# In-Class Problems Week 2, Mon.

**Problem 1.** [1] A set of propositions is *consistent* if there is a possible situation in which they are all true. This problem examines whether the following specifications are consistent.

1. If the file system is not locked, then
   (a) new messages will be queued.
   (b) new messages will be sent to the messages buffer.
   (c) the system is functioning normally, and conversely, if the system is functioning normally, then the file system is not locked.
2. If new messages are not queued, then they will be sent to the messages buffer.
3. New messages will not be sent to the message buffer.

**(a)** Begin by translating the parts of the specification into propositional formulas using four propositional variables:

$$\begin{aligned} L &::= \text{file system locked}, \\ Q &::= \text{new messages are queued}, \\ B &::= \text{new messages are sent to the message buffer}, \\ N &::= \text{system functioning normally}. \end{aligned}$$

**(b)** To be precise, the specification is consistent if there is a way to assign a truth value to each of the variables $L, Q, B, N$, so that every one of the propositional formulas from the previous part are true. Explain how to use a *truth table* to determine whether the specification is consistent.

**(c)** Use *simple reasoning by cases* to find a truth assignment that confirms that this system specification is consistent. Explain why there is only one such assignment.



[1] From Rosen, 5th edition, Exercise 1.1.36

**Problem 2.** If we raise an irrational number to an irrational power, can the result be rational? Show that it can by considering $\sqrt{2}^{\sqrt{2}}$ and arguing by cases.

**Problem 3.** Generalize the proof from lecture (reproduced below) that $\sqrt{2}$ is irrational, for example, how about $\sqrt[3]{2}$? Remember that an irrational number is a number that cannot be expressed as a ratio of two integers.

**Theorem.** $\sqrt{2}$ *is an irrational number.*

*Proof.* The proof is by contradiction. Assume for purpose of contradiction that $\sqrt{2}$ is rational.

Then we can write $\sqrt{2} = m/n$ where $m$ and $n$ are integers and the fraction is in lowest terms. Squaring both sides gives $2 = m^2/n^2$, so $2n^2 = m^2$. This implies that $m^2$ is even, and hence that $m$ is even; that is, $m$ is a multiple of $2$. But that means $m^2$ is actually a multiple of 4, say $m^2 = 4k$.

Now we have $2n^2 = m^2 = 4k$, so $n^2 = 2k$. So $n^2$ is even, and hence $n$ is even. But since $m$ and $n$ are both even, the fraction $m/n$ is not in lowest terms, a contradiction. $\square$