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# On the Coordinator's Rule for Fast Paxos

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## Abstract

Fast Paxos is an algorithm for consensus that works by a succession of rounds, where each round tries to decide a value  $v$  that is consistent with all past rounds. Rounds are started by a coordinator process and consistency is guaranteed by the rule used by this process for the selection of  $v$  and by the properties of process sets called quorums. We show a simplified version of this rule for the specific case where the quorums are defined by the cardinality of these process sets. This rule is of special interest for implementors of the algorithm.

## 1 Introduction

The problem of deciding a single value out of a set of values proposed by processes is known as the *consensus* problem. This problem is easy to solve in the absence of failures, but it is impossible to solve in an asynchronous distributed system even if a single process fails by permanently stopping [1]. A better approximation of the failures that processes of a real distributed system can suffer is the one where processes stop but may later recover. Unfortunately, the impossibility also holds for these systems. One of the ways to get around the impossibility is to design algorithms that do not violate what they are allowed to do, or dually, what they may not do while the system behaves asynchronously. But they are certain to make progress, that is, to do what they must do, if the system behaves partially synchronously for long enough periods. Designing consensus algorithms for the asynchronous crash-recovery model is a difficult task of practical interest and probably Paxos [2] and Fast Paxos [3] are the most studied solutions so far.

Fast Paxos solves the consensus problem through a succession of rounds that lead to the choice of the consensus value. In each round a distinguished process, the *coordinator*, is responsible for picking a single value using a rule that is based on quorums of processes. Quorums of successive rounds are used to guarantee that if a single value has been chosen or might ever be chosen in previous rounds then the same value is going to be chosen in the

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current round. Thus, quorums are fundamental to the correctness of Fast Paxos because they are ultimately responsible for the validity of consensus.

Lamport [3] shows how quorums can be characterized using the cardinality of sets of processes, defining what minimum number of processes represents a quorum. However, he defines the coordinator’s rule in terms of quorum sets and general set operations. This complete characterization of the coordinator’s rule is perfect for the purposes of his work, but it does not address thoroughly the needs of a programmer who wants to implement it. Therefore, the main contributions of this paper are (i) an interpretation of the Fast Paxos coordinator’s rule only in terms of the cardinalities of quorum sets, and (ii) its simplification. The simplified interpretation is efficient, easier to implement and test; it can help developers to create reliable implementations of Fast Paxos. This is important, as the use of Fast Paxos to build fault-tolerant applications is bound to require the execution of a very large number of consensus instances.

## 1.1 Fast Paxos

Before detailing the coordinator’s rule, it is useful to give a very brief overview of Fast Paxos; a complete description of it can be found in [3]. The algorithm is easier to explain in terms of reactive agents that represent a role, such that a single process can enact multiple agents, with each one of them playing a different role. An agent can enact one of the following main roles: a *proposer* that can propose values by sending them to acceptors, an *acceptor* that chooses a single value, or a *learner* that learn what value has been chosen.

To solve consensus, Fast Paxos agents execute multiple rounds, each round has a coordinator and may be either a *fast* round or a *classic* round. Positive integers are used to uniquely identify rounds, each identifier determines the coordinator and indicates the round type: fast or classic. Regardless of its type, each round progresses through two phases with two steps each:

- In Phase 1a the coordinator sends a message requesting every acceptor to participate in round  $i$ . An acceptor accepts the invitation if it has not already accepted to participate in round  $j < i$ , otherwise it declines the invitation by simply ignoring it.
- In Phase 1b every acceptor that has accepted the invitation answers to the coordinator with a reply that contains the round number and the value of the last vote it has cast for a value, or *null* if it has not voted.
- In Phase 2a, if the coordinator of round  $i$  has received answers from a quorum of acceptors then it executes its rule on the set of values suggested by acceptors in Phase 1b and picks a single value  $v$ . It then asks the acceptors to cast a vote for  $v$  in round  $i$ , if  $v$  is not *null*, otherwise, if the round is fast the coordinator sends a *any* message to the proposers indicating that any value can be chosen in round  $i$ . In this case, the proposers ask the acceptors to cast a vote for a value  $v$  of their choice in round  $i$ .
- In Phase 2b, after receiving a request to cast a vote from the coordinator or from one of the proposers, acceptors can either cast a vote for  $v$  in round  $i$ , if they have not

voted in any round  $j \leq i$ , otherwise, they can ignore the vote request. Votes are cast by sending them together with the round identifier to the learners.

- Finally, a learner learns that a value  $v$  has been chosen if, for some round  $i$ , it receives Phase 2b messages from a quorum of acceptors announcing that they have all voted for  $v$  in round  $i$ .

As Fast Paxos agents may crash and recover, they must save their state in stable memory so that agents, once recovered, can remember the votes they have cast earlier. The sequence of steps described above imply that a learner can only learn the value of consensus after a period of at least four message delays. If numerous executions of Fast Paxos are required, then it is possible to run Phase 1 and Phase 2a only once for all these instances. This factorization of phases is carried out immediately after the election of a coordinator. At this point, most of the consensus instances have not been started yet, allowing the coordinator to send Phase 2a *any* messages. The improvement brought about by this factorization allows consensus in two message delays, making Fast Paxos an optimal consensus algorithm [4]. Unfortunately, Fast Paxos cannot always be fast. Proposers can propose two different values concurrently, in this case, their proposals may collide. Also, process and communication failures may block a round from succeeding. Different recovery mechanisms can be implemented to deal with collisions and failures, but eventually the coordinator intervention may be necessary to start a new round [3]. Any process can act as the coordinator as long as it follows the rule for choosing a value, if any, from the values proposed in Phase 2a.

As already mentioned, quorums are fundamental for Fast Paxos. Quorums are set of processes and each round has a set of quorums associated with it, classic quorums for a classic round and fast quorums for a fast round. For the proper operation of the algorithm, quorums have to satisfy properties on the sets of processes that form them. Specifically, any two quorum sets must have a non-empty intersection and any quorum and any two fast quorums from the same round must also have a non-empty intersection [3]. There are many ways to define quorums, but a very interesting one from the point of view of process fault tolerance is the definition based only the number of processes contained in each quorum. The definition of quorum using this parameter is straightforward and Section 2 states, for the sake of completeness, fundamental results regarding the cardinality-based quorums defined by Lamport [3].

The coordinator's rule determines how a coordinator can consistently start a new round, after collecting information about previous rounds from the acceptors. That is, for each round  $i$  the coordinator is about to start, it must know if a value  $v$  had been decided or might have been decided in previous rounds  $j < i$ . The coordinator's rule of Fast Paxos must take into account that in a fast round, more than one value might have been proposed and voted concurrently. Quorums are defined to guarantee that only one, if any, of the conflicting proposed values is selected through the application of the coordinator's rule. Section 3 presents the original coordinator's rule as defined in [3] and then shows how this rule can be effectively implemented using cardinality-based definitions of quorums. Section 4 brings our derivation of a simplified cardinality-based coordinator rule, it is more strict than the one presented in Section 3, but it is easier to understand and to implement. Section 5 closes the

work by commenting on the practical value of our main result: a simplified coordinator's rule for Fast Paxos.

## 2 Choosing Quorums

The quorum requirements for Fast Paxos assert that: (a) any two quorums must have non-empty intersection, (b) any two fast quorums and any classic or fast quorum from the same round have a non-empty intersection [3]. We can satisfy these conditions by considering only the number of process in each quorum, where  $N$  is the number of acceptors, and  $F$  and  $E$  are the maximum number of failed acceptors in classic and fast rounds, respectively [3]. A classic quorum is formed by  $N - F$  acceptors and  $N - E$  acceptors form a fast quorum. As the requirements for fast quorums are always stricter than those for classic quorums, we can always assume that  $E \leq F$ . The quorum conditions[3] are then stated as:

$$N > 2F \quad (1)$$

$$N > 2E + F \quad (2)$$

In the next few paragraphs we show how these equations are derived. Condition (a) states that any two quorums must have non-empty intersection. If the sum of the cardinality of two quorums is larger than  $N$  then, by the pigeonhole principle, they must share an element. Thus we have:

$$N - F + N - F > N \Leftrightarrow N > 2F$$

$$N - E + N - E > N \Leftrightarrow N > 2E$$

$$N - F + N - E > N \Leftrightarrow N > E + F$$

As we also have  $E \leq F$  then these three inequalities are satisfied by  $N > 2F$ .

Condition (b) states that any two fast quorums and any classic or fast quorum from the same round must also have a non-empty intersection. We assume (a), that is, any two quorums have non-empty intersection, and we take the cardinality of the smallest intersection of two fast quorums to be  $N - 2E$ ; as the only processes that cannot be shared by two quorums are the ones that have failed. If the sum of the cardinality of a fast or classic quorum and the cardinality of the smallest intersection of two fast quorums is larger than  $N$  then, by the pigeonhole principle, they must share an element. Thus we have:

$$N - F + N - 2E > N \Leftrightarrow N > 2E + F$$

$$N - E + N - 2E > N \Leftrightarrow N > 3E$$

Once again, as we have  $E \leq F$ , these two inequalities are satisfied by  $N > 2E + F$ .

Now, it is easier to reason about how to choose quorums. For a fixed  $N$ ,  $F$  and  $E$  can be chosen in various different ways and a natural way of choosing them is by maximizing

one or the other [3]. As we have  $E \leq F$ , maximizing  $E$  leads to  $E = F$  and the quorum requirements are:

$$\begin{aligned} N &> 2F \\ N &> 3F \end{aligned}$$

Thus, we can satisfy the system only with  $N > 3F$  and:

$$N > 3F \Leftrightarrow F < N/3 \Leftrightarrow F \leq \lceil N/3 \rceil - 1$$

For this case, the cardinality of any classic quorum ( $|Q_c|$ ) or fast quorum ( $|Q_f|$ ), expressed only as a function of  $N$ , is:

$$|Q_c| = |Q_f| \geq N - \lceil N/3 \rceil + 1 \geq \lfloor 2N/3 \rfloor + 1$$

If instead we maximize  $F$ , the limit for its value is given by the Equation 1, thus:

$$N > 2F \Leftrightarrow F < N/2 \Leftrightarrow F \leq \lceil N/2 \rceil - 1$$

In this case  $E$  must be chosen to satisfy Equation 2, considering the value of  $F$  we have just chosen:

$$N > 2E + F \Leftrightarrow N > 2E + \lceil N/2 \rceil - 1 \Leftrightarrow 2E \leq N - \lceil N/2 \rceil \Leftrightarrow E \leq \lfloor N/4 \rfloor$$

For this case, the cardinality of any classic quorum ( $|Q_c|$ ) and fast quorum ( $|Q_f|$ ), expressed only as a function of  $N$ , is:

$$\begin{aligned} |Q_c| &\geq N - \lceil N/2 \rceil + 1 \geq \lfloor N/2 \rfloor + 1 \\ |Q_f| &\geq N - \lfloor N/4 \rfloor \geq \lceil 3N/4 \rceil \end{aligned}$$

### 3 Coordinator's Rule

The original coordinator's rule for Fast Paxos [3] is:

LET  $Q$  be any  $i$ -quorum of acceptors that have reported their last votes to the coordinator.

$vr(a)$  and  $vv(a)$  be the round and the value voted by acceptor  $a$ .

$k$  be the largest value of  $vr(a)$  for all  $a \in Q$ .

$V$  be the set of values  $vv(a)$  for all  $a \in Q$  with  $vr(a) = k$ .

$O4(v)$  be true iff there is a  $k$ -quorum  $R$  such that  $vr(a) = k$  and  $vv(a) = v$  for all  $a \in (Q \cap R)$ .

IF  $k = 0$  THEN let  $v$  be any proposed value.

ELSE IF  $V$  contains a single element

THEN let  $v$  equal that element.

ELSE IF there is some  $w \in V$  satisfying  $O4(w)$

THEN let  $v$  equal that  $w$  (unique).

ELSE let  $v$  be any proposed value.

We now show how this rule can be interpreted in terms of the cardinality-based quorum definitions presented in the previous section. When  $k = 0$  or  $V$  contains a single element the rule is trivial to evaluate no matter the quorum implementation used. However, the evaluation of  $O4(w)$  is more complex because it requires the evaluation of all possible intersections  $Q \cap R$  for all  $k$ -quorums  $R$ . Considering only the cardinality of the quorums involved we have that  $O4(w)$  is true if at least  $|Q \cap R|$  acceptors voted for  $w$  for some  $R$ . As we don't know, and don't want to know, all possible quorums  $R$ , we must consider the smallest possible  $|Q \cap R|$ , assuming as implied by  $O4(w)$  that all acceptors *outside* of  $Q \cap R$  also voted for  $w$ . Considering that  $V$  can only contain more than one element if  $k$  was a fast round, we have two situations:  $i$  is a classic quorum or  $i$  is a fast quorum. Let  $T$  be the number of votes the value  $w$  received from acceptors in  $Q$ . If we want  $T$  to be at least as large as the smallest  $|Q \cap R|$  then we have:

$$T \geq \begin{cases} N - E - F & \text{if } i \text{ is classic} \\ N - 2E & \text{if } i \text{ is fast} \end{cases}$$

$O4(w)$  can now be evaluated by simply counting the number of votes for  $w$  from members of  $Q$ . So, any value  $w$  that satisfies the condition above satisfies  $O4(w)$  and is by definition unique. Considering the values of  $E$  and  $F$  derived in the Section 2, we have for the case where  $i$  is classic:

$$T \geq \begin{cases} N - 2\lceil N/3 \rceil + 2 & \text{if } E = F \leq \lceil N/3 \rceil - 1 \\ N - \lfloor N/4 \rfloor - \lceil N/2 \rceil + 1 & \text{if } F \leq \lceil N/2 \rceil - 1 \text{ and } E \leq \lfloor N/4 \rfloor \end{cases}$$

In the case where  $i$  is fast, we have:

$$T \geq \begin{cases} N - 2\lceil N/3 \rceil + 2 & \text{if } E = F \leq \lceil N/3 \rceil - 1 \\ N - 2\lfloor N/4 \rfloor & \text{if } F \leq \lceil N/2 \rceil - 1 \text{ and } E \leq \lfloor N/4 \rfloor \end{cases}$$

## 4 Simplified Coordinator's Rule

In the last section we have shown that determining the cardinality of the quorums, using simple arithmetic operations, is enough to evaluate  $O4(w)$ . Despite the simplification of  $O4(w)$ , the actual evaluation of the coordinator's rule still depends on whether  $i$  identifies a classic or fast round and also on the values of  $E$  and  $F$ . Here we show that the coordinator's rule can be simplified further by exploiting the fact that the quorums are defined by cardinalities:

IF  $k = 0$  THEN let  $v$  be any proposed value.

ELSE IF there is some  $w \in V$  voted by the larger number of acceptors

THEN let  $v$  equal that  $w$ .

ELSE let  $v$  be any proposed value.

We derive this rule from the—more general—original rule. If  $k = 0$  both rules are the same. If  $V$  contains a single element, then this element has been voted by the larger number

of acceptors (at least one) and both rules are the same. The next case to be considered is whether  $V$  contains more than one element and there is some  $w \in V$  satisfying  $O4(w)$ . We are going now to express  $T$  as a function of the quorum sizes only, using the rule for evaluating  $O4(w)$  shown in the last section and the observation that  $E < (N - F)/2$  from Equation 1. For the case where  $i$  is a classic round, we have:

$$T \geq N - E - F \Rightarrow T > N - (N - F)/2 - F \Leftrightarrow T > (N - F)/2 \Leftrightarrow T \geq \lfloor |Q_c|/2 \rfloor + 1$$

Similarly, for the case where  $i$  is a fast round we have:

$$\begin{aligned} T \geq N - 2E &\Rightarrow T > N - E - (N - F)/2 \Leftrightarrow 2T > N - E + (F - E) \\ &\Rightarrow T > (N - E)/2 \Leftrightarrow T \geq \lfloor |Q_f|/2 \rfloor + 1 \end{aligned}$$

In all cases  $T$  is at least as large as  $\lfloor |Q|/2 \rfloor + 1$ , so if any value  $w$  satisfies  $O4(w)$ , then it has to have been voted by a majority of processes *inside* a quorum. In this case,  $w$  is unique by definition and our rule selects it when we pick  $v$  to be some  $w \in V$  voted by the larger number of acceptors. If no  $w$  satisfies  $O4(w)$ , we are free to choose any value, and our rule uses this freedom and selects the most frequent  $w$ , if any. Finally, if no acceptor that sent a last vote to the coordinator has cast a vote, we are free to choose any value as  $v$ , just as in the original rule.

## 5 Conclusion

We have showed how the coordinator's rule of Fast Paxos [3] can be simplified by resorting exclusively to counting the number of votes for each of the proposed values. Our rule is more restrictive than the original rule, as for some consensus instances it forbids the coordinator of freely choosing any value when he would be allowed otherwise by the original rule. However, the restriction imposed by the simplification does not lead to any disadvantage because if some value received votes in a previous round and the consensus value isn't decided yet, it is reasonable to consider that the coordinator will try to decide on that value first. Our simplified rule is easier to implement, has the advantage that it is independent of the type of a round (fast or classic), and it has to consider only the cardinality of the quorum  $Q$ .

## References

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