Optimum-Path Forest Classifier and Open-Set Optimum-Path Forest Classifier

Institute of Computing
University of Campinas

Pedro Ribeiro Mendes Júnior

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Optimum-path forest classifier

Overview

1. Proposed by Papa et al. (2007, 2009) from Unicamp.
2. Generalization of the Image Foresting Transform (IFT).
3. Graph based. Nodes are training samples.
4. Behavior similar to the kNN (see Souza et al. (2014)).
5. Inherently multiclass.
6. Parameter independent.
7. Closed set.
8. Good results in some classification problems.
9. General methodology.

OPF formalization

$$Z = \{(s_1, \theta(s_1)), (s_2, \theta(s_2)), \ldots, (s_m, \theta(s_m))\}, \tag{1}$$

whereby

- s_i, $i = 1, \ldots, m$, is a feature vector.
- $\theta(s_i)$ is the actual class of s_i, i.e., $\theta(s_i) \in \mathcal{L} = \{\ell_1, \ell_2, \ldots, \ell_n\}$.
- $\mathcal{L}$ is a set of labels representing n distinct classes.

OPF formalization

$$S = \{(s_1^p, \theta(s_1^p)), \ldots, (s_k^p, \theta(s_k^p))\}, \tag{2}$$

such that

- $S \subseteq Z$
- $k \leq m$

- S is the set of *prototypes*.
- How to obtain S?
  - We are going to see.

OPF formalization

- Based on Z, we have the complete graph $A = Z \times Z$.
- Based on S and A, we create an optimum-path forest.
- Each tree of the forest is rooted in a sample in S.

- What is an optimum-path forest?
  - What is an optimum path?
    - How to measure a path?
- How to create the optimum-path forest?

Optimum-path forest creation

How to create the optimum-path forest?

- Based on the IFT algorithm.
    - It is similar to Dijkstra's algorithm.
    - It is for multi-sources.
    - S is the set of sources.

OPF's path cost function

How to measure a path?

$$f_{cost}(\langle s \rangle) = \begin{cases} 0 & \text{if } s \in S \\ +\infty & \text{otherwise,} \end{cases}$$
$$f_{cost}(\pi_{s,u} \cdot (u,t)) = \max\{f_{cost}(\pi_{s,u}), w(u,t)\}. \tag{3}$$

whereby

- $\pi_{s,u} \cdot (u,t)$ denotes the concatenation of a path $\pi_{s,u}$ ending at u and an arc (u,t).
- $w(s,t)$ denotes the Euclidean distance between s and t in the feature space.

Optimum-path definition

What is an optimum path?

- A path $\pi_{s,t}$ with terminus t is said optimum if
  - $f_{cost}(\pi_{s,t}) \leq f_{cost}(\pi_{u,t})$ for all $u \in Z$.
- We say that $P^*(t) = \pi_{s,t}$.

Optimum-path forest definition

What is an optimum-path forest?

- Every optimum path is from $s \in S$, $S \subseteq Z$, according to the f_{cost}.

- Now, we know $P^*(t)$ for every sample $t \in Z$.
- Then, every sample t holds its minimum cost $C(t)$:

$$C(t) = f_{cost}(P^*(t)) \tag{4}$$

Defining S

How to obtain S?

- Create the Minimum Spanning Tree (MST) of $A = Z \times Z$.
- $S = \{s \text{ such that } s \in Z \text{ and } \theta(s) \neq \theta(t) \text{ for some } t \in N(s)\}$ whereby $N(s)$ are the neighbors of s in the MST.

Algorithm 1 OPF algorithm.

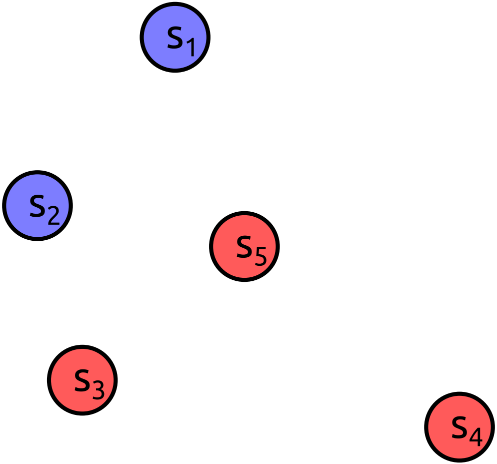
```
Require: Set of training samples $Z$
Require: Set of prototypes $S$
Ensure: $S \subseteq Z$
Output: Optimum-path forest $P$
Output: Cost map $C$
Output: Label map $L$
  for all $s \in Z \backslash S$ do
      $C(s) \leftarrow +\infty$
  end for
  for all $s \in S$ do
      $C(s) \leftarrow 0$; $P(s) \leftarrow nil$; $L(s) \leftarrow \theta(s)$
      Insert $s$ in $Q$
  end for
  while $Q$ is not empty do
      Remove $s$ from $Q$ such that $C(s) \leq C(t)$ for all $t \in Q$, $s \neq t$
      for all $t \in Z$ such that $s \neq t$ do
          $cst \leftarrow max\{C(s), w(s,t)\}$
          if $cst < C(t)$ then
              if $C(t) \neq +\infty$ then
                  Remove $t$ from $Q$
              end if
              $P(t) \leftarrow s$; $L(t) \leftarrow L(s)$; $C(t) \leftarrow cst$
              Insert $t$ in $Q$
          end if
      end for
  end while
```

Prediction phase

$$L(s) = L(\mathcal{R}(\arg\min_{\forall t \in Z}\{\max\{C(t), w(s,t)\}\})) \qquad (5)$$

whereby

- $\mathcal{R}(s) \in S$ is the root of the tree.
- $L(s)$ is the predicted class of the sample s.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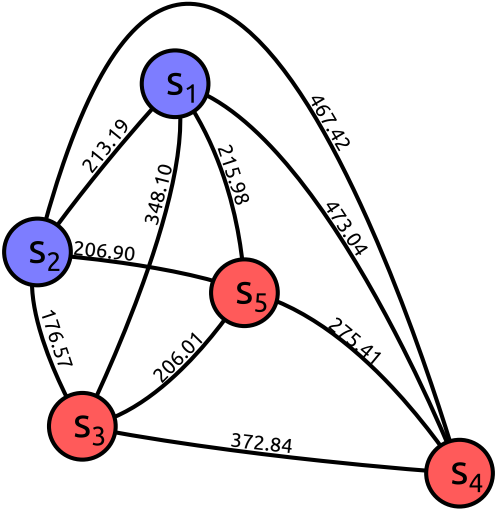
s_2
s_5
s_3
s_4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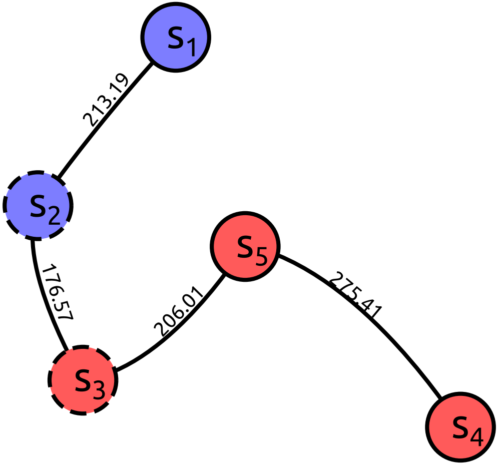
206.01
275.41
372.84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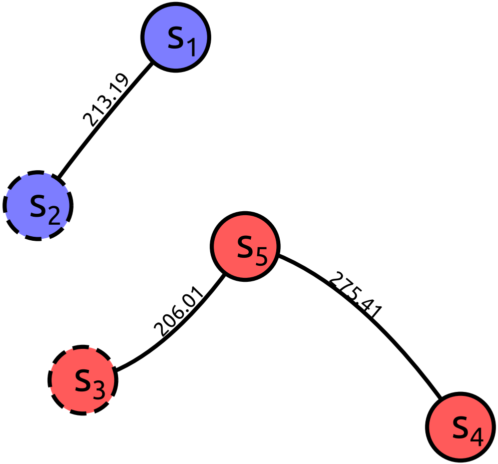
206.01
275.41
s_3
s_4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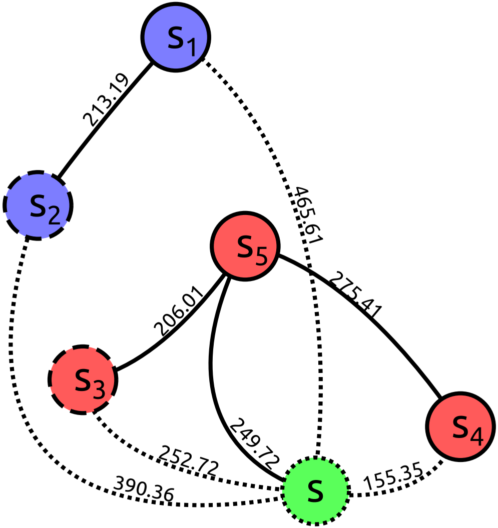
s_5
206.01
275.41
s_3
s_4


s_1
s_2
s_3
s_4
s_5
s
213.19
465.61
206.01
275.41
249.72
252.72
390.36
155.35

Open-set recognition

Open-set recognition problem

- Recognition vs. classification
- A limited amount of classes are available for training

Openness of a problem (Scheirer et al., 2013)

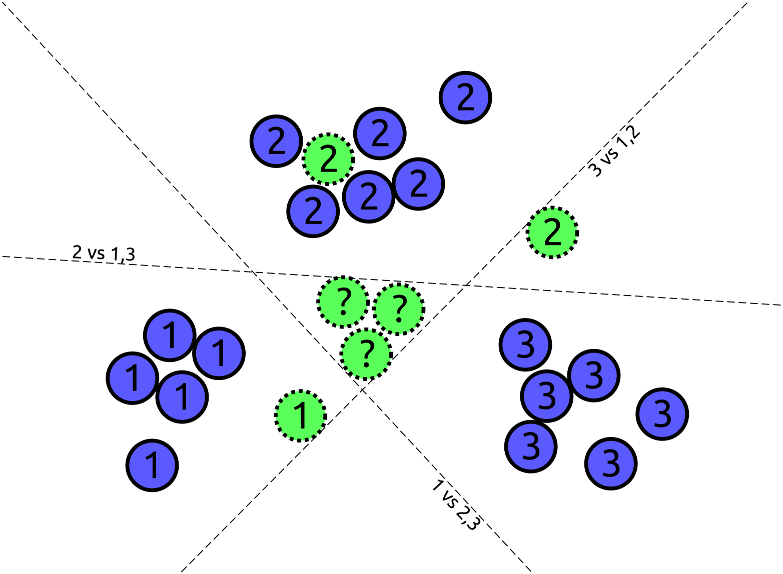
$$\mathit{openness} = 1 - \sqrt{\frac{|\text{training classes}|}{|\text{testing classes}|}}$$

Types of error

- Misclassification
- False unknown
- False known

Traditional approach

- Threshold on the best similarity score
- Multiclass-from-binary SVM using one-vs-all approach


2
2
2
2
2
2
2
2
3 vs 1,2
2 vs 1,3
?
?
?
1
1
1
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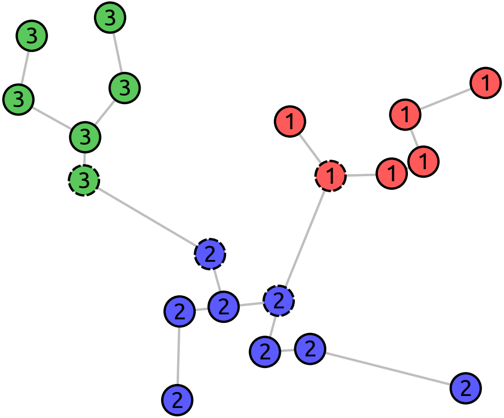
3
3
3
3
3
1 vs 2,3

Open-set optimum-path forest classifier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3
3
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3
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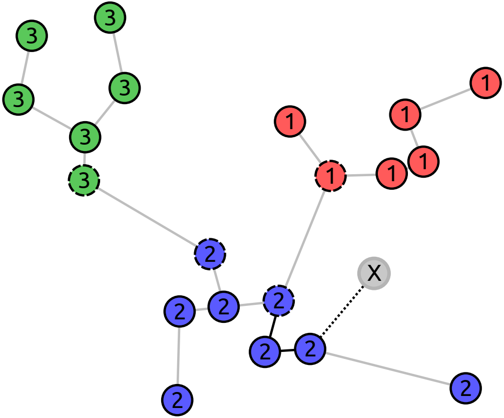
1
1
1
1
1
1
2
2
2
2
2
2
2
2
2


3
3
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3
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1
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1
1
1
X
2
2
2
2
2
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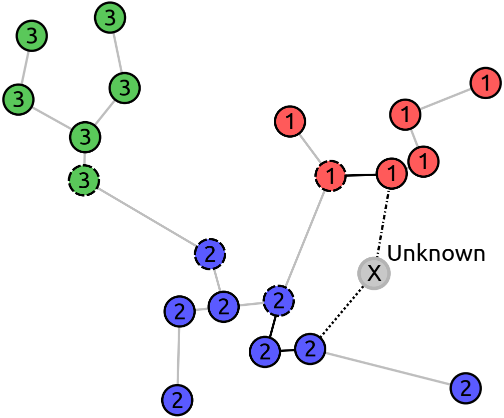
2
2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1
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X
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2
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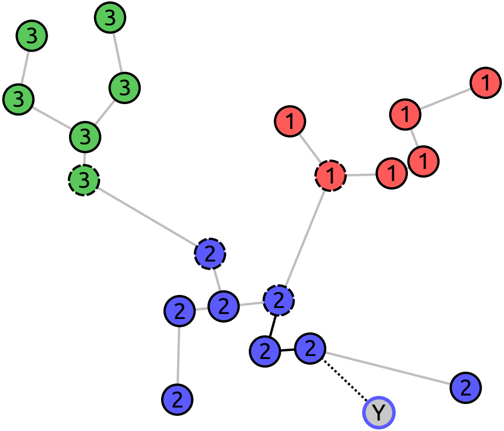
2
2
2
2


3
3
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3
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1
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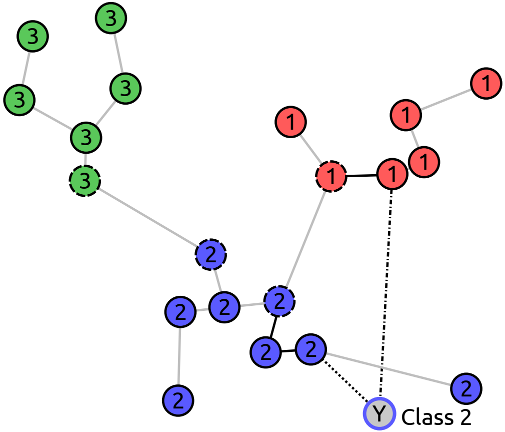
1
1
1
2
2
2
2
2
2
2
2
Y


3
3
3
3
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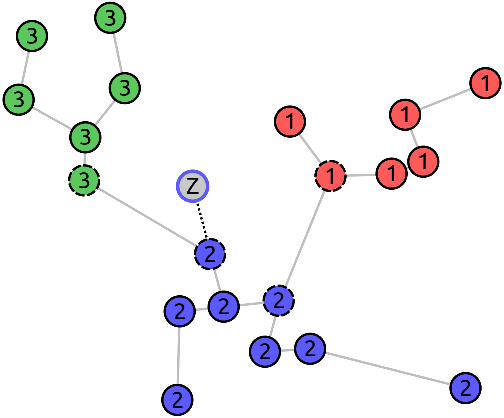
3
1
1
1
1
1
1
2
2
2
2
2
2
2
2
Y
Class 2


3
3
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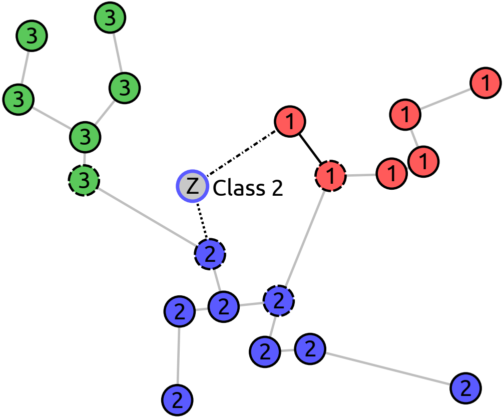
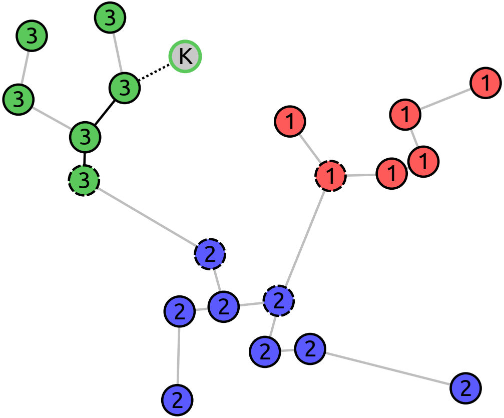
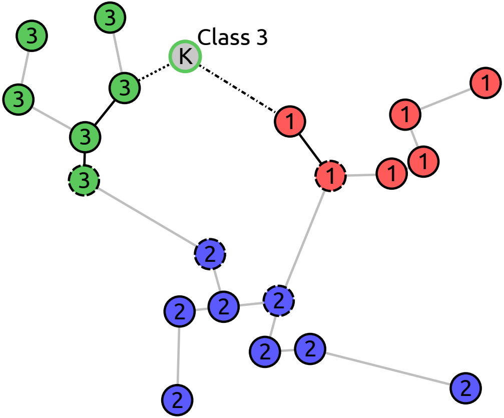
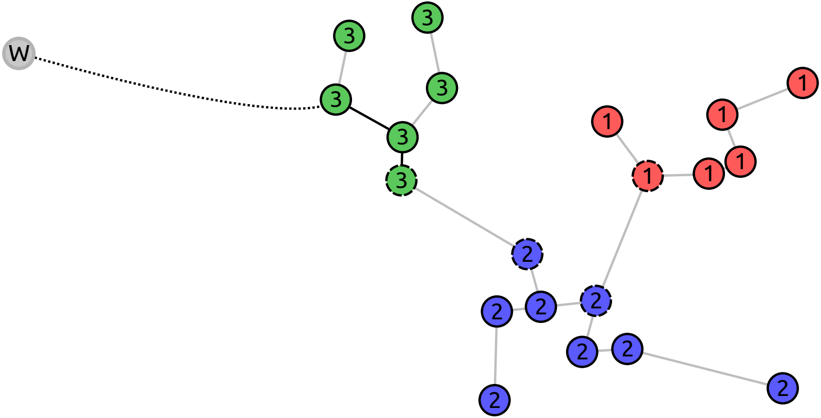
3
Z
1
1
1
1
1
1
2
2
2
2
2
2
2
2
2


3
3
3
3
3
3
1
1
1
1
1
1
Z
Class 2
2
2
2
2
2
2
2
2
2


3
3
K
3
3
3
3
1
1
1
1
1
1
2
2
2
2
2
2
2
2


Class 3
K
3
3
3
3
3
3
3
1
1
1
1
1
1
2
2
2
2
2
2
2
2
2


W
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3
3
1
1
1
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1
1
2
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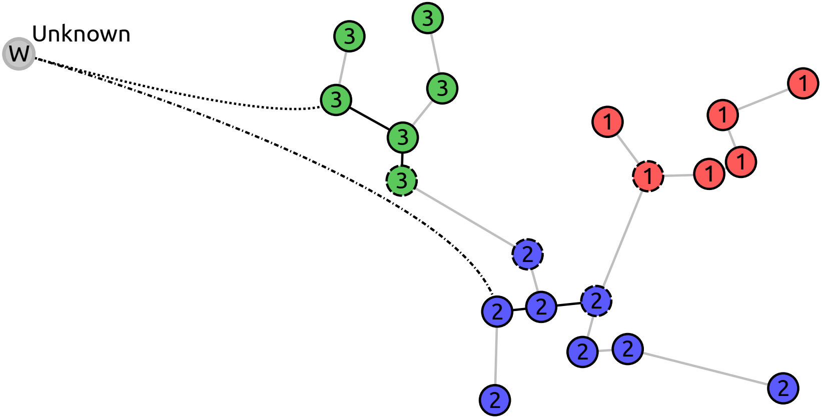
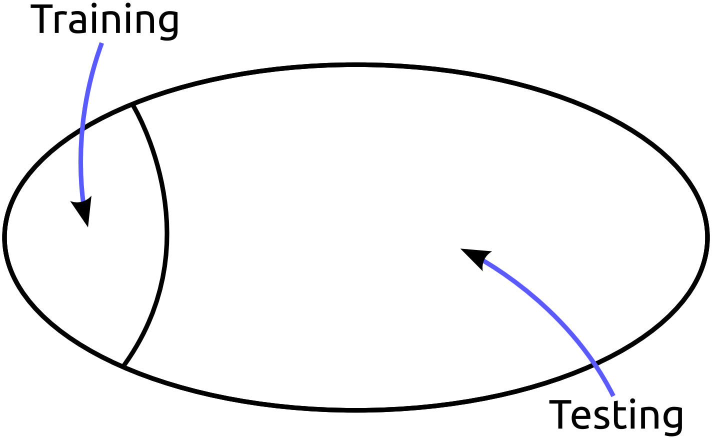
2
2
2
2


Unknown
W
3
3
3
3
3
3
1
1
1
1
1
1
2
2
2
2
2
2
2
2

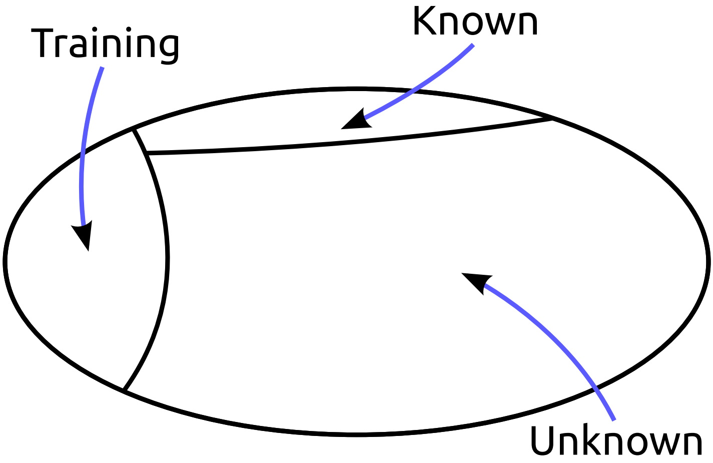
Parameter optimization of OSOPF^2

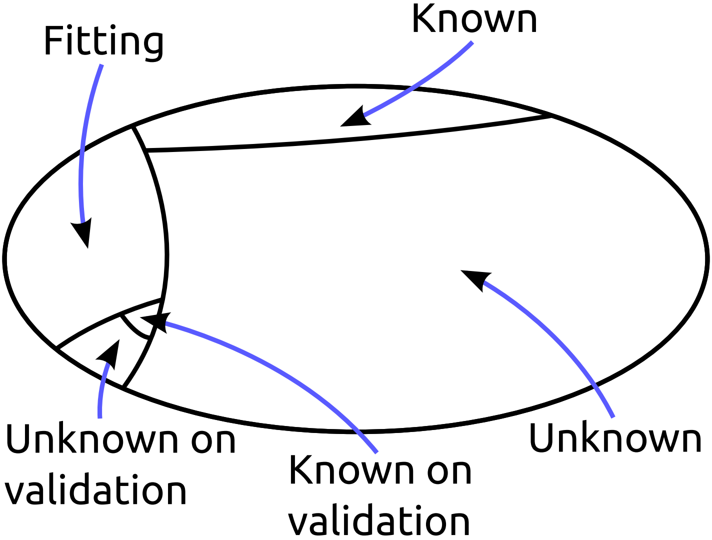
- Find the threshold T
- Simulation of the open-set scenario



Training

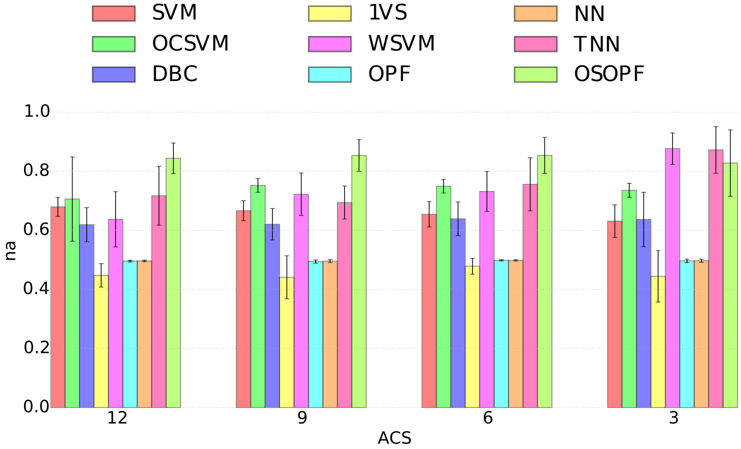
Testing


Training
Known
Unknown

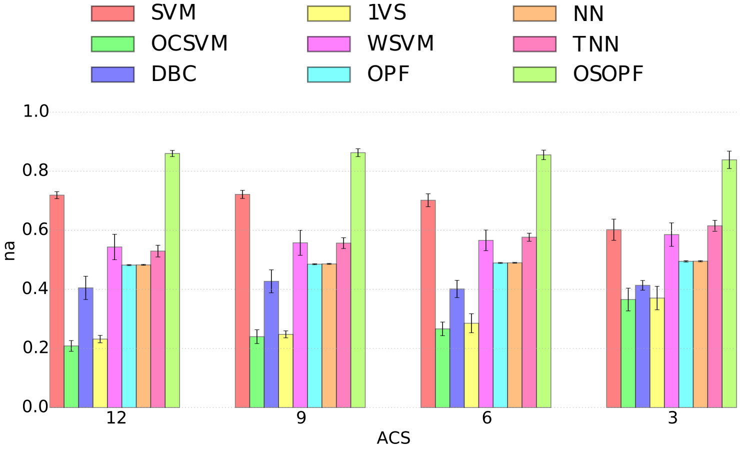

Fitting
Known
Unknown on validation
Known on validation
Unknown

Open set vs. closed set

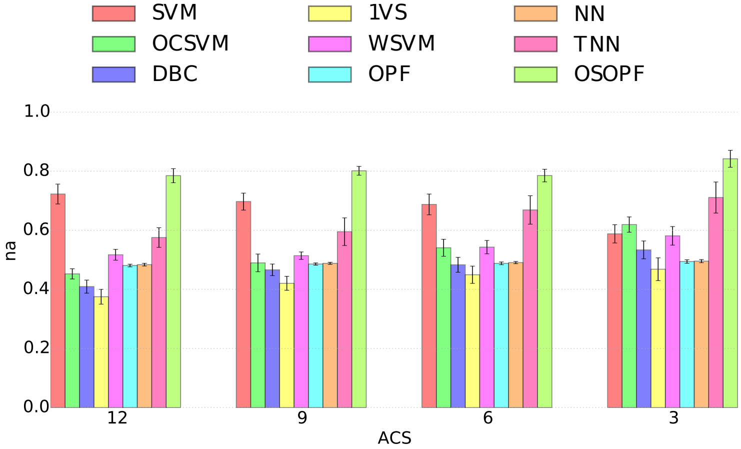
Classification performance – *ALOI* – NA



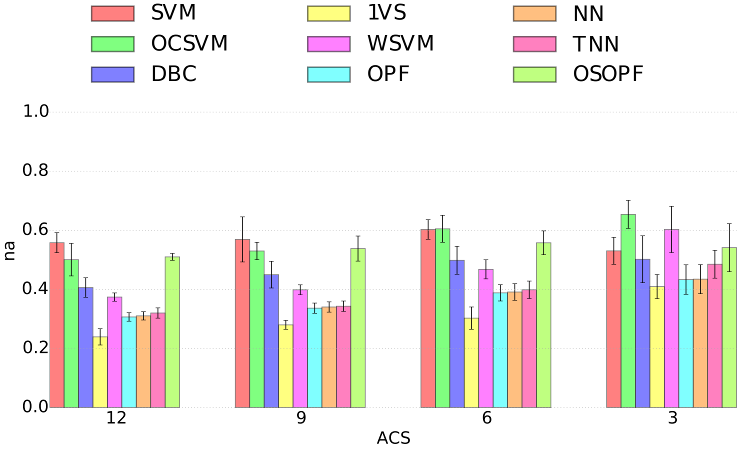
Classification performance – *Auslan* – NA



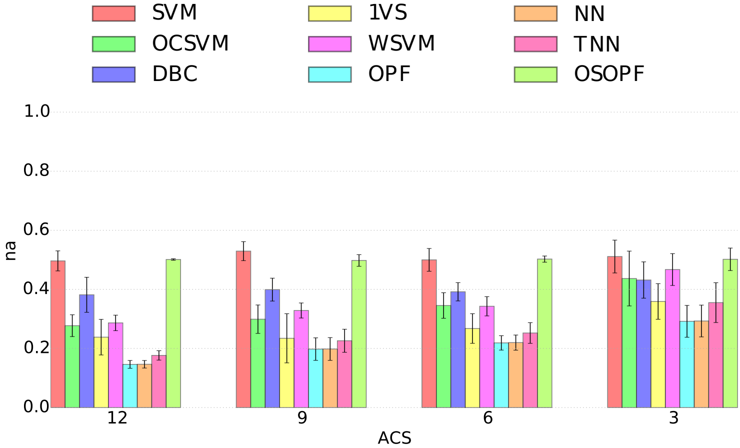
Classification performance – *letter* – NA



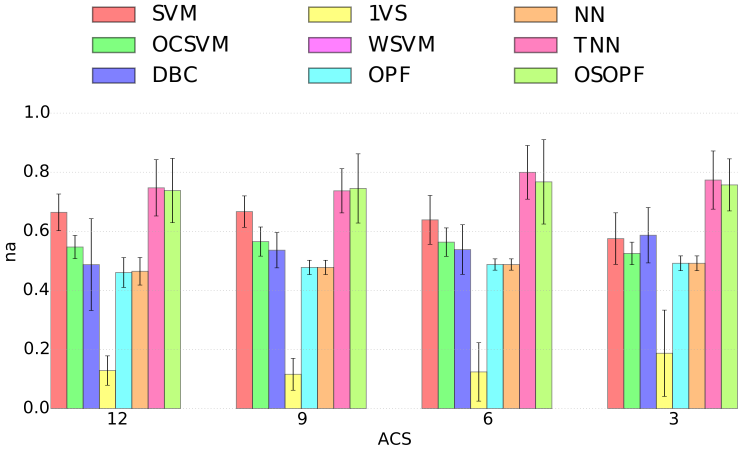
Classification performance – *15-Scene* – NA

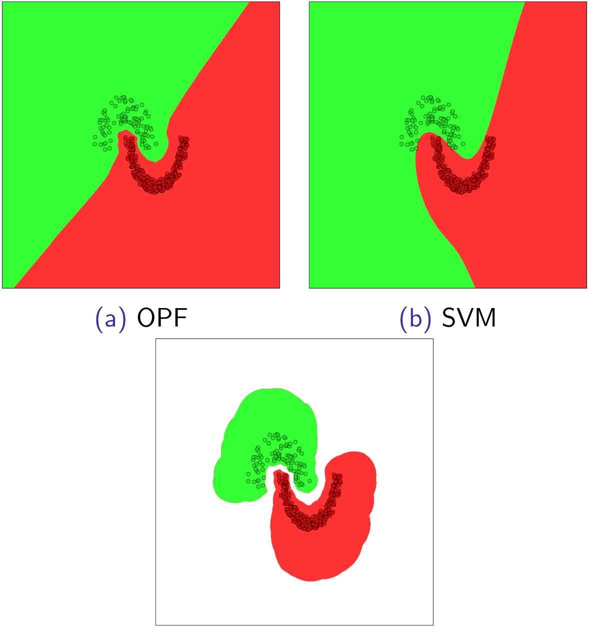


Classification performance – *Caltech-256* – NA



Classification performance – *ukbench* – NA

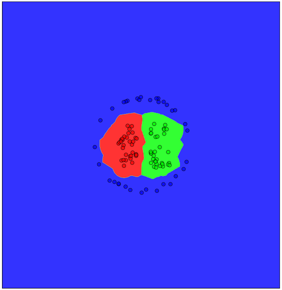




(a) OPF

(b) SVM

(c) OSOPF



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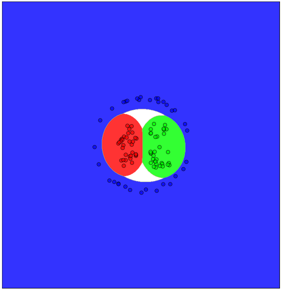

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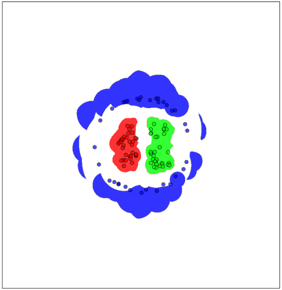

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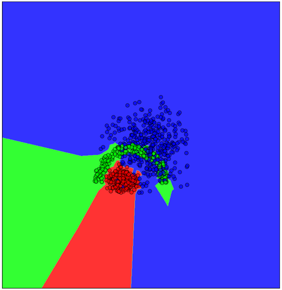

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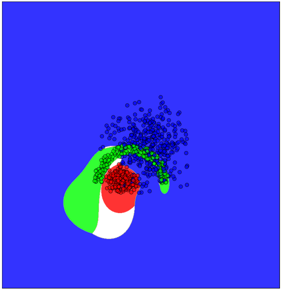

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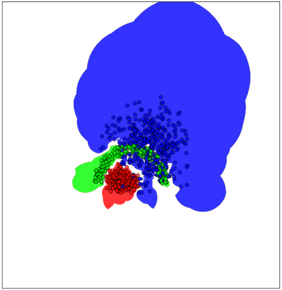

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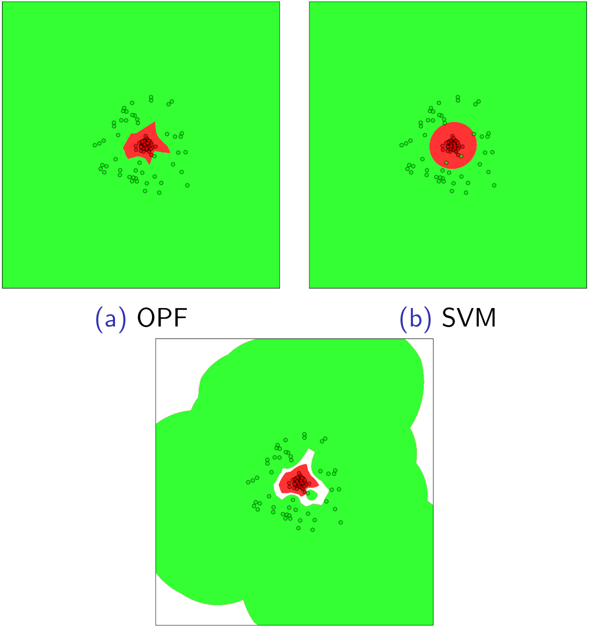
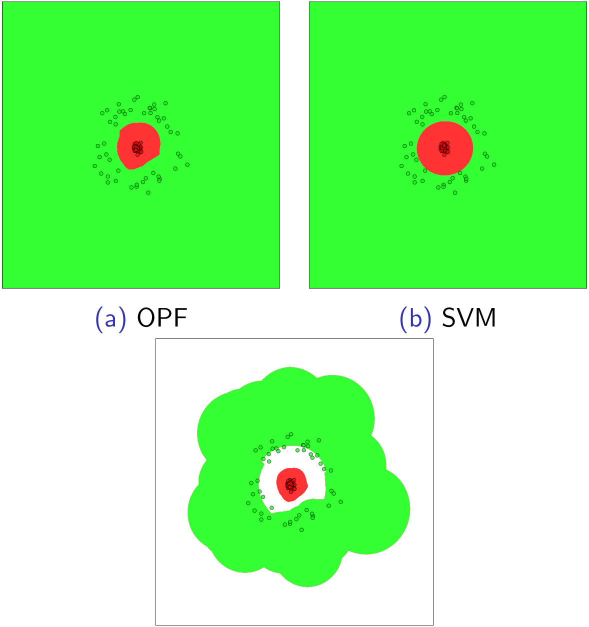


(a) OPF

(b) SVM

(c) OSOPF



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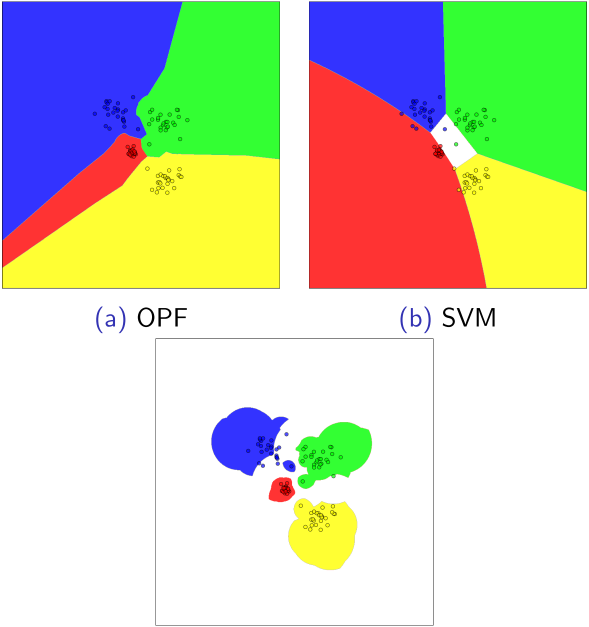
(b) SVM

(c) OSOPF



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(b) SVM

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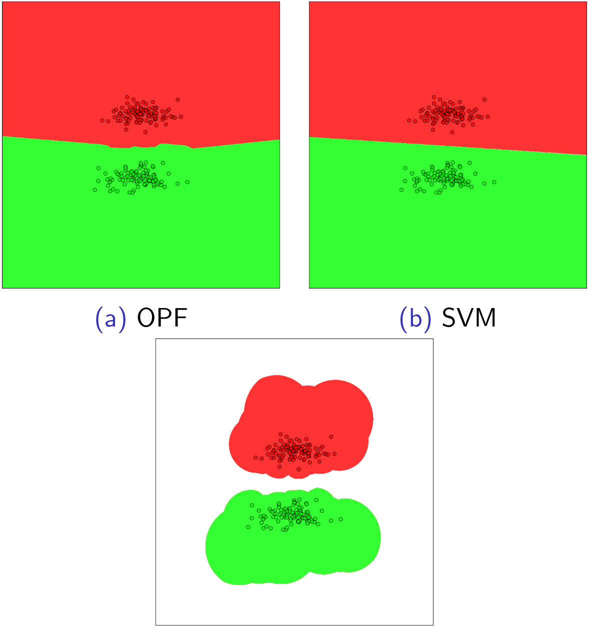

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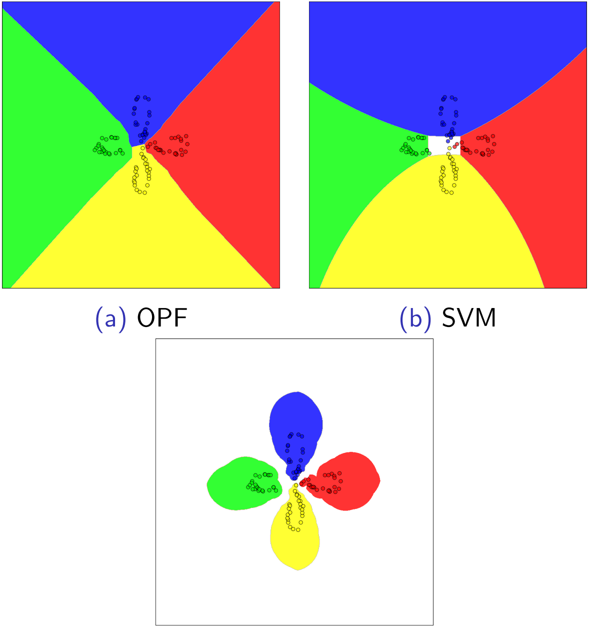

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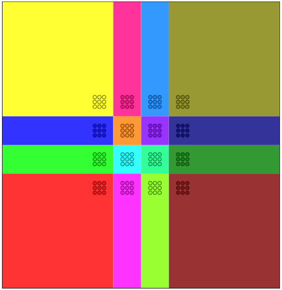

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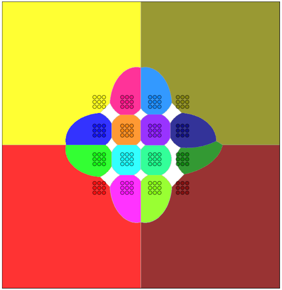

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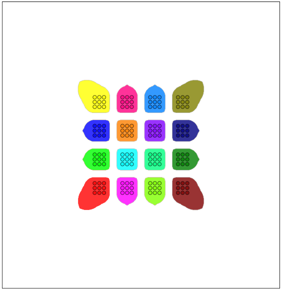

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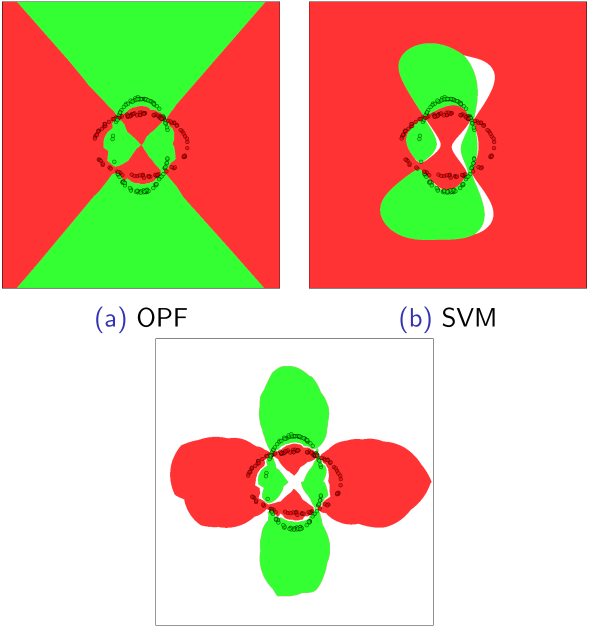

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(b) SVM

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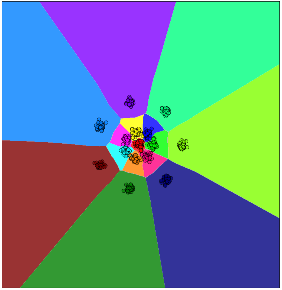

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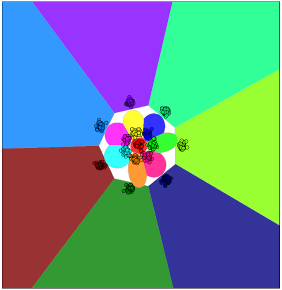

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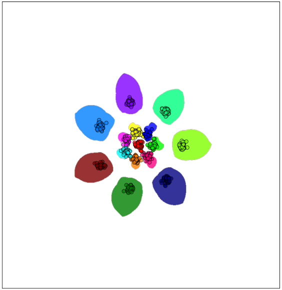

(c) OSOPF

Mendes Júnior, P. R., de Souza, R. M., Werneck, R., Stein, B., Pazinato, D., de Almeida, W., Penatti, O., Torres, R., and Rocha, A. d. R. (2014a). Method for multiclass classification in open-set scenarios and uses thereof. USPTO 14532580.

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