Mini-Quiz Apr. 27

Your name:____

Circle the name of your TA/LA:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is **twenty (20)** minutes.
- There are four (4) problems totalling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	4		
2	4		
3	4		
4	3		
Total	15		

Problem 1 (4 points)**.** Albert needs to pick a 10 digit numeric password containing *each* of the digits $0, 1, \ldots, 9$. He recognizes that it clearly should not contain either of the subwords "6042" or "18062". Just to be safe he decides that it should also not contain the subword "624" (his office number in the Gates Tower).

Use the Inclusion-Exclusion Principle to find a simple formula for the number of such passwords that do not contain "6042", "18062" or "624".

Problem 2 (4 points)**.** For given positive integers i and m, how many different solutions[1] over the nonnegative integers are there to the following inequality?

$$x_1 + x_2 + x_3 + \ldots + x_m \leq i.$$

Your answer should be a simple formula (no indexed sums or three dots) which may involve factorials and binomial coefficients.

Problem 3 (4 points)**.** Provide the combinatorial identity implied by the following:

Suppose it's 4:30pm Friday and you must choose a team of m people with 1 leader among them from a pool of n people.

Your first inclination is to select m people and let them pick the leader. Then you remember how much time it will take to do your hair for your hot date at 7pm, so instead you choose the leader from the pool and let that person choose the rest of the team after you leave.

You know the same teams could potentially result from either method and so the numbers of ways to choose teams either way are equal. What is the identity?

[1] A ***solution*** is a specification of m nonnegative integer values for the successive variables $x_1, x_2, \ldots, x_m$. Two solutions are different if they specify different values for some variable.

Problem 4 (3 points). Find the coefficient of x^{15} in $(2x^2 - x)^{11}$.

1 Appendix

Rule (Inclusion-Exclusion for Three Sets).

$$\begin{aligned}|S_1 \cup S_2 \cup S_3| = &|S_1| + |S_2| + |S_3| \\ &- |S_1 \cap S_2| - |S_1 \cap S_3| - |S_2 \cap S_3| \\ &+ |S_1 \cap S_2 \cap S_3|\end{aligned}$$

Theorem 4.1 (Binomial Theorem). *For all* $n \in \mathbb{N}$ *and* $a, b \in \mathbb{R}$*:*

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$