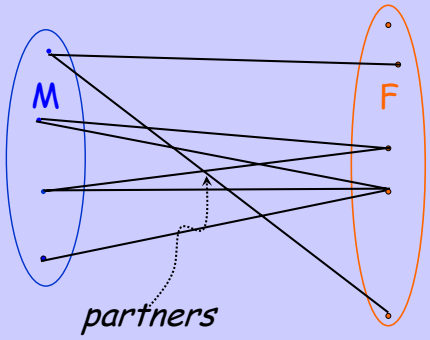
Sex in America: Men more Promiscuous?

Study claims:

Men average many more partners than women.

Graph theory shows this is nonsense

Sex Partner Graph



Counting pairs of partners

$$\sum_{m \in M} \deg(m) = |E| = \sum_{f \in F} \deg(f)$$

divide by both sides by $|M|$

$$\underbrace{\left(\frac{\sum_{m \in M} \deg(m)}{|M|}\right)}_{\text{avg-deg}(M)} = \underbrace{\left(\frac{\sum_{f \in F} \deg(f)}{|F|}\right)}_{\text{avg-deg}(F)} \frac{|F|}{|M|}$$

Average number of partners

$$(\text{avg-deg}(M)) = (\text{avg-deg}(F))\,1.035$$

Averages differ solely by *ratio of females to males.*

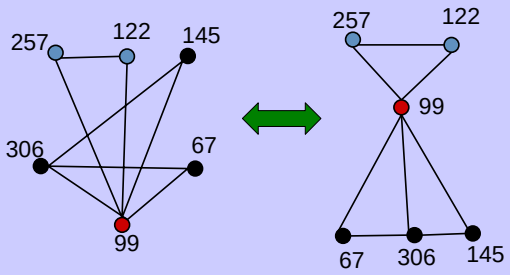
No big difference

Nothing to do with promiscuity.

Isomorphism

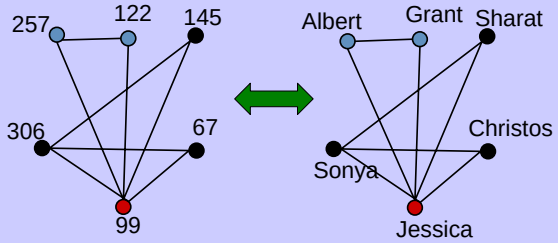
The Graph Abstraction

Same graph (different *layouts*)



The Graph Abstraction

Same graph (different *labels*)



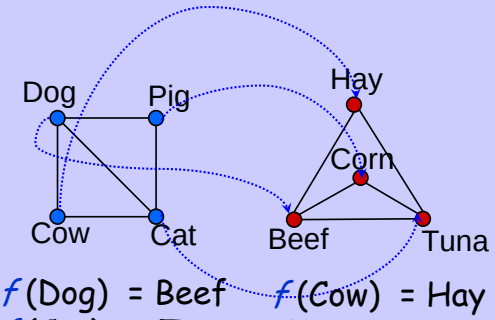
The Graph Abstraction

All that matters is the *connections.* Graphs with the same connections are *isomorphic.*

Isomorphism

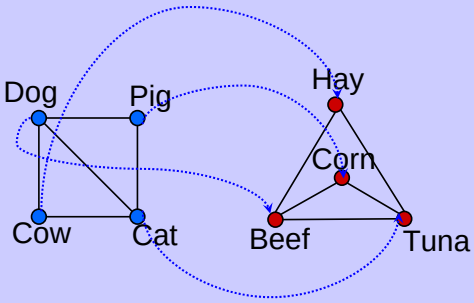
G_1 *isomorphic* to G_2 means there is an *edge-preserving vertex matching.*

Are these Isomorphic?

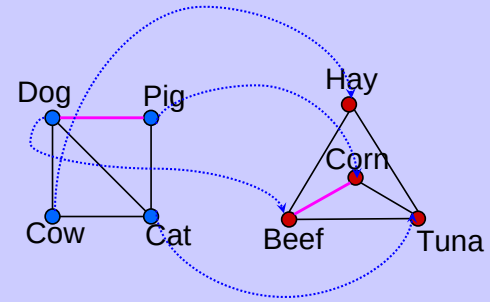


f(Dog) = Beef f(Cow) = Hay
f(Cat) = Tuna f(Pig) = Corn

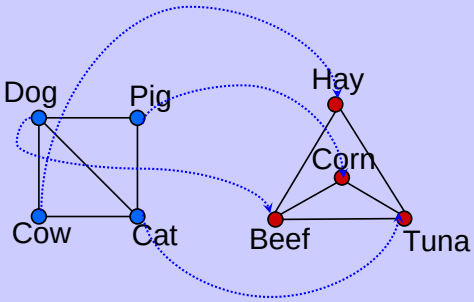
Edges Preserved?



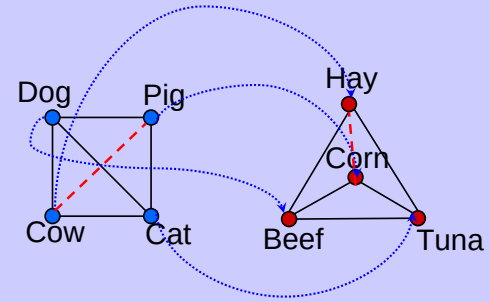
Edges Preserved? YES!



NonEdges Preserved?



NonEdges Preserved? YES!



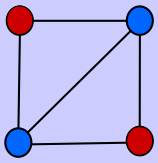
Graph Isomorphism

G_1 *isomorphic* to G_2 means there is an *edge-preserving vertex matching.*

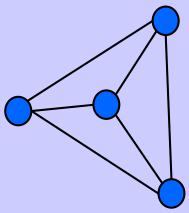
$\exists$ *bijection* $f: V_1 \to V_2$

$u - v$ in E_1 iff $f(u) - f(v)$ in E_2

Non-isomorphism



degree 2



all degree 3

Finding the Mapping?

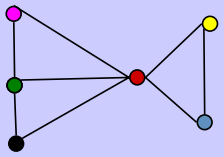
Not easy --many possible mappings.
Can test for *properties preserved under isomorphism:*

- # of nodes, # edges,
- degree distributions,
- length of paths & cycles …

Connectedness

Paths

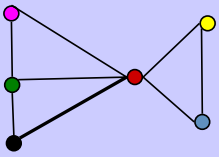
Path: sequence of *adjacent* vertices



(●)

Paths

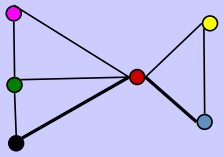
Path: sequence of *adjacent* vertices



(● ●)

Paths

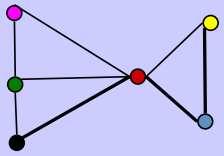
Path: sequence of *adjacent* vertices



(● ● ●)

Paths

Path: sequence of *adjacent* vertices



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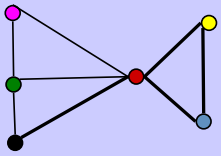
Paths

Path: sequence of *adjacent* vertices



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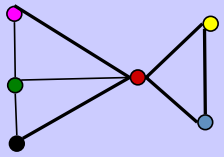
Paths

Path: sequence of *adjacent* vertices



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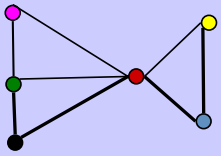
Simple Paths

Simple Path: all vertices different



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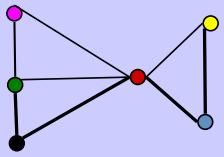
Simple Paths

Simple Path: (doesn't cross itself)



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Connectedness

vertices *v*, *w* are *connected* iff there is a path starting at *v* and ending at *w*.

A *graph* is *connected* iff every pair of vertices are connected.

Team Problems

Problems 2,3,1,4