

## In-Class Problems Week 1, Wed.

**Problem 1.** Identify exactly where the bugs are in each of the following bogus proofs.<sup>1</sup>

(a)  $1/8 > 1/4$ .

*Bogus proof.*

$$\begin{aligned}3 &> 2 \\3 \log_{10}(1/2) &> 2 \log_{10}(1/2) \\ \log_{10}(1/2)^3 &> \log_{10}(1/2)^2 \\ (1/2)^3 &> (1/2)^2,\end{aligned}$$

and the claim now follows by the rules for multiplying fractions. □

(b)  $1\text{¢} = \$0.01 = (\$0.1)^2 = (10\text{¢})^2 = 100\text{¢} = \$1$ .

**Problem 2.**

**Proposition (Arithmetic-Geometric Mean Inequality).** For all nonnegative real numbers  $a$  and  $b$

$$\frac{a+b}{2} \geq \sqrt{ab}.$$

What is wrong with the following proof of this proposition?

*Bogus proof.*

$$\begin{aligned}\frac{a+b}{2} &\stackrel{?}{\geq} \sqrt{ab} \\ a+b &\stackrel{?}{\geq} 2\sqrt{ab} \\ a^2 + 2ab + b^2 &\stackrel{?}{\geq} 4ab \\ a^2 - 2ab + b^2 &\stackrel{?}{\geq} 0 \\ (a-b)^2 &\geq 0\end{aligned}$$

The last statement is true because  $a - b$  is a real number, and the square of a real number is never negative. This proves the claim. □