Massachusetts Institute of Technology
6.042J/18.062J, Spring '07: Mathematics for Computer Science
Prof. Albert R. Meyer
revised May 4, 2007, 584 minutes

Mini-Quiz May. 4

Your name:_____

Circle the name of your TA/LA:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is 25 minutes.
- There are three (3) problems totaling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	5		
2	5		
3	5		
Total	15		

Problem 1 (5 points)**.** You would like to give a bouquet for Mother's Day, but you know nothing about flower arrangement. You google and find an online service where you just enter the number of each type of flower you want, and they make a nice bouquet and send it to your home. You decide to buy a bouquet with some number of lilies and red and white roses, but with the following restrictions:

- there must be at most 3 lilies,
- there must be at least 4 red roses,
- there must be an odd number of white roses.

Let f_n be the number of ways to compose a bouquet of n flowers satisfying the restrictions.

Find a simple closed form for $F(x)$, the generating function for the sequence $f_0, f_1, f_2, \ldots$.

Problem 2 (5 points)**.** A sequence a_n is defined recursively by the following rules

$$\begin{aligned} a_0 &= 0 \\ a_1 &= 2 \\ a_n &= 4a_{n-1} - 3a_{n-2} + 2, \text{for } n > 1 \end{aligned}$$

Find a simple closed form for $A(x)$, the generating function for the sequence $a_0, a_1, a_2, \ldots$.

Problem 3 (5 points)**.** The following is the generating function of an infinite sequence $\langle g_0, g_1, g_2, g_3, \ldots \rangle$. Find a simple closed form for the value of g_n.

$$G(x) = \frac{e}{(1-ex)^2} + \frac{2}{(1-x)^3} - \frac{2}{1-x}$$

Appendix

Definition 3.1. The *generating function* for the infinite sequence $\langle g_0, g_1, g_2, g_3, \cdots \rangle$ is the power series:

$$G(x) = g_0 + g_1 x + g_2 x^2 + g_3 x^3 + \cdots .$$

Useful series expansion

$$\frac{1}{(1-x)^k} = \sum_{n=0}^{\infty} \binom{n+k-1}{k-1} x^n, \text{for } |x| < 1.$$