In-Class Problems Week 1, Fri.

Problem 1. Albert announces that he plans a surprise 6.042 quiz next week. His students wonder if the quiz could be next Friday. The students realize that it obviously cannot, because if it hadn't been given before Friday, everyone would know that there was only Friday left on which to give it, so it wouldn't be a surprise any more.

So the students ask whether Albert could give the surprise quiz Thursday? They observe that if the quiz wasn't given *before* Thursday, it would have to be given *on* the Thursday, since they already know it can't be given on Friday. But having figured that out, it wouldn't be a surprise if the quiz was on Thursday either. Similarly, the students reason that the quiz can't be on Wednesday, Tuesday, or Monday. Namely, it's impossible for Albert to give a surprise quiz next week. All the students now relax having concluded that Albert must have been bluffing.

And since no one expects the quiz, that's why, when Albert gives it on Tuesday next week, it really is a surprise!

What do you think is wrong with the students' reasoning?

Problem 2. Identify the antecedents and conclusions of each of the following deductions and translate them into propositional logic notation using logical operators:

$$
\begin{aligned}
\wedge &::= \text{AND},\\
\vee &::= \text{OR},\\
\neg &::= \text{NOT},\\
\longrightarrow &::= \text{IMPLIES},\\
\longleftrightarrow &::= \text{IFF (if and only if)}
\end{aligned}
$$

This may require that you "pin down" a statement that could be interpreted in more than one way.

Identify which of the deductions are sound ones.

(a) Jane and Pete won't both win the math prize. Pete will win either the math prize or the chemistry prize. Jane will win the math prize. Thus, Pete will win the chemistry prize.

(b) The main course will be beef or fish. The vegetable will be peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.

(c) Either John or Bill is telling the truth. Either Sam or Bill is lying. Thus, either John is telling the truth or Sam is lying.

(d) Either sales will go up and the boss will be happy, or expenses will go up and the boss won't be happy. Therefore, sales and expenses will not both go up.

Problem 3. Boolean logic comes up in digital circuit design using the convention that **T** corresponds to 1 and **F** to 0. For example, suppose we want to describe a circuit with $n+1$ inputs $a_n, a_{n-1}, \ldots, a_1, a_0$ which are the $n+1$ bits of the binary representation of an integer, k, between 0 and $2^{n+1}-1$. We want outputs $o_{n+1}, o_n, \ldots, o_1, o_0$ to be the bits of $k+b$ where b is a single bit.

For example, for $n = 1$, the formulas

$$
\begin{aligned}
o_0 &::= a_0 \oplus b \\
c_1 &::= a_0 \wedge b && \text{the carry into column 1} \\
o_1 &::= c_1 \oplus a_1 \\
c_2 &::= c_1 \wedge a_1 && \text{the carry into column 2} \\
o_2 &::= c_2
\end{aligned}
$$

do the job. Here $\oplus$ is the "mod 2 sum" operator: $a \oplus b$ is 1 iff $a+b$ is odd.

(a) Generalize the example above for any $n \geq 0$. That is, give simple formulas for o_i and c_i for $0 \leq i \leq n+1$.

(b) Write similar definitions for the $n+1$ bits of the sum of two binary numbers $a_n, a_{n-1}, \ldots, a_1, a_0$ and $b_n, b_{n-1}, \ldots, b_1, b_0$.

(c) How many Boolean operations does your system use to calculate the sum?