

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science 6.042J/18.062J

WELCOME!

Prof. Albert R. Meyer

<http://theory.lcs.mit.edu/classes/6.042>

“Proof, Proofs & More Proofs”

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Quick Summary

1. Fundamental Concepts of Discrete Mathematics.
2. Discrete Mathematical Structures
(like *trees* or *lists*)
3. Discrete Probability Theory.

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Vocabulary

Quickie:

What does “discrete” mean?
(≠ “discreet”)

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Online Tutor Problems 1

Due Friday, 6pm:
Part 1.1: Course Registration

Due Monday, 6pm:
Part 1.2: Diagnostic Questionnaire

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Reading Assignment

Reading: Notes for week 1;
Week 2 also available
(see course calendar)

Email comments on **week 1&2** Notes:
due next Wednesday, 11am

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Course Organization

- **Web site:** All course handouts.
- **Problem Sets:** *up to 30%* of grade (see **course info**).

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Course Organization

- **Studio-Lecture Style:**
mix of mini-lectures &
team problem-solving;
preparation & attendance
required (25% of grade)

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Active Lectures

Say “hello” to your
neighbors -- you’ll be
working with them .

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Active Lectures

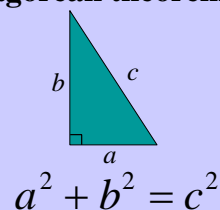
Quickie question:
Where was your neighbor
born?

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Getting started: Pythagorean theorem



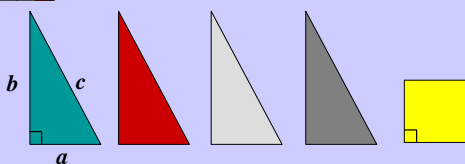
Familiar? Yes!
Obvious? No!

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A Cool Proof



Rearrange into: (i) a $c \times c$ square, and then
(ii) an $a \times a$ & a $b \times b$ square

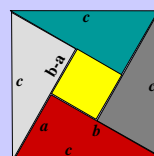
(Many many proofs: <http://www.cut-the-knot.com>)

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A Cool Proof



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A Cool Proof

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A Cool Proof

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A False Proof: Getting Rich By Diagram

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A False Proof: Getting Rich By Diagram

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Getting Rich

The bug:

1 1 are not right triangles!

The top and bottom line of the “rectangle” is not straight!

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Another False Proof

Theorem:

Every polynomial, $ax^2 + bx + c$, has two roots over \mathbb{C} .

Proof (by calculation):

The polynomial $ax^2 + bx + c$ has roots

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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Another False proof

Counter-examples:

$0x^2 + 0x + 1$ has 0 roots.

$0x^2 + 1x + 1$ has 1 root.

The bug: divide by zero error.

The fix: assume $a \neq 0$.

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Another false proof

Counter-example:

$1x^2 + 0x + 0$ has 1 root.

The bug: $r_1 = r_2$

The fix: need hypothesis $D \neq 0$ where

$$D ::= \sqrt{b^2 - 4ac}$$

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Another false proof

Ambiguity when $D < 0$:

$x^2 + 1$ has roots $i, -i$.

Which is r_1 , which is r_2 ?

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$1 = -1$?

The ambiguity causes problems:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = (\sqrt{-1})^2 = -1$$

Moral: “mindless” calculation not safe.

1. Be sure rules are properly applied.
2. Calculation is a risky substitute for understanding.

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Consequences of $1 = -1$

$\frac{1}{2} = -\frac{1}{2}$ (multiply by $\frac{1}{2}$)

$2 = 1$ (add $\frac{3}{2}$)

“Since 1 and the Pope are clearly 2,
we conclude that

1 and the Pope are 1.

That is, I am the Pope.”

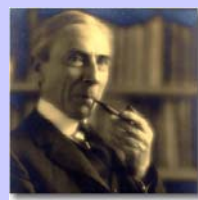
-- Bertrand Russell

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Consequences of $1 = -1$



Bertrand Russell (1872 - 1970)

(Picture source: <http://www.uneca.drexel.edu/~jlane/brs.html>)

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In-class Problems

PROBLEMS 1 & 2

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