In-Class Problems Week 5, Mon.

Problem. See if you can come up with a stable marriage assignment given the following preferences. You are not expected to know/remember the Dating Protocol that solves this problem and which is about to be covered in lecture. (And if you do remember the protocol, don't spoil your teammates' fun by telling them.)

$boys$	$girls$
$1 : CBEAD$	$A : 35214$
$2 : ABECD$	$B : 52143$
$3 : DCBAE$	$C : 43512$
$4 : ACDBE$	$D : 12345$
$5 : ABDEC$	$E : 23415$

Problem 1. Four Students want separate assignments to four VI-A Companies. Here are their preference rankings:

Student	Companies
Albert:	HP, Bellcore, AT&T, Draper
Jeff:	AT&T, Bellcore, Draper, HP
Tina:	HP, Draper, AT&T, Bellcore
Jay:	Draper, AT&T, Bellcore, HP

Company	Students
AT&T:	Jay, Albert, Tina, Jeff
Bellcore:	Tina, Jeff, Albert, Jay
HP:	Jay, Tina, Albert, Jeff
Draper:	Jeff, Jay, Tina, Albert

(a) Use the Mating Ritual (given in the Appendix) to find *two* stable assignments of Students to Companies.

(b) Describe a simple procedure to determine whether any given stable marriage problem has a unique solution, that is, only one possible stable matching.

Problem 2. You heard that an invariant of the Mating ritual is:

> For every girl, G, and every boy, B, if G is crossed off B's list, then G has a favorite suitor and she prefers him over B.

Use the invariant to prove that the Mating Algorithm produces stable marriages. (Don't look up the proof in the Course Notes.)

Problem 3. Suppose there are more boys than girls.

(a) Define what a stable matching should mean in this case.

(b) Explain why applying the Mating Ritual in this case will yield a stable matching in which every girl is married.

Appendix: The Mating Ritual

The *Mating Ritual* takes place over several days. The following events happen each day:

Morning: Each girl stands on her balcony. Each boy stands under the balcony of his favorite among the girls on his list, and he serenades her. If a boy has no girls left on his list, he stays home and does his 6.042 homework.

Afternoon: Each girl who has one or more suitors serenading her, says to her favorite suitor, "We might get engaged. Come back tomorrow." To the others, she says, "No. I will never marry you! Take a hike!"

Evening: Any boy who is told by a girl to take a hike, crosses that girl off his list.

Termination condition: When every girl has at most one suitor, the ritual ends with each girl marrying her suitor, if she has one.