Mathematics for Computer Science
MIT 6.042J/18.062J

Introduction to Probability Theory

Counting in Probability

What is the *probability* of getting exactly two jacks in a poker hand?

Counting in Probability

Outcomes: $\binom{52}{5}$ 5-card hands

Event: $\binom{4}{2} \cdot \binom{52-4}{3}$ hands w/2Jacks.

$$\Pr\{2J\} ::= \frac{\binom{4}{2} \cdot \binom{48}{3}}{\binom{52}{5}} \approx 0.04$$

Probability: 1st Idea

- set of basic experimental outcomes,
- subset of outcomes considered a noteworthy event,
- probability{event}

$$::= \frac{\#\text{ outcomes in event}}{\#\text{ possible outcomes}}$$

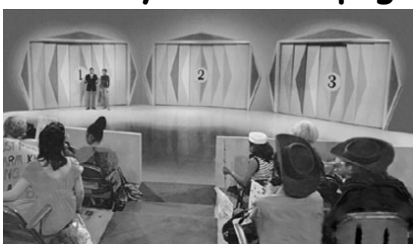
The Monty Hall Game

Applied Probability:

Let's Make A Deal

(1970's TV Game Show)

Monty Hall Webpages



http://www.letsmakeadeal.com

Analyzing Monty Hall

Marilyn Vos Savant explained Game in magazine -- bombarded by letters (even from PhD's) debating:

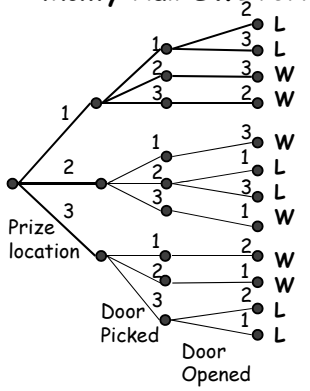
1) sticking & switching equally good
2) switching better

Analyzing Monty Hall

Determine the outcomes.

-- a tree of possible steps can help

Monty Hall SWITCH Strategy



SWITCH
Wins: 6

Lose: 6

Monty Hall STICK Strategy

Win by sticking iff
Lose by switching.

STICK
Lose: 6

Wins: 6

Analyzing Monty Hall

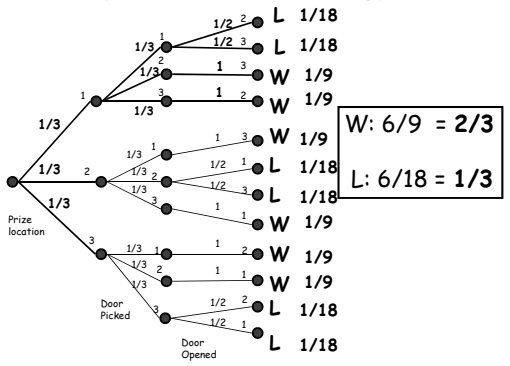
Sticking and Switching have same # winning outcomes.

False conclusion:
Contestant has same probability of winning: $\frac{1}{2}$

Analyzing Monty Hall

What's wrong?
Let's look at the outcome tree more carefully.

Monty Hall SWITCH Strategy



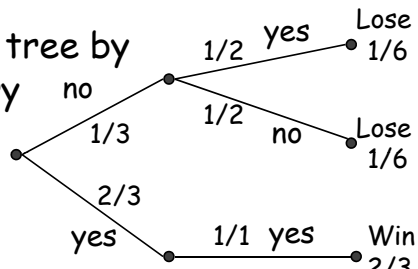
Probability: 2nd Idea

Outcomes may have differing probabilities!

Not always uniform.

Monty Hall SWITCH Strategy

Simplify tree by symmetry



Contestant picks goat

Carol opens 1st possible door

Finding Probability

Intuition is important but dangerous.

Stick with 4-part method:

1. Identify outcomes (*tree helps*)
2. Identify event (*winning*)
3. Assign outcome probabilities
4. Compute event probabilities

Probability Spaces

1) Sample space, $\mathcal{S}$, whose elements are called outcomes.

2) Probability function,

Pr: $\mathcal{P}(\mathcal{S}) \to [0,1]$

(a) $\Pr\{\mathcal{S}\} = 1$,

(b) the Sum Rule:

(Disjoint) Sum Rule

If A_1, A_2 are disjoint,

$$\Pr\{\mathcal{A}_1 \cup \mathcal{A}_2\} = \Pr\{\mathcal{A}_1\} + \Pr\{\mathcal{A}_2\}$$

Sum Rule (Infinite)

For pairwise disjoint $A_0, A_1, \ldots$

$$\Pr\{\mathcal{A}_0 \cup \mathcal{A}_1 \cup \cdots\} = \Pr\{\mathcal{A}_0\} + \Pr\{\mathcal{A}_1\} + \cdots$$

Inclusion-Exclusion

$$\Pr\{\mathcal{A} \cup \mathcal{B}\} = \Pr\{\mathcal{A}\} + \Pr\{\mathcal{B}\} - \Pr\{\mathcal{A} \cap \mathcal{B}\}$$

The Union Bound

$$\Pr\{\mathcal{A} \cup \mathcal{B}\} \leqslant \Pr\{\mathcal{A}\} + \Pr\{\mathcal{B}\}$$

Team Problems

Problems 1–4