In-Class Problems Week 5, Fri.

Problem 0. (From Wednesday's class problems.)

(a) Prove that in every graph, there are an even number of vertices of odd degree.

Hint: The Handshaking Theorem.

(b) Conclude that at a party where some people shake hands, the number of people who shake hands an odd number of times is an even number.

Problem 1. Procedure $Mark$ starts with a connected, simple graph with all edges unmarked and then marks some edges. At any point in the procedure a path that traverses only marked edges is called a *fully marked* path, and an edge that has no fully marked path between its endpoints is called *eligible*.

Procedure $Mark$ simply keeps marking eligible edges, and terminates when there are none.

Prove that $Mark$ terminates, and that when it does, the set of marked edges forms a spanning tree of the original graph.

Problem 2. Prove that K_n is $(n-1)$-connected for $n > 1$.

Problem 3. Prove that a graph is a tree iff it has a unique simple path between any two vertices.