In-Class Problems Week 9, Fri.

Problem 1. A license plate consists of either:

- 3 letters followed by 3 digits (standard plate)
- 5 letters (vanity plate)
- 2 characters – letters or numbers (big shot plate)

Let L be the set of all possible license plates.

(a) Express L in terms of

$$\mathcal{A} = \{A, B, C, \ldots, Z\}$$
$$\mathcal{D} = \{0, 1, 2, \ldots, 9\}$$

using unions ($\cup$) and set products ($\times$).

(b) Compute $|L|$, the number of different license plates, using the sum and product rules.

Problem 2. **(a)** How many of the billion numbers in the range from 1 to 10^9 contain the digit 1? (*Hint:* How many don't?)

(b) There are 20 books arranged in a row on a shelf. Describe a bijection between ways of choosing 6 of these books so that no two adjacent books are selected and 15-bit sequences with exactly 6 ones.

Problem 3. A *numbered tree* is a tree whose vertex set is $\{1, 2, \ldots, n\}$ for some $n \geq 2$. We define the *code* of the numbered tree to be a sequence of $n - 2$ integers from 1 to n obtained by the following recursive process:

> If $n = 2$, stop—the code is the empty sequence. Otherwise, write down the *father* of the largest leaf[1], delete this *leaf*, and continue the process on the resulting smaller tree.

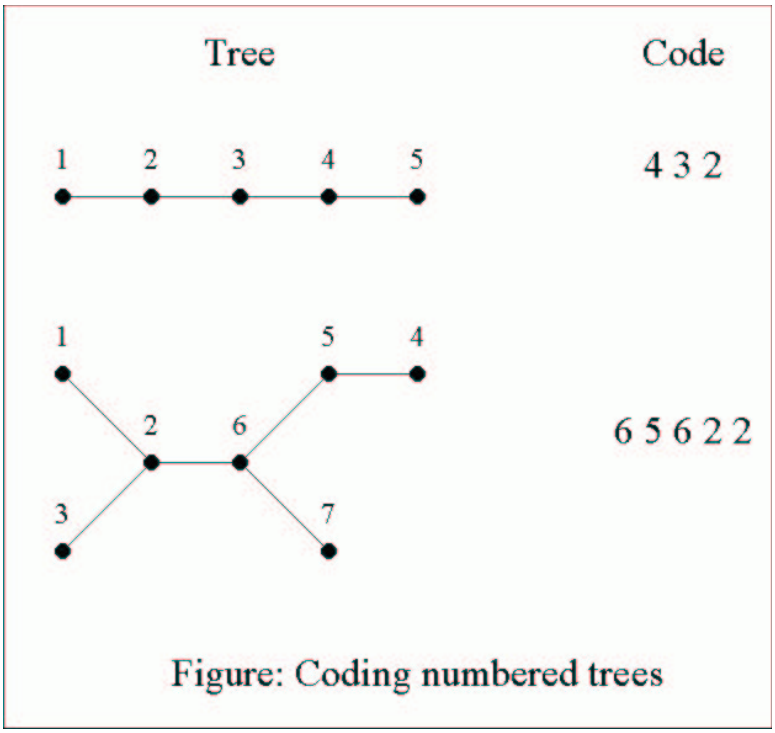

Figure 1:

For example, the codes of a couple of numbered trees are shown in the Figure 1.

(a) Describe a procedure for reconstructing a numbered tree from its code.

(b) How many numbered trees with n vertices are there? Justify your answer assuming the result of the previous problem part.

[1]The necessarily unique node adjacent to a leaf is called its *father*.