Problem Set 8

Due: April 18

Reading:

- Week 8, pp.15–20: Asymptotic Notation
- Week 9: Rules for Counting, §1& §2.

Problem 1. Prove that $\sum_{k=1}^{n} k^6 = \Theta(n^7)$.

Problem 2. Determine which of these choices

$$\Theta(n), \quad \Theta(n^2 \log n), \quad \Theta(n^2), \quad \Theta(1), \quad \Theta(2^n), \quad \Theta(2^{n \ln n}), \quad \text{none of these}$$

describes each function's asymptotic behavior. Full proofs are not required, but briefly explain your answers.

(a)

$$n + \ln n + (\ln n)^2$$

(b)

$$\frac{n^2 + 2n - 3}{n^2 - 7}$$

(c)

$$\sum_{i=0}^{n} 2^{2i+1}$$

(d)

$$\ln(n^2!)$$

(e)

$$\sum_{k=1}^{n} k\left(1-\frac{1}{2^k}\right)$$

Problem 3. **(a)** Prove that the relation, R, on functions such that fRg iff $f = o(g)$ is a strict partial order.

(b) Describe two incomparable elements in this partial order.

Problem 4. **(a)** Describe a bijection between the set of paths from $(0, 0)$ to $(10, 20)$ consisting of right-steps (which increment the first coordinate) and up-steps (which increment the second coordinate) and the set of 30-bit sequences with 10 zeros and 20 ones.

(b) Mr. and Mrs. Grumperson have collected 13 identical pieces of coal as Christmas presents for their beloved children, Lucy and Spud. Describe a bijection between the set of all ways of distributing the 13 coal pieces to the two children and the set of 14-bit sequences with exactly 1 one.

(c) On Christmas Eve, Mr. and Mrs. Grumperson remember that they have a third child, little Bottlecap, locked in the attic. Describe a bijection between the set of all ways of distributing the 13 coal pieces to the three children and the set of 15-bit sequences with exactly 2 ones.

(d) On reflection, Mr. and Mrs. Grumperson decide that each of their three children should receive *at least two* pieces of coal for Christmas. Describe a bijection between the set of all ways of distributing the 13 coal pieces to the three Grumperson children given this constraint and the set of 9-bit sequences with exactly 2 ones.

(e) Describe a bijection between the set of solutions over the natural numbers to the inequality:

$$x_1 + x_2 + \cdots + x_{10} \leq 100, \tag{1}$$

and the set of 110-bit sequences with exactly 10 ones.

(f) Describe a bijection between solutions to the inequality (1) and sequences $(y_1, y_2, \ldots, y_{10})$ such that:

$$0 \leq y_1 \leq y_2 \leq \cdots \leq y_{10} \leq 100.$$

Student's Solutions to Problem Set 8

Your name:

Due date: April 18

Submission date:

Circle your TA/LA: Chiyoun Jay Jeffrey Jessica Tina

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	
2	
3	
4	
Total	

[1] People other than course staff.

[2] Give citations to texts and material other than the Spring '07 course materials.