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**GRASP Strategies for Scheduling Activities at  
Oil Wells with Resource Displacement**

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# GRASP Strategies for Scheduling Activities at Oil Wells with Resource Displacement

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## Abstract

Before promising locations at petroliferous basins become productive oil wells, it is often necessary to complete drilling activities at these locations. The scheduling of such activities must satisfy several conflicting constraints and attain a number of goals. Moreover, resource displacements between wells are also important. We describe a Greedy Randomized Adaptive Search Procedure (GRASP) for the scheduling of oil well development activities with resource displacement. The results are compared with schedules produced by a well accepted constraint programming implementation. Computational experience on real instances indicates that the GRASP implementation is competitive, outperforming the constraint programming implementation.

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## 1 Introduction

Today, oil and gas are fossil fuels of wide use in our society. They are important ingredients in the processes of making plastics, dyes, kerosene, gasoline, gas and many other products. A significant amount of these fossil fuels is extracted from oceanic basins, *e.g.*, from the offshore Marlin basin in Rio de Janeiro, Brazil. Petrobras is a company with recognized expertise in oil exploration in deep sea waters, being also one of the twenty biggest oil companies in the world. Usually, Petrobras explores diverse petroliferous basins, each with hundreds of promising spots where productive oil wells could be located. However, before these places are turned into productive wells they must be developed, that is, a sequence of engineering activities must be completed at each promising spot, to render them ready for oil extraction. Oil derricks and ships are used to complete these activities. These resources have to move from one spot to another and such displacements must be considered when sequencing the activities. Furthermore, such resources are limited and expensive, either in acquisition or rent value, and must be used efficiently.

The oil *well development with resource displacement problem* (WDRDP) can be summarized thus: given a set of promising spots, the activities to be executed at each location, and the available resources, find a scheduling of the activities and resources, fulfilling several conflicting engineering and operational constraints, including the displacements of the resources, in such a way as to optimize some objective criteria. In this work, the specific WDRDP faced by Petrobras is studied.

This WDRDP imposes much more realistic constraints than other similar studies [7, 21]. Actually, [21] is an early version of this paper, which did not consider the displacement of the resources. Here, we will take this special characteristic into account. In addition, in this paper, we discuss other advanced techniques not explored in [21].

In the sequel, the constraints are presented in detail and our GRASP heuristic strategies are developed, maximizing oil production within a given time horizon. The GRASP implementations were built and tested over several real instances provided by Petrobras. The best implementation indicated a gain of almost 16 million barrels of oil, or more than US\$ 830 millions, summing up over the average solutions of all the instances, when compared to solutions generated by the constraint programming scheduler in use by the company. As another advantage, our methods rely on non commercial libraries, whereas the constraint programming implementation relied on expensive third party constraint programming solvers.

The next section describes the WDRDP. Section 3 discusses GRASP implementations for the WDRDP. Section 4 presents our computational results and compares them to other results derived from the constraint programming implementation presently running at Petrobras. Finally, some concluding remarks are offered in the last section.

## 2 Scheduling the Development of Oil Wells

When a spot is considered a promising oil well, oil derricks are sent there to accomplish the due *drilling* operations. After a well is drilled, the preparation for oil extraction develops in several stages.

First, in the stage called *completion*, oil derricks place Wet Christmas Trees (or WCTs, structures where hydraulic valves are attached) at the mouth of the wells in order to avoid oil leakage. Later, boats connect pipelines between WCTs and manifolds, this stage being called *interconnection*. Manifolds are metallic structures installed by boats at the sea floor. Their use prevents the need for exclusive pipelines connecting each well to the surface, which would be prohibitively expensive. Once this stage is completed, oil extraction can begin. For that, Stationary Units of Production (SUPs) are anchored at specific locations in the surface and boats interconnect manifolds to them. SUPs are used to process, and possibly store, the extracted products. Later, ships fetch the products from SUPs to land storage sites or other processing units. If the oil outflow is very high or a SUP does not have storage capacity, a petroliferous platform may be installed at the surface.

There are two types of wells that may be developed: (i) *productive wells*, which are those who have an oil yield; and (ii) *injection wells*, where only maintenance activities are executed and oil is not extracted from them.

## 2.1 Oil Yield and Objectives

The oil yield is calculated as follows. Each well has an associated outflow and an activity that marks the beginning of its production. When this last activity is concluded, the well is considered in production. The yield is obtained by multiplying the oil outflow by the period between that instant and the established time horizon of production. If the start production activity is set for a time after the horizon, the corresponding yield is disregarded. The objective is to obtain a scheduling of all activities, satisfying all constraints, while maximizing the oil yield.

Other goals to be attained by automating the scheduling of the activities are:

1. *Faster solutions.* Human made solutions take many hours, even days, to be constructed. A faster method would permit the analysis of different scenarios for the same problem, for example, by adding or removing resources. Furthermore, modifications in already committed plans would not result in new hours, or days, spent in rescheduling.
2. *Better resource allocation.* With an automated scheduling, all highly skilled engineers responsible for the manual scheduling can receive other duties.
3. *Savings.* The automated schedule will, usually, result in a better use of boats and derricks, thereby saving considerable operational resources.

## 2.2 The WDRDP Constraints

The constraints involved in the scheduling of oil well development activities are:

- C1. *Technological Precedence:* sets an order between pairs of activities. When considering the precedence between the start and finish of the activities in each pair, any of the four possible combinations can be present.

- C2. *Mark-Activity*: an activity must finish before or initiate after a fixed date, or *mark*, with or without lag time. This date is often related to some external event, *e.g.*, the installation of a petroliferous platform.
- C3. *Baseline*: sets the start date of the activities.
- C4. *Use of Resources*: to execute an activity, due to its intrinsic nature, a resource used must match some operational characteristics. For a boat, it must be verified if the on-board equipments can operate at the specified depth. For an oil derrick, its type and capabilities must be verified, as well as its maximum and minimum depth of operation and drilling.
- C5. *Concurrence*: two activities at the same well, or executed by the same resource, can not be simultaneous.
- C6. *Unavailability*: resources may be unavailable for a period of time, either for maintenance reasons or due to contract expiration.
- C7. *User Defined Sequences*: the user can specify a sequence for the drilling activities or for the “start production” activities of different wells, depending to the type of this sequence. These sequences are specified by engineers in order to avoid loss of pressure in the oil field. If well *A* appears before well *B* in the sequence, then the activity of well *A* must terminate before the start of activity of well *B* can be scheduled.
- C8. *Surface Constraints*: represented by a polygonal security area defined around a well. When the center of a well is inside the restricted area of another well, activities executed at both wells cannot be simultaneous. These constraints must be verified between pairs of mobile and pairs of mobile and anchored oil derricks.
- C9. *Cluster Constraints*: an activity may be part of a cluster, which is a set of activities that must use the same resource.
- C10. *Oil Derrick Displacements*: when an oil derrick moves between two wells, a set-up time<sup>1</sup> will be considered. Therefore, unnecessary displacements must be avoided, for example, by making the same oil derrick execute as much activities at a well as possible. For more details, see Subsection 2.4.

From the above description, it can be seen that the WDRDP is a difficult combinatorial optimization problem. In fact, it is simple to devise a polynomial-time reduction from the classical *Job Shop Scheduling Problem (JSP)* to the WDRDP, thus showing that the WDRDP is NP-hard. The WDRDP treated here shows several differences from similar problems studied in the literature [7, 21].

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<sup>1</sup>Time to unanchor, move and anchor in a new place.

### 2.3 The Constraint Programming Solution

To tackle the same problem, a project team from Petrobras developed a Constraint Programming (cf., [20]) model using ILOG’s Solver and Scheduler<sup>2</sup> [18]. After four years of development and testing, the tool, named ORCA<sup>3</sup>, became operational and very successful. Nowadays, the ORCA solver is often used by engineers both to define a good scheduling for the drilling activities and, also, to analyze the need for acquiring or renting new resources. They confirmed that ORCA generates better solutions than those made by humans. In one real instance, ORCA showed that buying a third oil derrick was unnecessary and that it was better to add a new LSV ship instead. As a result, an expenditure of US\$ 15 million was avoided, while anticipating oil production by 26 days. Despite the good performance of ORCA, searching for even better solutions is still important, since a tenth of a percent improvement in the oil production may represent an increase of millions of dollars in the company’s revenue.

### 2.4 The Resource Displacement

As already mentioned, here we will deal with a new facet of the problem, the resource displacement. This constraint forces a period of “inactivity” for the resource, so that it can unanchor, travel between two wells and anchor in the new spot. We call this period the set-up time. It is set between activities of different wells which are scheduled consecutively. In Figure 1.a we see a typical schedule with no resource displacement. Each square in the “Schedule” area represents a different well. A square in dark gray represents a group of activities of the same well which are executed by the resource, the light gray square, which is in the same horizontal line. In Figure 1.b, we present a schedule with resource displacement. Note that between each well there is a void period of time, which is the set-up time.

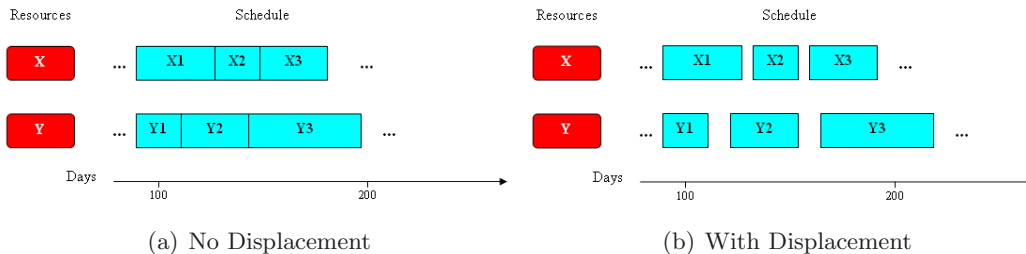


Figure 1: Schedules without and with resource displacement.

The set-up time is a constant for each resource. It is calculated by engineers, based on the average speed of the resource and the average distance between spots in the field, plus the average time to anchor and unanchor. As engineers noted when scheduling the activities at wells of an oil basin, the average distance between these wells does not differ

<sup>2</sup>Registered trade marks of ILOG Inc.

<sup>3</sup>Portuguese acronym for “Optimization of Critical Resources in the Production Activity”.



much from the average distance traveled by the resources. The weather, which influences the speed of the resource, was also considered in the average speed.

However, different resources may have different set-up times. Consider, for example, resources  $X$  and  $Y$  in Figure 1.b. Note that the set-up time of  $Y$  is twice longer than the set-up time of  $X$ . This may alter the selection of which resource will execute activities at a well. For example, wells  $X2$  and  $Y2$  have their “end of execution” activities scheduled at the same instant. This would not be the case if both resources  $X$  and  $Y$  had the same set-up times. Thus, a good selection of which resource will execute the activity is even more important when resource displacement is considered.

The constraint programming solver had many difficulties when treating the resource displacement. A first approach used created virtual activities to represent the set-up time. But, since as it is not known in advance if an activity from another well will be scheduled consecutively, it would be necessary to create virtual activities dynamically, during the search for solutions. Of course they would have to be destroyed when backtracking. This process was quite complex and slowed down the search considerably, and so this approach was abandoned. Instead, an *if-then constraint* was set, imposing that if the previous activity executed by the resource was from another well, then the current activity was shifted by the corresponding set-up time. Of course, as this constraint needed to hold between each pair of activities, the number of constraints added to the problem was  $n * (n - 1) / 2$ , where  $n$  is the number of activities. This increase in the number of constraints also slowed down the execution of ORCA, and it did not generate very good solutions for many instances.

### 3 GRASP Strategies for the WDRDP

Our search for alternatives to compete with ORCA started with an implementation of a tabu search strategy [16] for a simpler version of the WDRDP [7]. However, some issues proved to be particularly difficult to treat, especially the definition of an adequate neighborhood and ways to explore it. After some investigation, the GRASP [10] method seemed to be a more appropriate approach for the WDRDP. Contrary to what occurs with other meta-heuristics, such as tabu search or genetic algorithms which use a large number of parameters in their implementations, the basic GRASP version requires the adjustment of fewer parameters. Despite its simplicity, GRASP is a well studied meta-heuristic which has been successfully applied to a wide variety of optimization problems (cf. [11]). In particular, applications of GRASP to scheduling problems can be found in [2, 8, 9, 3, 4]. It is worth mentioning that GRASP proved itself very valuable in dealing with the well development problem without resource displacement [21].

The next paragraphs review some GRASP basics and describe our specific implementation designed to solve the WDRDP, named GRASP-WDRDP (or GRASPW, for short). In the sequel, we present some advanced GRASP techniques that were used to improve our solver. The model and its algorithms are also shown in detail in the subsequent paragraphs.

### 3.1 Greedy Randomized Adaptive Search Procedure (GRASP)

In the GRASP methodology each iteration consists of two phases: *construction* and *local search* [10]. Figure 2 illustrates a generic implementation of GRASP, in pseudo-code. The input includes parameters for setting the candidate list size (*ListSize*), the maximum number of iterations (*MaxIter*), and the seed (*Seed*) for the random number generator. The iterations are carried out in lines 2-6. Each iteration consists of the construction phase (line 3), the local search phase (line 4) and, if necessary, the incumbent solution update (line 5). In the construction phase, a feasible solution is built, updating the variable *Solution*. Then the local search algorithm seeks a better solution in the neighborhood of *Solution*, according to a given criterion, and updates *Solution*. This process of construction, search and update is executed *MaxIter* times.

```

1: procedure GRASP(ListSize, MaxIter, Seed)
2:   for k = 1 to MaxIter do
3:     Solution ← Construct_Solution(ListSize, Seed);
4:     Solution ← Local_Search(Solution);
5:     Update_Solution(Solution, Best_Solution_Found);
6:   end for
7:   return Best_Solution_Found;
8: end GRASP

```

Figure 2: Pseudo-code of the GRASP Meta-heuristic.

In the construction phase, a feasible solution is built one element at a time. Figure 3 illustrates a generic implementation of the construction phase, in pseudo-code. Input includes the candidate list size (*ListSize*) and the seed (*Seed*). The iterations are carried out in lines 2-8. At each iteration, the next element is selected from all possible elements added to the candidate list. These elements are ordered with respect to a greedy function that measures the, maybe myopic, benefit of selecting each element. This list is called the *Restricted Candidate List* (RCL). The adaptive component of the heuristic arises from the fact that the benefits associated with every element are updated at each iteration to reflect the changes brought on by the selection of the candidate in the previous iteration. The probabilistic component is present by the random choice of one of the best candidates in the RCL, but usually not the best one. This way of choosing elements allows for different solutions to be obtained at each iteration, while not necessarily jeopardizing the adaptive greedy component.

The solutions generated by the construction phase are not guaranteed to be locally optimal. Hence, it is almost always beneficial to apply a local search procedure to improve the constructed solution. The search phase is a standard deterministic local search algorithm that seeks to optimize the solution built in the construction phase.

```

1: procedure Construct_Solution(ListSize, Seed)
2: Solution  $\leftarrow$  0;
3: Evaluate the incremental costs of the candidate elements;
4: while Solution is not a complete solution do
5:   Build the restricted candidate list, RCL(ListSize);
6:   Select an element s from the RCL at random;
7:   Solution  $\leftarrow$  Solution  $\cup$  {s};
8:   Reevaluate the incremental costs;
9: end while
10: return Solution;
11: end Construct_Solution

```

Figure 3: Pseudo-code of the Construction Phase of GRASP.

### 3.2 GRASP Advanced Techniques

As the problem increased in complexity, compared to the problem with no resource displacement studied in [21], we decided to consider some improvements and alternative techniques to be introduced in the basic GRASP procedure.

**Bias Function:** In the construction procedure of the basic GRASP, the next element to be introduced in the solution is chosen at random from the candidates in the RCL. The elements of the RCL are assigned equal probabilities of being chosen. However, any probability distribution can be used to bias the selection toward some particular candidates. Bresina [5] proposed a construction mechanism based on the *rank*  $r(\alpha)$  assigned to each candidate element  $\alpha$ , according to its value, measured by the greedy function,  $v(\alpha)$ . However, as the sorting of elements to obtain the *rank* has time complexity  $O(n \log n)$ , where  $n$  is the number of elements in the RCL, and since the sorting must be repeated at each choice of an element, this process could slow down the implementation. Thus, we decided to use directly the value  $v(\alpha)$  of the candidates to create the following bias functions:

- uniform:  $bias(\alpha) = 1$
- linear:  $bias(\alpha) = v(\alpha)$
- log:  $bias(\alpha) = \ln v(\alpha)$
- exponential:  $bias(\alpha) = e^{v(\alpha)}$
- quadratic:  $bias(\alpha) = v(\alpha)^2$
- square root:  $bias(\alpha) = \sqrt{v(\alpha)}$

Once the value of the bias function is evaluated for all elements of the RCL, the probability of the candidate  $\alpha$  being chosen is:

$$\frac{bias(\alpha)}{\sum_{\alpha' \in RCL} bias(\alpha')}$$

**Proximate Optimality Principle (POP):** This technique is based in the idea that “good solutions at one level are likely to be found ‘close to’ good solutions at an adjacent level” [16]. Fleurent and Glover [12] provided a GRASP interpretation of this principle. They suggested that imperfections introduced during steps of the GRASP construction phase can be ironed out by applying local search during (and not only at the end of) the GRASP construction phase. Due to efficiency considerations, a practical use of the POP in a GRASP implementation would be to apply a local search during a few points in the construction phase, and not at the end of each construction iteration. Binato et al. [4], when dealing with a JSP, applied a local search when 40% and 80% of the construction moves have been taken, as well as at the end of the construction phase. Due to the relation between the JSP and the WDRDP (see Section 2.2), and after some promising tests, the same values were used in our GRASP solver.

One possible shortcoming of the basic GRASP method is the independence of its iterations, *i.e.*, the fact that it does not learn from the history of solutions found in previous iterations. This is so because the standard algorithm discards information about any solution encountered that does not improve the incumbent solution. Information gathered from good solutions can be used to implement memory-based procedures. We show some of these strategies below.

**Intensification:** Fleurent and Glover [12] observed that the standard GRASP does not use long term memory and proposed a scheme to use this kind of memory in the heuristic. Long term memory is one of the pillars of tabu search. The idea of the approach is to maintain a set of elite solutions. To be included in this set, a solution must either be better than all elite solutions according to some objective function, or must be better than the worst solution of the set, while being sufficiently different from all elite solutions. In our problem, a solution is considered sufficiently different from another if the number of activities that have different start dates in both, or the number of different resources allocated to both is higher than the number of activities divided by the number of resources. As the number of resources is usually smaller than five, two solutions will be sufficiently different if at least 20% of their activities have a different start date or resource allocated to both.

We show next how the elite solutions will bias the selection of the candidates in the GRASP construction phase. To each candidate  $\alpha$  we evaluate the intensity function  $Int(\alpha)$  as follows:

$$Int(\alpha) = \frac{\sum_{e_i \in S} Prod(e_i)}{Max_{e_j \in E} Prod(e_j)}.$$

In this function,  $S$  is the set of elite solutions in which the element  $\alpha$  has the same start date and resource allocated when compared to the solution being built;  $Prod()$  is the value of oil yield associated with a solution,  $E$  is the set of elite solutions, and  $Max$  returns the maximum value in the indicated set.

Let  $iter$  be the amount of iterations executed and  $k$  a parameter, we define the bias function using the intensity function as follows:

$$biasInt(\alpha) = bias(\alpha) + \frac{Int(\alpha) \cdot iter}{k}$$

The fraction  $iter/k$  is used to give emphasis to the intensity function as the number of iterations increases, and thus the quality of the elite solutions in which the intensity function is based also possibly increases.

**Path-Relinking (P-R):** This technique was originally proposed by Glover [13] as an intensification strategy exploring trajectories connecting elite solutions obtained by tabu search or scatter search methods [14, 16, 15]. Starting from one or more elite solutions, paths in the solution space leading towards other elite solutions are generated and explored in the search for better solutions. This is accomplished by selecting moves that introduce, in the moving solution, attributes contained in the target solution. Path-relinking may be viewed as a strategy that seeks to incorporate attributes of high quality solutions, by favoring these attributes in the selected moves.

The use of path-relinking within a GRASP procedure was first proposed by Laguna and Mart [15]. It was followed by several extensions, improvements and successful applications [1, 6, 22]. Two basic strategies are:

- path-relinking is applied as a post-optimization step to all pairs of elite solutions.
- path-relinking is applied as an intensification strategy to each local optimum obtained after the local search phase.

According to [23] the second strategy is more promising. Another consideration, also from [23], is that exploring the two trajectories, in both directions, between two solutions results in small gains and demands twice the time. Thus, it is usually more appropriate to explore only one path, the one from the best solution to the other, as better solutions are more often found near the neighborhood of the best solutions [23].

**Selective Local Search (SLS):** Another approach that can be used within the standard GRASP procedure is to apply local search only in those solutions sufficiently different from the elite solutions, or that have a good quality, *i.e.*, better than the worst of the elite solutions. In other words, we try to apply local search only where there are higher chances of obtaining better solutions, improving the efficiency of the algorithm.

### 3.3 The New GRASP Solver Implementation: GRASPW

The GRASPW implementation was constructed using the C/C++ programming language. Our model uses two types of integer variables. One represents the beginning of execution of each activity in the corresponding well. These values range between a minimum and a maximum start time, depending on the current partial solution being constructed. The second type of variables represents which resource will execute each activity in a well. Their

domains are characterized by a set of the possible resources, of whose one must be chosen to execute the corresponding activity.

All the constraints described in Section 2 were enforced. Three constraints, namely, C2, C3 e C4, were set while reading the problem data, before the search begins. Note that, in these cases, all values needed to set the constraints are already defined. The other constraints were dealt with during the search for solutions, the variables involved being assigned single values.

Constraint C10, that deals with resource displacement and which was responsible for a loss of performance of the ORCA search algorithm, was treated here in a simple way. As we do not use a mechanism of constraint propagation, we displace the current activity by at least the amount of the set-up time, if the previous activity scheduled in the same resource is from a different well.

Of course, we could also have imposed constraint C10 during the search in ORCA instead of writing such constraints directly in the model. However this was not done for two reasons:

- First, this goes against the fundamental design principle in constraint programming, which dissociates the representation of the problem from strategies for its resolution [19, 17]. This approach is known as *declarative programming*. In other words, this separation of the *model* from the *search* has a number of practical implications: it shortens development time, decreases maintenance problems and heightens adaptability of the application. Furthermore, such separation makes it easier to experiment with different strategies and different algorithms without redesigning the model, and even allows for the addition of further constraints to the problem without having to rewrite the search method.
- Besides generating solutions to the WDRDP, ORCA has also the functionality of validating the model. If the data of the problem generates inconsistencies, ORCA will inform the user that there is no feasible solution to the problem. In order to validate the import data, all the constraints must be imposed in the model, including the C10 constraints. Our GRASP solver does not have this functionality.

### 3.4 The Construction Phase

The following two adaptations were made to the procedure illustrated in Figure 2:

- The search procedure was interrupted by a time limit instead of by the number of iterations; and
- During a complete run of the GRASPW meta-heuristic, the value of *ListSize* can be monotonically incremented by a fixed amount when a predefined interval of time is reached with no improvement on the best solution. This allows the algorithm to explore larger regions of the search space. Alternatively, during a run of GRASPW, the value of *ListSize* can be monotonically decremented between iterations, thus focusing into a greedier heuristic. With this scheme we obtain a *dynamic sized RCL*, in opposition to the original *static sized RCL*. Note that, as GRASP iterations are independent, one could imagine that there is no difference between increasing and

decreasing the RCL size. However, as we do not know in advance the amount of time the algorithm will execute in each run, or when the RCL size will be altered, we can not anticipate the result of a GRASPW run, when increasing or decreasing the RCL size.

As in the ORCA implementation, we seek solutions with the highest oil yield. To this end, the construction phase illustrated in Figure 3 was modified thus:

1. The first time the construction phase is initiated, we use *ListSize* equals to one, when the algorithm behaves like a greedy heuristic. With few constraints obstructing the greedy heuristic, it tends to generate good or even very good solutions. For example, in six out of seventeen real instances, the best solution was found in the first pass of the construction phase.
2. The candidates are defined by the production wells that are available (meaning that there are no wells yet not scheduled which must precede them), or the injection wells that have activities of production wells succeeding them. The activities of injection wells that do not have activities of production wells succeeding them are left to be scheduled after all others. Note that injection wells are not productive and therefore must not be scheduled before production wells, unless there are technological constraints forcing such a schedule. We schedule the wells and not single activities because otherwise the solution would have a huge number of displacements of resources between wells, which would affect the oil production negatively.
3. The evaluation of incremental costs (line 3 of Figure 3) assesses how much oil a well can offer until the end of the time horizon. The RCL is built with those wells that offer the highest yields of oil. Actually, not only the oil offer is considered, but also the oil offer of the constrained successors of a well.
4. In the construction phase, the next element to be introduced in the solution is chosen uniformly from the candidates in the RCL (line 5 of Figure 3). However, any probability distribution can be used to bias the selection (see Section 3.2). We tried some bias functions in the selection of the candidates, as will be discussed later.
5. To schedule the candidate well (line 7 of Figure 3), the routine is as follows.

As long as there are activities not yet scheduled in the well:

- (a) choose any activity available in the well, *i.e.*, one not yet scheduled and such that there is no other activity not yet scheduled in the wells that must precede it;
- (b) choose a resource for this activity that can execute it, and that can complete the activity the earliest;
- (c) set the start time of the activity at the earliest possible time, *i.e.*, the maximum between the earliest time the resource is available to execute the activity (considering the constraints, including the set-up time) and the minimum start time of the activity; and



- (d) all activities that are constrained to succeed the chosen one must have their minimum start times updated so that all constraints are satisfied.

The scheduling of a well is done so as to satisfy all constraints, including the seven ones not yet enforced while reading the problem data. In case of violations, and this can be tested after each activity is scheduled, the construction of this solution is aborted and a new one is started. We could, instead, backtrack a few steps, but this would slow down this phase, especially if the first steps were not appropriate.

6. After a well is scheduled, any activities that must succeed it have their minimum start times updated to satisfy all constraints. If that is not possible, the construction of this solution is also aborted.

### 3.5 The Local Search Phase

As we have no guaranties that the solution found in the construction phase is locally optimum, a local search is used to improve the solution. For the search phase, an appropriate neighborhood was defined, so as to permit explorations that quickly lead to better solutions. The 2-exchange local search algorithm based on the disjunctive graph model [24] was used. The same neighborhood was used in [4] for a Job Shop Scheduling problem. In order to apply the 2-exchange local search to the WDRDP, we swap two elements in the scheduling. For example, consider the schedule presented in Figure 4.a. The elements (light gray squares) are executed by the resource (dark gray squares) which is depicted in the same horizontal line. The size of the squares represents the execution time of the corresponding elements. We swap elements *A* and *B*, which results in the schedule shown in Figure 4.b. Since the execution time of elements *A* and *B* can be different, all activities after them may have their start times updated.

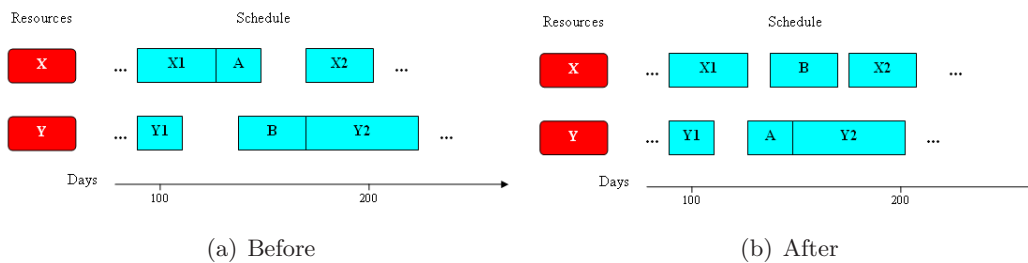


Figure 4: 2-exchange swap.

We need to decide, of course, what an element stands for. Some options are:

1. *An activity.* Very small granularity, giving rise to huge neighborhoods [7] and, worse, moving an activity to another position would possibly force its predecessors and successors in the same well to be moved as well, in order to avoid the displacements that may result from constraints of type C10;



2. *A well.* With higher granularity. But since the sequence of activities in a well may be splitted in the present schedule due to constraints of type C1, moving all activities takes time to verify all constraints, and exchanging the whole well may not be possible even though exchanging only part of it could be;
3. *Part of a well.* That means a maximal set of activities of the same well scheduled consecutively in the same resource. With medium granularity, it avoids the displacements that may result from constraints of type C10.

In our implementation, we chose the last alternative, where the local search algorithm exchanges all pairs of parts of wells, no matter on what resource they have been scheduled. That neighborhood is of size  $O(n^2)$ , where  $n$  is the number of parts of wells. For practical instances, this is one order of magnitude smaller than the neighborhood that uses activities as the moving elements.

To fully specify the local search phase we need a rule that defines how the neighborhood is searched and which solution replaces the current one. This rule is called the *pivoting rule* [25], and examples of it are the *first improvement rule* (FIR) and the *best improvement rule* (BIR). In the first case, the algorithm moves to a neighboring solution as soon as it finds a better solution; in the second case, all neighbors are checked and the best one is chosen. In either case, the worst case running time of each iteration is bounded by  $O(n^2)$ , where  $n$  is the number of elements in the neighborhood. In the next section we present a comparison between these two alternatives.

Besides the improvements proposed above to the standard GRASP procedure, all the advanced techniques presented in Section 3.2 were implemented and tested.

## 4 Computational Results

In this section, computational results for the GRASPW implementation are discussed. They are also compared with results obtained with the ORCA implementation over the same real instances. All tests were run on a platform equipped with a Sun SPARC Ultra 60 processor, running a Solaris 9 operating system at 450 MHz and with 1024 MB of RAM. Both the GRASPW and the ORCA implementations were allowed to run for 1800 seconds on each instance.

### 4.1 Typical Instances

Twenty two real instances provided by Petrobras were used in our tests. Table 1 summarizes the dataset. Columns with the same numerical data, like columns 8 and 9, refer to distinct instances that differ in the number of other constraints not shown in the table, like C9. The first part of that table displays the instances where no C7 constraints were found. In order to reduce the amount of time spent in testing, in some experiments we used only 7 of these instances, eliminating instances that differed only by a few constraints. The lower part of Table 1 shows the ten instances where C7 constraints were present. It is worth mentioning that, as GRASP makes use of randomization, each instance was tested five times, and the results being reported always reflect the average of the tests.

The horizon of production ranges from a thousand to three thousand days. Note that, after every well has been scheduled, the oil yield is the same between all solutions until the horizon of production. Thus, any gains in production, when comparing two solutions, happen before all wells are scheduled. If we considered the full horizon to compute yields, these gains, in percent, would be smaller. Therefore, for each instance we use as the horizon of production the end date of the last activity of the well scheduled the latest, among all solutions of the solvers.

Instance	1	2	3	4	5	6	7	8	9	10	11	12
# wells	29	22	29	29	17	22	22	29	29	22	29	22
# activities	98	107	98	98	111	107	128	98	98	107	98	107
# boats	1	1	2	1	1	2	1	1	1	2	1	2
# derricks	3	2	3	3	2	2	3	3	3	2	3	2
# C7	0	0	0	0	0	0	0	0	0	0	0	0

Instance	13	14	15	16	17	18	19	20	21	22
# wells	22	22	22	22	22	22	22	22	29	65
# activities	107	107	107	107	107	107	107	107	98	338
# boats	2	2	2	1	2	2	2	2	1	1
# derricks	2	2	2	2	2	2	2	2	3	2
# C7	1	1	1	1	1	1	1	1	2	2

Table 1: Tested Instances.

## 4.2 Setting GRASPW Parameters

In Section 3 we presented the idea of a *dynamic sized RCL*. There are at least two ways we could exploit this idea: we may decrease monotonically the number of candidates using a greedier heuristic; or we may increase monotonically the number of candidates in order to drive away from a local optimum into new regions of the search tree. In the first case, the initial RCL size is set to  $\max(13, w)$ ,  $w$  being the number of wells, and is decreased by one every 300 seconds without improvement. In the second case, we start with  $\max(5, w)$  for the initial RCL size and increase it by one every 300 seconds without improvement. The first approach did not yield good results when applied to the WDRDP, generating the same or worse solutions than those found by GRASPW with a static sized RCL. However, the second approach proved promising. Figure 5.a shows the algorithm with dynamic sized RCL generating better solutions after 150 thousand iterations, when the RCL is increased. The same happens in Figure 5.b after 50 thousand iterations, when another real instance is tested. Amongst twelve scenarios tested, four had better solutions with the dynamic sized RCL, summing up an increase of around 261 thousand barrels of oil. In the other eight scenarios, the same solutions were found.

We also considered two options for searching the neighborhood and selecting a new neighbor: the *first improvement rule* (FIR) and the *best improvement rule* (BIR) strategy. Tests were executed and FIR has proved to be better. The BIR heuristics found solutions whose production was equal to a predefined target value with the least number of iterations

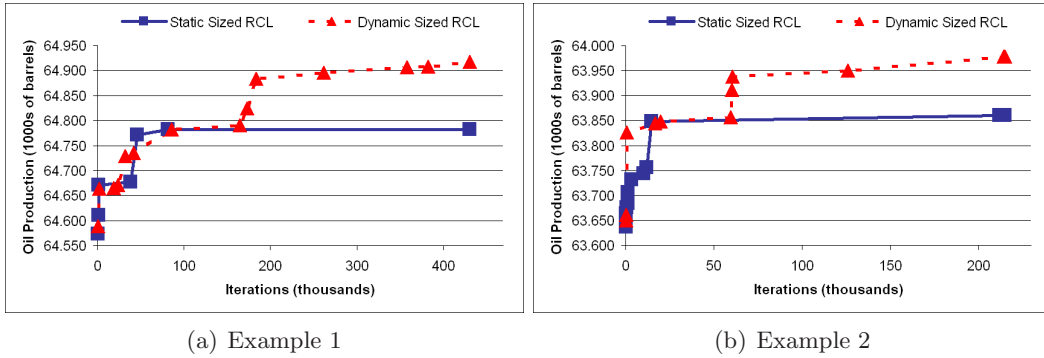


Figure 5: Static Sized RCL Solver x Dynamic Sized RCL Solver.

(see Figure 6.a). On average, to find a solution with a predefined production, the BIR strategy used about 60% of the number of iterations of the FIR strategy. On the other hand, the FIR strategy was faster in most instances (see Figure 6.b). On the average, to find a solution with a predefined production, the FIR strategy used 66% of the time used by the BIR strategy. That is because, on the average, a FIR iteration was almost seven times faster than a BIR iteration. Note that, to users, running time is deemed important. Note also that the FIR solver built solutions with a slightly higher oil production, as can be seen in Figures 6 and 7. Summing up all instances, the gain was almost 80 thousand barrels of oil. We concluded that the FIR strategy was the one that better suited this problem.

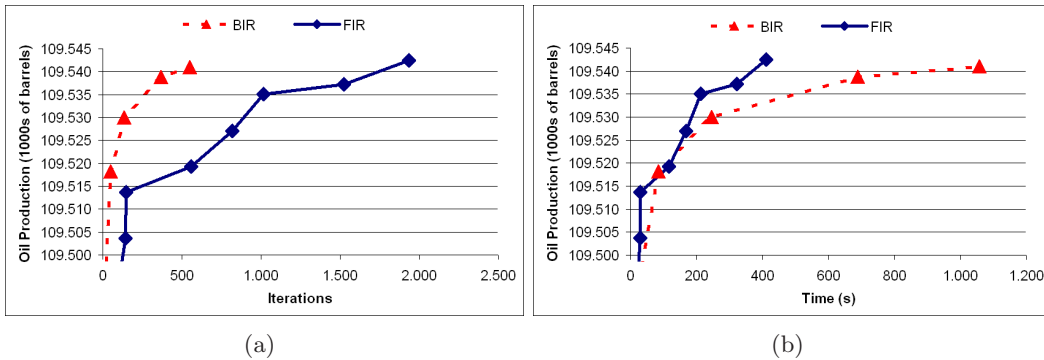


Figure 6: BIR x FIR: Example 1

Another technique tested was to bias the selection towards some particular candidates, those with the highest oil yield. Five probability distributions, besides the uniform distribution, were considered, as presented in Section 3.2. Comparative experiments showed that the exponential and quadratic bias functions generated much worse solutions, with production of more than 2 millions barrels of oil smaller than the other approaches, summing up over all instances. Among the other bias functions, the function based on square root

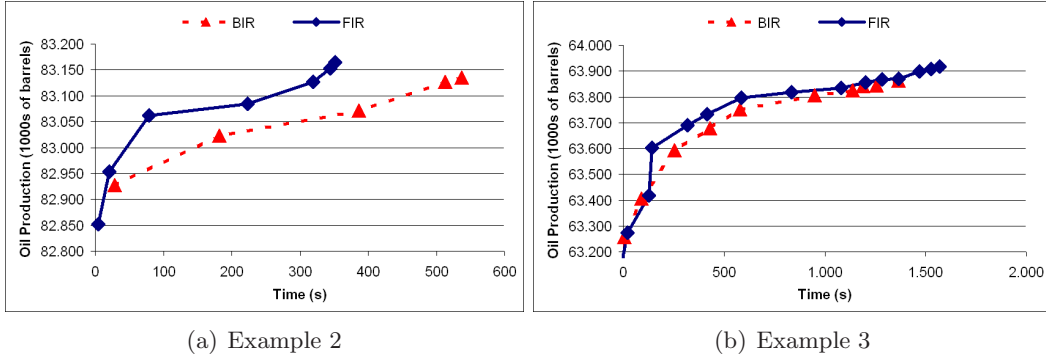


Figure 7: BIR x FIR.

proved to be the best one for this problem, as can be seen in Figures 8.a and 9.a. For a better view over this two examples, we zoom comparing square root to others bias functions on Figures 8.(b,c,d) and 9.(b,c,d).

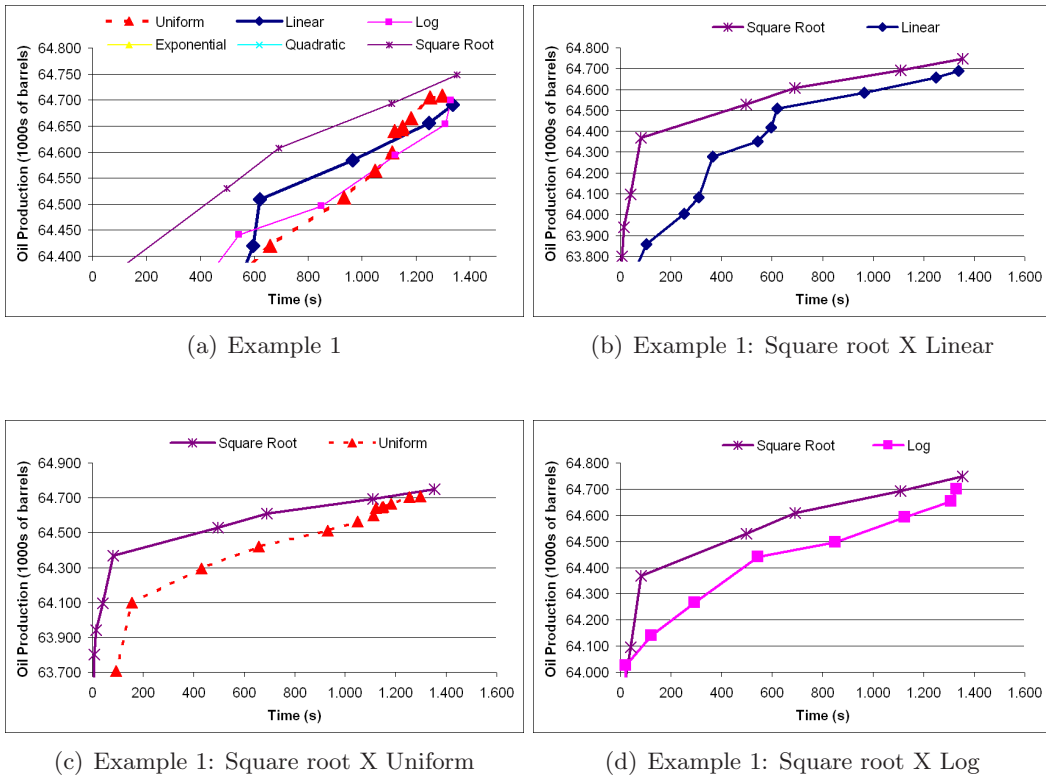


Figure 8: Bias Functions, I.

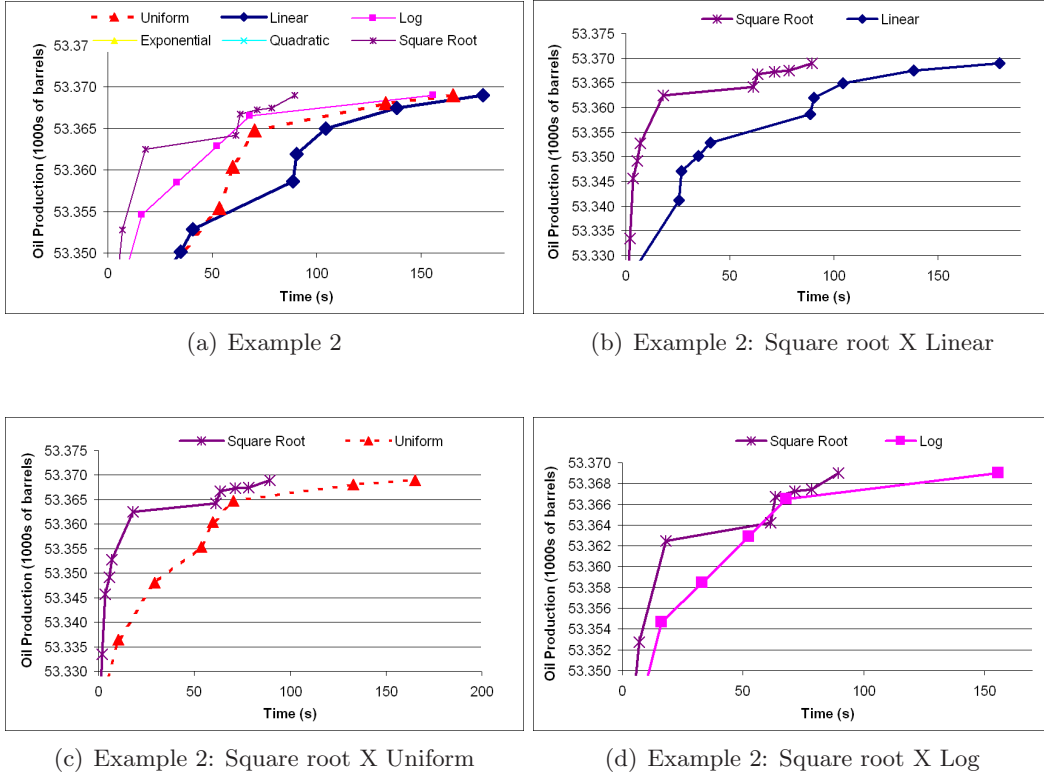


Figure 9: Bias Functions, II

### 4.3 The GRASPW Implementations

In Section 3.2, we presented some advanced techniques that can be included in the basic GRASP procedure. Many combinations of these were tested, but not all possible combinations. With 6 distinct techniques (five advanced plus the pivoting rule), all possible combinations would number 64 distinct solvers. It was simply too time consuming to execute broad tests with each and every one of these solvers. We choose the best combinations based on faster tests, selecting a pool of 27 solvers. To simplify the visualization and data analysis, we show only 14 solvers, besides the ORCA solver. See Table 2.

As there are many solvers and in many tests there were ties between them, we elaborated five criteria to analyze which would be the best solver. We list these criteria below:

1. Sum of the oil production of the best solutions for all instances.
2. Sum of the oil production of the average solutions for all instances.
3. Average of the rank obtained sorting decreasingly the solver solutions of each instance using the *Ranking1 Rule* (R1), explained below. The sorting was according to the average oil production and average time of execution to find the best solution. We ranked all solvers.

Techniques	C.P.	GRASP	POP	Bias	Int	P-R	SLS	BIR
ORCA	•							
G1		•						
G2		•		•				
G3		•		•	•			
G4		•				•		
G5		•					•	
G6		•						•
G7		•	•					
G8		•	•				•	
G9		•	•					•
G10		•	•	•				
G11		•	•	•			•	
G12		•	•	•			•	•
G13		•	•	•	•		•	
G14		•	•	•	•	•	•	

C.P.	Constraint Programming
GRASP	Greedy Randomized Adaptive Search Procedure
POP	Proximate Optimality Principle
Bias	Bias Function (Square Root)
Int	Intensification
P-R	Path-Relinking
SLS	Selective Local Search
BIR	BIR

Table 2: Solvers

4. Average of the rank obtained sorting decreasingly the solver solutions of each instance using the *Ranking2 Rule* (R2), explained below. The sorting was according to the average oil production and average time of execution to find the best solution. We ranked all solvers.
5. Number of instances for which the solver obtained the best solution known to that instance.

*Ranking1 Rule* is a rank where, if  $s$  solvers are drawn in rank  $r$ , the next solvers would come in rank  $r + 1$ . *Ranking2 Rule*, in the same example, would put the next solvers in rank  $r + s$ .

Table 3 presents the values of the criteria achieved by the 15 solvers created for the WDRDP. Values in boldface are the best results among the solvers for each criteria. Note that G14 stands out being the best solver according to three criteria, followed closely by G7 which was the best in two criteria. However we can not say yet that G14 is the best solver, because if it behaves poorly in the other two criteria, another solver could prove to be more appropriate for the WDRDP. According to these values, we elaborated Table 4, where we show the rank of the solvers in each criterion. Note that there are significant variations in the rank of the solvers when using different criteria.

In order to make a decision about which would be the best solver, we again ranked the solvers according to the average rank they obtained in each criterion. We made this rank

Criterion	1	2	3	4	5
ORCA	331.043.827	331.043.827	9,588235	23,47059	2
G1	354.428.768	346.325.637	7,682353	10,47059	7
G2	354.421.217	346.255.908	8,035294	11,11765	6
G3	354.528.275	346.318.220	7,470588	10,23529	6
G4	354.379.849	346.255.508	8	11,41176	6
G5	354.428.768	346.342.803	6,823529	9,176471	7
G6	354.432.875	346.377.716	4,623529	8,235294	7
G7	354.838.320	346.807.529	<b>2,941176</b>	<b>4,352941</b>	8
G8	354.838.320	346.744.587	4,352941	5,764706	8
G9	354.838.320	346.801.864	3,470588	5,294118	8
G10	354.929.689	346.782.341	4,411765	6,588235	7
G11	354.929.689	346.762.940	4,294118	6,411765	<b>9</b>
G12	354.929.689	346.762.060	4,588235	6,941176	7
G13	<b>354.932.391</b>	346.765.955	4,058824	6	7
G14	354.888.020	<b>346.814.967</b>	<b>2,941176</b>	4,764706	<b>9</b>

Table 3: Values obtained by the Solvers in the Comparative Criteria

using both R1 and R2 type rules, as can be seen in Table 5. Using this table, we obtained the rank of the best solvers, presented in Table 6. We conclude that G14 is actually the best GRASP solver that we developed for the WDRDP.

This solver makes use of many advanced techniques for GRASP heuristics, like POP, bias function based on square root, intensification, *Path-Relinking* and SLS (see Section 3.2). Our second best solver for the problem was G7, which makes use only of the POP technique. As can be seen in Table 3, and by the average rank in Table 5, G7 is almost as good as G14. Hence, the other advanced techniques used in solver G14, besides POP, give only marginal gains to G14. POP, on the other hand, improves drastically the performance of a solver for this problem. This can be seen comparing G1 (basic GRASP) to G7 (basic GRASP with POP) in the same tables mentioned above, or comparing the group of solvers G1 until G6 (solvers without POP) to the group of G7 and the other solvers, that is, the group of solvers that implemented POP. When POP is applied, a great leap in quality can be seen. Note, for example, that G1 is just the eleventh best solver, while G7 is the second best.

Note also that G7 is better than G10, G11, G12 and G13, solvers which combine POP and other advanced techniques. This happened because these advanced techniques required longer processing times from the solvers, and the gains were not sufficient to justify their use. Solver G7 could only be overcome when all the advanced techniques were combined in G14, and even so it was just slightly overcome.

With the above facts in mind, we concluded that POP is the best advanced technique to solve the WDRDP.

#### 4.4 Comparative Results

As can be seen in Section 4.3, the G14 solver was considered the best GRASPW implementation. Note that the constraint programming solver, ORCA, was much worse than all the

Criterion	1	2	3	4	5
	G13	G14	G7-G14	G7	G11-G14
	G10-G11-G12	G7	G9	G14	G7-G8-G9
	G14	G9	G13	G9	G1-G5-G6-G10-G12-G13
	G7-G8-G9	G10	G11	G8	G2-G3-G4
	G3	G13	G8	G13	ORCA
	G6	G11	G10	G11	
	G1-G5	G12	G12	G10	
	G2	G8	G6	G12	
	G4	G6	G5	G6	
	ORCA	G5	G3	G5	
		G1	G1	G3	
		G3	G4	G1	
		G2	G2	G2	
		G4	ORCA	G4	
		ORCA		ORCA	

Table 4: Rank of the Solvers

GRASPW solvers, even when compared to the standard GRASP procedure, G1.

Compared to ORCA, the G14 solver generated gains of more than 16 millions barrels of oil, summing up the average solutions of all instances. This means an increase of 4,5% in the oil production. Note that, with the oil barrel price around US\$ 52, the use of the G14 solver, instead of the ORCA solver, would yield gains of almost US\$ 832 million. Besides, even generating better solutions, G14 takes, to find its best solution, only 95% of the time ORCA takes to find its best solution. And to find better solutions than the ORCA’s best one, G14 takes only 45% of the time taken by ORCA. Among the 17 instances tested, G14 generates, within the first second of execution, better solutions than ORCA’s best solution, in 14 instances. Two examples of that can be seen in Figure 10. In Figure 11, we show two instances where G14 does not generate, within the first second of execution, solutions better than the ORCA best solution.

However, there is an instance where ORCA was competitive compared to G14, as can be seen in Figure 12. Here, ORCA’s best solution was found in 5 seconds and G14 only generated a better solution than that after 1134 seconds. Nevertheless, after 1522 seconds, G14 generated a solution with an oil production almost 63 thousand barrels superior than ORCA’s best solution.

We conclude that our GRASPW implementation is much more efficient and generate better solutions than the ORCA solver.

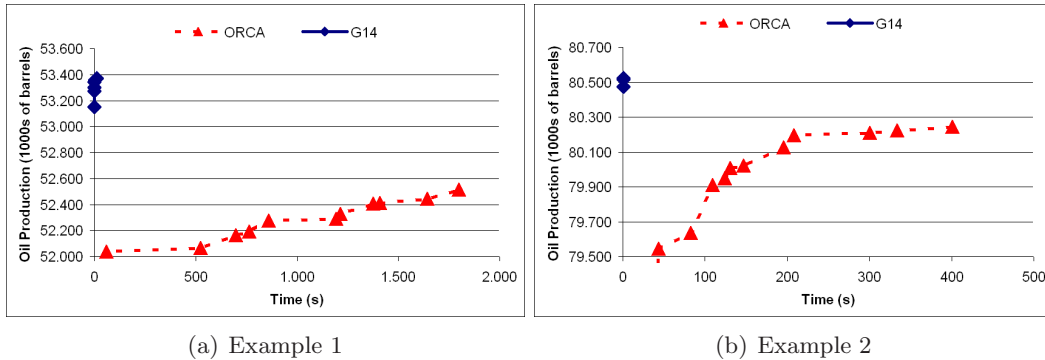
## 5 Conclusions

Scheduling activities efficiently is of paramount importance to the industry, in general. Petrobras, a leading company in deep water oil exploration, presented us the WDRDP, a scheduling problem related to oil well development. Here we contrast two approaches to the WDRDP: the constraint programming tool ORCA and a GRASP implementation, dubbed GRASPW.



Analysis of Criteria	R1	R2	Average R1-R2
ORCA	11,8	15	13,4
G1	8,8	10,4	9,6
G2	10,2	13	11,6
G3	8,4	11	9,7
G4	10,6	13,4	12
G5	7,8	9,4	8,6
G6	7	8,6	7,8
G7	2	2,6	2,3
G8	4,6	5,4	5
G9	2,8	3,6	3,2
G10	4,4	5,2	4,8
G11	3,8	4	3,9
G12	5,4	6,2	5,8
G13	3,4	4,2	3,8
G14	<b>1,6</b>	<b>2</b>	<b>1,8</b>

Table 5: Analysis of the Rank of the Solvers

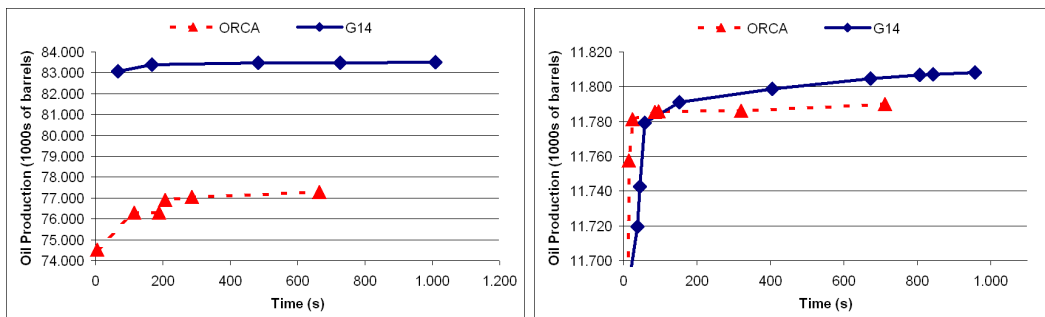
Figure 10: ORCA  $\times$  G14, I

Computational experiments were carried out on several real instances. We conclude that GRASPW greatly outperforms ORCA. Not only it generates solutions with higher oil production, but often it outputs solutions with the same oil production as ORCA, but in much less time. We recall that ORCA already produced better results than the manual solutions.

It is worth mentioning that ORCA is built using the ILOG Constraint Programming suite, a set of expensive and sophisticated libraries with years of development. Using GRASPW, which was entirely programmed from the ground up, these costs could be averted. In opposition, the ILOG suite favors easiness of development, of maintenance and of understanding of the source code.

Solver
G14
G7
G9
G13
G11
G10
G8
G12
G6
G5
G1
G3
G2
G4
ORCA

Table 6: Rank of the Best Solvers



(a) Example 3

(b) Example 4

Figure 11: ORCA × G14, II

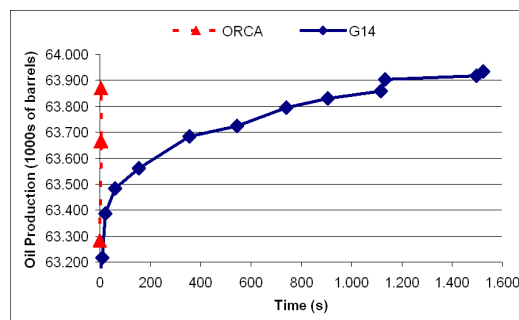


Figure 12: ORCA×G14: Example 5

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