Mini-Quiz May 11

Your name: ______________________________

Circle the name of your TA/LA:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is 25 minutes.
- There are four (4) problems totaling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	5		
2	4		
3	3		
4	3		
Total	15		

Problem 1 (5 points)**. [A Baseball Series]** The New York Yankees and the Boston Red Sox are playing a two-out-of-three series. (In other words, they play until one team has won two games. Then that team is declared the overall winner and the series ends.) Assume that the Red Sox win each game with probability $3/5$, regardless of the outcomes of previous games.

Answer the questions below. (Partial credit may be awarded based on work you show.)

(a) (2 points) What is the probability that the Red Sox win the series?

(b) (3 points) Given that the Red Sox won the series, what is the probability they lost the first game?

Problem 2 (4 points)**.** There is a rare and serious disease called *Beaver Fever* which afflicts about 1 person in 1000. Victims eventually start telling math jokes at cocktail parties, thinking others will find the jokes amusing.

Doctor X says he can test for the disease.

- If a person has Beaver Fever, he says "yes" with probability 0.99.
- If a person doesn't have it, he says "no" with probability 0.97.

(a) (2 points) What is the probability that Doctor X says "yes"?

(b) (2 points) If Doctor X says someone has the disease, what is the probability that person really does have the disease?

Problem 3 (3 points)**.** You want to estimate the fraction of voters in the entire nation that will prefer Donald Duck in the upcoming elections. To do this, you pick a suitable number, n, perform n successive independent selections of a random voter, and ask each selected voter whether they will vote for the Donald. You find that a fraction of 0.53 of your selections will vote for the Donald. You also find that Mickey Mouse happened to be one of the voters you selected.

You also calculate that if you independently flip a fair coin n times, the fraction of Heads flipped will be between 0.49 and 0.51 with probability at least 0.97.

Circle all the following statements that are true:

- Assuming no voter was selected more than once, the probability is 0.53 that a randomly chosen voter *among those you selected* will vote for the Donald.
- The probability is 0.53 that a randomly chosen voter *among all the voters in the nation* will vote for the Donald.
- The probability is 0.53 that Mickey Mouse will vote for the Donald.
- You are 97% confident that the probability is 0.53 that Mickey Mouse will vote for the Donald.
- You are 97% confident that the Donald will get between 0.52 and 0.54 of the vote.
- Assuming no voter was selected more than once, the probability is at least 0.97 that the fraction of voters in the nation who will vote for the Donald is between 0.52 and 0.54.
- Even if several voters were sampled more than once, you can still say that the probability is at least 0.97 that the fraction of voters in the nation who will vote for the Donald is between 0.52 and 0.54.

Problem 4 (3 points)**.** Independently flip two biased coins, each with probability p of coming up Heads, where $0 < p < 1$. Let H_1 be the indicator variable for a Head occurring on the first coin, and likewise H_2 for a Head on the second coin. Define $S ::= H_1 \oplus H_2$ where $\oplus$ denotes addition modulo 2.

Prove that if H_1 and S are independent, then $p = 1/2$.

Hint: $[H_1 = 1 \text{ and } H_2 = 1]$ and $[H_1 = 1 \text{ and } S = 0]$ are the same event.

Appendix

The probability of an event A given an event B, is

$$\Pr\{A \mid B\} ::= \frac{\Pr\{A \cap B\}}{\Pr\{B\}} \text{ where } \Pr\{B\} \neq 0$$

The Total Probability Law is

$$\Pr\{A\} = \Pr\{A \mid E\} \cdot \Pr\{E\} + \Pr\left\{A \mid \overline{E}\right\} \cdot \Pr\left\{\overline{E}\right\}.$$

The complement law says $\Pr\{E\} + \Pr\left\{\overline{E}\right\} = 1$.

The inclusion-exclusion formula says:

$$\Pr\{E_1 \cup \cdots \cup E_n\} = \sum_i \Pr\{E_i\} - \sum_{i,j} \Pr\{E_i \cap E_j\} + \sum_{i,j,k} \Pr\{E_i \cap E_j \cap E_k\} - \cdots.$$

Two events A and B are independent iff $\Pr\{A \cap B\} = \Pr\{A\}\Pr\{B\}$.

Random variables R_1 and R_2 are ***independent*** iff

$$\Pr\{R_1 = r_1 \text{ and } R_2 = r_2\} = \Pr\{R_1 = r_1\} \cdot \Pr\{R_2 = r_2\}$$

for all $r_1, r_2, \cdots \in \mathbb{R}$.