# Mini-Quiz Feb. 21

**Your name:**________________________________________

**Circle the name of your TA/LA**:

Chiyoun Jay Jeffrey Jessica Tina

- This quiz is **closed book**. Total time is 25 minutes.
- There are six (6) problems totalling 15 points.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

**DO NOT WRITE BELOW THIS LINE**

| Problem | Points | Grade | Grader |
|---|---|---|---|
| 1 | 3 | | |
| 2 | 2 | | |
| 3 | 3 | | |
| 4 | 2 | | |
| 5 | 2 | | |
| 6 | 3 | | |
| Total | 15 | | |

**Problem 1 (3 points).** Next to each of the following propositional formulas, indicate whether it is valid (V), satisfiable but not valid (S), or not satisfiable (N).

$$
\begin{array}{rcll}
(P \wedge Q) & \longrightarrow & (P \vee Q \vee R) & \underline{\qquad\quad} \\
(P \vee Q \vee R) & \longrightarrow & (\overline{P} \wedge \overline{Q} \wedge \overline{R}) & \underline{\qquad\quad} \\
(P \wedge Q \wedge R) & \longleftrightarrow & (P \vee Q \vee R) & \underline{\qquad\quad}
\end{array}
$$

**Problem 2 (2 points).** Let $A$ and $B$ be finite sets.

**(a)** **(1 point)** What relation among $\leq, <, =, \geq$, or $>$, best describes the relationship between

$$|A| + |B| \text{ and } |A \cup B|\,? \quad \underline{\qquad\quad}$$

**(b)** **(1 point)** One of the following conditions holds if and only if $|A| + |B| = |A \cup B|$. Circle this condition.

- $A \cap B = \emptyset$
- $A \cup B = \emptyset$
- $A - B = \emptyset$
- $A \subseteq B$

**Problem 3 (3 points).** Circle those logical formulas below, if any, that are implied by the formula $\forall x\, \exists y. P(x, y)$. (Heads up: the last formula ends with $P(y, x)$.)

$$\exists y\, \forall x. P(x, y)$$

$$\forall y\, \exists x. P(x, y)$$

$$\forall y\, \exists x. P(y, x)$$

**Problem 4 (2 points).** Define a bijection between the nonnegative integers and all the integers.

**Problem 5 (2 points).** The following proof ends with a contradiction, so unless Math is broken, this proof must be mistaken. In a sentence or two, explain what the mistake is.

Let $x$ be a variable ranging over all sets, and define

$$W ::= \{x \mid x \notin x\} .$$

So by definition,

$$x \in W \quad \text{iff} \quad x \notin x,$$

for every set, $x$. Letting $x$ be $W$, we get the contradiction

$$W \in W \quad \text{iff} \quad W \notin W.$$

**Problem 6 (3 points).** Prove that $\sqrt[3]{2}$ is irrational. You may assume the fact that if a positive integer power of an integer is even, then the integer is even. Your answer will be graded primarily on the clarity of your argument.

## Glossary

| symbol | meaning |
|---|---|
| $\wedge$ | AND |
| $\vee$ | OR |
| $\longrightarrow$ | IMPLIES |
| $\neg$ | NOT |
| $\longleftrightarrow$ | EQUIVALENT |
| $\overline{P}$ | NOT $P$ |
| $\cup$ | SET UNION |
| $\cap$ | SET INTERSECTION |
| $\lvert S\rvert$ | size of finite set $S$ |
| $\exists$ | EXISTS |
| $\forall$ | FOR ALL |
| $\in$ | is a member of |
| $\subseteq$ | is a subset of |