Team Problems

Problems 1&2

 May 9, 2007 lec 13w.1

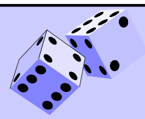
Mathematics for Computer Science

MIT 6.042J/18.062J

Great Expectations

 May 9, 2007 lec 13w.2

Carnival Dice



Choose a number from 1 to 6, then roll 3 fair dice:

win $1 if any die matches num

lose $1 if no match. *Example:*

choose number 2, roll 2,4,2,

win $1

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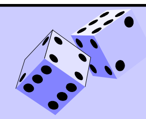
Carnival Dice



Is this a fair game?

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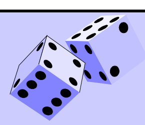
Carnival Dice



Clearly NOT fair:

$\text{pr}\{\text{win}\} = 1-(5/6)^3 < 0.43 < 1/2$

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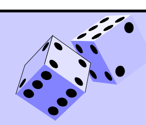
Carnival Dice, II



Choose a number from 1 to 6, then roll 3 fair dice:

win $1 for **each** match

lose $1 if no match. *Example:*

choose number 2, roll 2,4,2,

win $2

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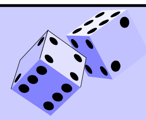
Carnival Dice, II



Is this now a fair game?

Carnival Dice, II

		win
$\Pr\{0 \text{ matches}\}$	$= \left(\frac{5}{6}\right)^3$	-1
$\Pr\{1 \text{ match}\}$	$= \binom{3}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$	1
$\Pr\{2 \text{ matches}\}$	$= \binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)$	2
$\Pr\{3 \text{ matches}\}$	$= \binom{3}{3}\left(\frac{1}{6}\right)^3$	3

Carnival Dice, II

Average win:

$$\frac{\left(5^3\cdot -1\right)+3\cdot 5^2\cdot 1+3\cdot 5\cdot 2+3}{6^3}$$

$$= -\frac{17}{216} \approx -8 \text{ cents}$$

NOT fair!

Carnival Dice, II

You can "expect" to lose 8 cents per play.
Notice that you never actually lose 8 cents on any single play.
Rather, this is what you expect to lose on average.

Expectation

The expected value of a random variable D is: the *average value* of D --with values weighted by their probabilities.

Expectation

expected value also called
mean value,
mean, or expectation

Expectation

The expected value of a random variable D is:

$$E[D] ::= \sum_{v} v \cdot \Pr\{D = v\}$$

Sum or Integral?

In the most general probability spaces, the sum would have to be an integral. We can get away with sums because we assume the sample space is countable:

$$\mathcal{S} = \{\omega_0, \omega_1, \ldots, \omega_n, \ldots\}$$

Sum or Integral?

$$\Pr\{D = v\} ::= \sum_{D(\omega)=v} \Pr\{\omega\}$$

Expectation

So

$$E[D] ::= \sum_{v} v \cdot \Pr\{D = v\}$$

$$= \sum_{\omega \in \mathcal{S}} D(\omega) \cdot \Pr\{\omega\}$$

Mean Time to Failure

Biased coin with pr{Head} = p.
Flip until a Head comes up.
Expected #flips?

Mean Time to Failure

pr{1st Head on flip 1} = p,
pr{1st Head on flip 2} = $(1-p)p$,
pr{1st Head on flip 3} = $(1-p)^2p$,
⋮
pr{1st Head on flip n} = $(1-p)^{n-1}p$.

Mean Time to Failure

E [# flips till 1st Head]

$$= \sum_{1}^{\infty} n \cdot (1-p)^{n-1} p$$

$$= p \left(\sum_{0}^{\infty} (n+1) \cdot (1-p)^{n} \right)$$

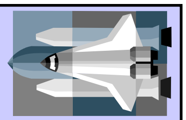
Mean Time to Failure

E [# flips till 1st Head]

$$= p \left(\frac{1}{(1-(1-p))^2} \right)$$

$$= \frac{1}{p}$$

Mean Time to Failure



application: Space station Mir say had 1/150,000 chance of exploding in any given hour. After how may hours did we expect it to explode?

150,000 hours $\approx$ 17 years

Team Problems

Problems 3—5