In-Class Problems Week 12, Mon.

Problem 1. [A Baseball Series] The New York Yankees and the Boston Red Sox are playing a two-out-of-three series. (In other words, they play until one team has won two games. Then that team is declared the overall winner and the series ends.) Assume that the Red Sox win each game with probability $3/5$, regardless of the outcomes of previous games.

Answer the questions below using the four-step method. You can use the same tree diagram for all three problems.

(a) What is the probability that a total of 3 games are played?

(b) What is the probability that the winner of the series loses the first game?

(c) What is the probability that the *correct* team wins the series?

Problem 2. [The Four-Door Deal] Suppose that *Let's Make a Deal* is played according to different rules. Now there are **four** doors, with a prize hidden behind one of them. The contestant is allowed to pick a door. The host must then reveal a different door that has no prize behind it. The contestant is allowed to stay with his or her original door or to pick one of the other two that are still closed. If the contestant chooses the door concealing the prize in this second stage, then he or she wins.

(a) Contestant Stu, a sanitation engineer from Trenton, New Jersey, stays with his original door. What is the probability that he wins the prize?

The tree diagram is awkwardly large. This often happens; in fact, sometimes you'll encounter *infinite* tree diagrams! Try to draw enough of the diagram so that you understand the structure of the remainder.

(b) Contestant Zelda, an alien abduction researcher from Helena, Montana, switches to one of the remaining two doors with equal probability. What is the probability that she wins the prize?

Problem 3. Suppose that the rules of Ultimate Frisbee were revised as follows:

> *Representatives of the two teams each flip a disc. The representative of one team predicts the orientation of his or her own disc by calling "up" or "down" while the discs are in the air. However, if both discs land the same way, then the call does not count and the process starts over.*

Assume that a disc lands face-up with probability p. What is the probability that the caller wins by always saying "up"?

Suggestions: The tree diagram is infinite, so draw only enough to see a pattern. Summing all the winning outcome probabilities directly is difficult. However, a neat trick solves this problem and many others. Let s be the sum of all winning outcome probabilities in the whole tree. Notice that *you can write the sum of all the winning probabilities in certain subtrees as a function of* s. Use this observation to write an equation in s and then solve.

Problem 4. Here are some handy rules for reasoning about probabilities that all follow directly from the Sum Rule for the probabilities of disjoint events. Prove them:

(a) The *Difference Rule*:

$$\Pr\{A - B\} = \Pr\{A\} - \Pr\{A \cap B\}$$

(b) The *Complement Rule*:

$$\Pr\left\{\overline{A}\right\} = 1 - \Pr\{A\}$$

(c) *Inclusion-Exclusion*:

$$\Pr\{A \cup B\} = \Pr\{A\} + \Pr\{B\} - \Pr\{A \cap B\}$$

(d) The *Union Bound*:

$$\Pr\{A \cup B\} \leq \Pr\{A\} + \Pr\{B\}.$$

(e) *Monotonicity*:

$$\text{If } A \subseteq B, \text{ then } \Pr\{A\} \leq \Pr\{B\}.$$

The Four-Step Method

This is a good approach to questions of the form, "What is the probability that ——-?" Intuition *will* mislead you, but this formal approach gives the right answer every time.

1. Find the sample space. (Use a tree diagram.)
2. Define events of interest. (Mark leaves corresponding to these events.)
3. Determine outcome probabilities:
   (a) Assign edge probabilities.
   (b) Compute outcome probabilities. (Multiply along root-to-leaf paths.)
4. Compute event probabilities. (Sum the probabilities of all outcomes in the event.)