Mini-Quiz Mar. 14

Your name:____

Circle the name of your TA/LA:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is 25 minutes.
- There are five (5) problems totalling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	3		
2	3		
3	3		
4	4		
5	2		
Total	15		

Problem 1 (3 points). Circle all of the properties below that are invariants of the *Mating Ritual* for finding a stable matching (see the Appendix if you need to look up the Ritual). Assume that the numbers of boys and girls are the same.

a. If a girl is crossed off a boy's list, she has a suitor that she prefers to that boy.

b. If a girl is crossed off a boy's list, the girl and the boy will not be a rogue couple.

c. There is a girl with no suitor (boy who is serenading her).

d. If a girl is crossed off a boy's list, he prefers that girl to the girl he is serenading.

e. All the boys have the same number of girls left uncrossed in their list.

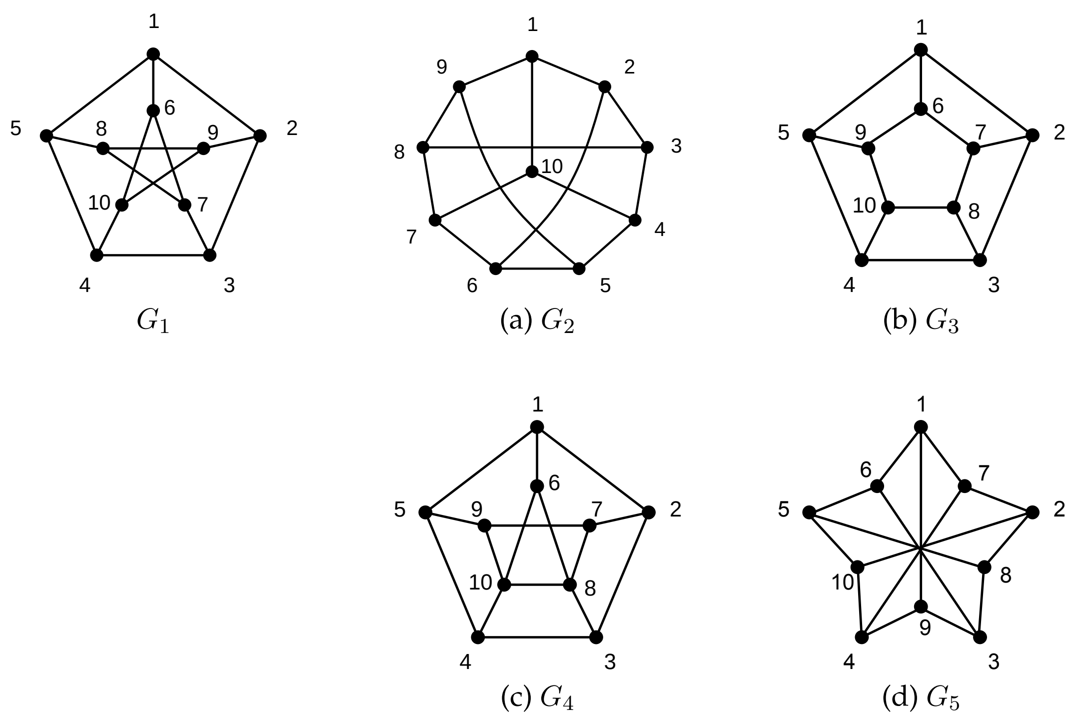
f. If a boy has only one girl left on his list, she will be his wife on the last day of the Ritual.

Problem 2 (3 points). Prove that in every graph, there are an even number of vertices of odd degree. You may assume the Handshaking Theorem (see Appendix). Your proof will be graded primarily on the clarity of your argument.

Problem 3 (3 points). Circle all the properties below that are preserved under isomorphism.

- Some vertex is unlabeled.
- There is an edge incident to a degree 8 vertex and a degree 4 vertex.
- Two of the edges form a right angle.
- The negation of a property that is preserved under isomorphism.
- There are two simple cycles that share at least one vertex.
- There are two connected components.

Problem 4 (4 points). Circle all the graphs from G_2 to G_5 below that are isomorphic to G_1.



G_1 (a) G_2 (b) G_3

(c) G_4 (d) G_5

Figure 1: Which graphs are isomorphic to G_1?

Problem 5 (2 points). One of the properties below is not a property of all trees. Circle that property.

- Any connected subgraph is a tree.
- Adding an edge between two vertices creates a cycle.
- The number of vertices is one less than twice the number of leaves.
- Removing any edge disconnects the graph.
- If it has at least two vertices, then it has at least two leaves.

Appendix

Stable marriage

A *marriage assignment* or *perfect matching* is a bijection, w : Boys $\rightarrow$ Girls.

A *rogue couple* is a boy, B, and a girl, G, such that B prefers G to his wife, and G prefers B to her husband.

An assignment is *stable* if it has no rogue couples.

The Mating Ritual

The Mating Ritual takes place over several days. The following events happen each day:

Morning: Each girl stands on her balcony. Each boy stands under the balcony of his favorite among the girls on his list, and he serenades her. If a boy has no girls left on his list, he stays home and does his 6.042 homework.

Afternoon: Each girl who has one or more suitors serenading her, says to her favorite suitor, "We might get engaged. Come back tomorrow." To the others, she says, "No. I will never marry you! Take a hike!"

Evening: Any boy who is told by a girl to take a hike, crosses that girl off his list.

Termination condition: When every girl has at most one suitor, the ritual ends with each girl marrying her suitor, if she has one.

Graph theory

A *simple graph*, G, consists of a nonempty set, V, called the *vertices* of G, and a collection, E, of two-element subsets of V. The members of E are called the *edges* of G.

Two vertices in a simple graph are said to be *adjacent* if they are joined by an edge, and an edge is said to be *incident* to the vertices it joins. The number of edges incident to a vertex is called the *degree* of the vertex.

Theorem. *(Handshaking) The sum of the degrees of the vertices in a graph equals twice the number of edges.*

A *path* in a graph, G, is a sequence of $k \geq 0$ vertices, $v_0, \ldots, v_k$, such that v_i—v_{i+1} is an edge of G for all i where $0 \leq i < k$. The path is said to *start* at v_0, to *end* at v_k, and *length* of the path is defined to be k. An edge, e, is *traversed n times* by the path if there are n different values of i such that edge v_i—v_{i+1} is e.

The path is *simple* iff all the v_i's are different, that is, $v_i = v_j$ only if $i = j$.

A *cycle* is a path that begins and ends with the same vertex. A *simple cycle* is a positive length cycle without repeated vertices except for the beginning and end vertices.

Two vertices in a graph are *connected* iff there is a path that begins with one of the vertices and ends with the other. The *shortest* path between two vertices is always simple.

A graph is *connected* iff every pair of vertices is connected.

A graph with no simple cycles is called *acyclic*. *Trees* are acyclic connected graphs.

A *subgraph*, G', of a graph, G, is a graph whose vertices, V', are a subset of the vertices of G and whose edges are a subset of the edges of G.

A vertex of degree one is called a *leaf*.