Mini-Quiz Apr. 20

Your name:_____

Circle the name of your TA/LA:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is 25 minutes.
- There are four (4) problems totalling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	5		
2	2		
3	4		
4	4		
Total	15		

Problem 1 (5 points)**.**

Circle each of the true statements below:

- $3n = o(n^2)$
- $\ln n = O(n^k)$, $k > 1$ a constant
- $3^{n/2} = O(3^n)$
- $(3n-7)/(n+4) = \Theta(1)$
- $(3n-7)/(n+4) \sim 1$
- $n^k = O(k^n)$, $k > 1$ a constant
- $k^n = O(n^k)$, $k > 1$ a constant
- $3^n = O(2^n)$
- $2^n = o(3^n)$
- $\sum_{i=1}^{n} i = O(n)$

Problem 2 (2 points)**.** Show that

$$\ln(n^2!) = \Theta(n^2 \ln n)$$

Problem 3 (4 points)**.** A license plate consists of either:

- 3 letters followed by 3 digits (standard plate)
- 5 letters (vanity plate)
- 2 characters – letters or numbers (big shot plate)

Let L be the set of all possible license plates.

(a) (2 points) Express L in terms of

$$\mathcal{A} = \{A, B, C, \ldots, Z\}$$
$$\mathcal{D} = \{0, 1, 2, \ldots, 9\}$$

using unions ($\cup$) and set products ($\times$ or the compact exponent notation).

(b) (2 points) Compute $|L|$, the number of different license plates, using the sum and product rules.

Problem 4 (4 points)**.** There are 20 different books arranged in a row on a shelf.

(a) (1 point) Describe a bijection between ways of choosing x distinct books, $0 \leq x \leq 20$, and 20-bit sequences with exactly x ones.

(b) (1 point) How many ways are there of choosing at least 2 books from the 20? (Hint: How many ways are there of not choosing at least 2 books?)

(c) (1 point) Describe a bijection between ways of choosing 6 of these books so that no two adjacent books are selected and 15-bit sequences with exactly 6 ones.

(d) (1 point) Using your answer from part (c), give the bit representation corresponding to the selection of the 1st, 3rd, 5th, 10th, 13th, and 19th books.

1 Appendix

Lemma (Stirling's Formula)**.**

$$n! \sim \left(\frac{n}{e}\right)^n \sqrt{2\pi n},$$

For functions $f, g : \mathbb{R} \to \mathbb{R}$, we say f is *asymptotically equal* to g, in symbols,

$$f(x) \sim g(x)$$

iff

$$\lim_{x\to\infty} f(x)/g(x) = 1.$$

For functions $f, g : \mathbb{R} \to \mathbb{R}$, we say f is *asymptotically smaller* than g, in symbols,

$$f(x) = o(g(x)),$$

iff

$$\lim_{x\to\infty} f(x)/g(x) = 0.$$

Given functions $f, g : \mathbb{R} \mapsto \mathbb{R}$, with g nonnegative, we say that[1]

$$f = O(g)$$

iff

$$\limsup_{x\to\infty} |f(x)|/g(x) < \infty.$$

An alternative, equivalent, definition is

$$f = O(g)$$

iff there exists a constant $c \geq 0$ and an x_0 such that for all $x \geq x_0$, $|f(x)| \leq cg(x)$.

Finally, we say

$$f = \Theta(g) \quad \text{iff} \quad f = O(g) \wedge g = O(f).$$

[1]

$$\limsup_{x\to\infty} h(x) ::= \lim_{x\to\infty} \text{lub}_{y\geq x} h(y).$$