INSTITUTO DE COMPUTAÇÃO
UNIVERSIDADE ESTADUAL DE CAMPINAS

A Linear Approach to Enforce the Minimal Characterization of the Rollback-Dependency Trackability Property

Islene C. Garcia *Luiz E. Buzato*

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Islene C. Garcia   Luiz E. Buzato
Instituto de Computação
Universidade Estadual de Campinas
Caixa Postal 6176
13083-970 Campinas, São Paulo, Brasil
{islene, buzato}@ic.unicamp.br

Abstract

A checkpointing protocol that enforces rollback-dependency trackability (RDT) during the progress of a distributed computation must take forced checkpoints to *break* non-trackable dependencies. Breaking just non-visibly doubled dependencies instead of breaking all non-trackable dependencies leads to fewer forced checkpoints, but seemed to require the processes of a computation to maintain and propagate $O(n^2)$ control information. In this paper, we prove that this hypothesis is false by presenting a protocol that breaks the minimal set of non-visibly doubled dependencies necessary to enforce RDT, called "non-visibly doubled PMM-paths", using only $O(n)$ control information.

Keywords: fault-tolerance, rollback recovery, distributed checkpointing, distributed algorithms, algorithm complexity.

1 Introduction

A checkpointing protocol that enforces rollback-dependency trackability (RDT) during the progress of the computation must take forced checkpoints to *break* non-trackable dependencies [1, 12]. Although it is not possible to design an RDT protocol that will take the minimum number of forced checkpoints for all checkpoint and communication patterns [11], RDT protocols based on stronger induction conditions usually take fewer forced checkpoints than RDT protocols based on weaker conditions [3]. The inconvenience of the strongest approach, based on breaking only non-visibly doubled paths, was that it seemed to require $O(n^2)$ control information [3], while a weaker approach based on breaking all non-trackable

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requires only $O(n)$ control information [12], where n is the number of processes in the computation. The main contribution of this paper is to present a simple RDT protocol that implements the stronger approach in $O(n)$.

A checkpoint is a recording in stable memory of a process' state that can be used for rollback recovery. The set of all checkpoints taken by a distributed computation and the dependencies established among these checkpoints due to message exchanges form a checkpoint and communication pattern (CCP). CCPs that satisfy RDT present only checkpoint dependencies that are on-line trackable using dependency vectors, and allow efficient solutions to the determination of the maximum and minimum consistent global checkpoints that include a set of checkpoints [12]. Many applications can benefit from these algorithms: rollback recovery, software error recovery, and distributed debugging [12].

Netzer and Xu have determined that checkpoint dependencies are created by sequences of messages called *zigzag paths* [10]. Two types of zigzag paths can be identified: causal paths (C-paths) and non-causal paths (Z-paths). C-paths are on-line trackable through the use of dependency vectors; Z-paths, on the contrary, cannot be on-line tracked. However, a CCP may present Z-paths and still satisfy RDT. In this case, all Z-paths must be *doubled* by a C-path; a Z-path is doubled by a causal one if the pair of checkpoints related by that Z-path is also related by a C-path [2, 3].

Baldoni, Helary and Raynal have established properties that could reduce the set of Z-paths that must be doubled in a CCP that satisfies RDT. They have concentrated their study on visible properties, that is, properties that can be tested on-line by an RDT protocol [2]. They have also proved that a process does not need to break a Z-path if it is able to detect that it is already causally doubled (a *visibly doubled* path). Additionally, they have conjectured that a specific set of Z-paths, named "non-visibly-doubled-EPSCM-paths", determines the smallest set of Z-paths that must be tested for breaking by an RDT protocol [2]. Based on this set, they have proposed an RDT protocol that enforces this characterization using $O(n^2)$ control information, claiming that this protocol is optimal with respect to the size of control information [3].

Recently, we have proved that their conjecture was false and the set of Z-paths that must be tested for breaking by an RDT protocol can be further reduced to the set of "non-visibly-doubled-PMM-paths" [5]. In this paper, extending the approach used to prove the conjecture false, we describe a protocol that enforces this minimal characterization of RDT requiring only $O(n)$ control information.

This paper is structured as follows. Section 2 describes the computational model adopted. Section 3 introduces rollback-dependency trackability. Section 4 describes a quadratic approach to enforce the minimal characterization of RDT, similar to the one suggested by Baldoni, Helary, Mostefaoui, and Raynal [1, 3]. Section 5 presents a linear approach to enforce the minimal characterization of RDT. Section 6 concludes the paper.

2 Computational model

A distributed computation is composed of n sequential processes $\{p_0, \ldots, p_{n-1}\}$ that communicate only by exchanging messages. Messages cannot be corrupted, but can be delivered out of order or lost. The local history of a process p_i is modeled as a possibly infinite sequence of events $(e_i^1, e_i^2, \ldots)$, divided into internal events and communication events.

A checkpoint is an internal event that records the process' state in stable memory. Each process takes an initial checkpoint immediately after execution begins and a final checkpoint immediately before execution ends. Let c_i^γ denote the γth checkpoint taken by p_i. Two successive checkpoints $c_i^{\gamma-1}$ and c_i^γ, $\gamma > 0$, define a checkpoint interval I_i^γ. An event e_i^ι belongs to I_i^γ ($e_i^\iota \in I_i^\gamma$) if it occured in p_i after $c_i^{\gamma-1}$, but not after c_i^γ. Figure 1 illustrates a checkpoint interval I_i^γ and an event e_i^ι that belongs to I_i^γ.

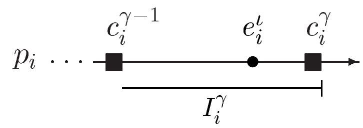


Figure 1: A checkpoint interval

The set of all checkpoints taken by a distributed computation and the dependencies established among these checkpoints due to message exchanges form a checkpoint and communication pattern (CCP). Figure 2 illustrates a CCP using a space-time diagram [9] augmented with checkpoints (black squares).

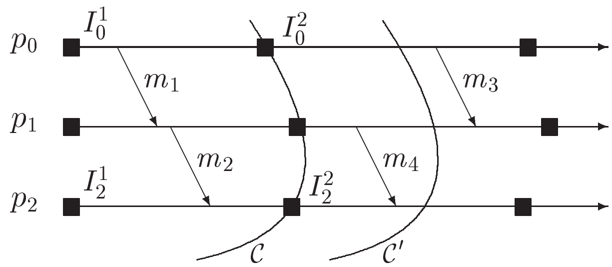


Figure 2: A distributed computation

2.1 Consistency

The concept of causal precedence is fundamental for a better understanding of consistency [9].

Definition 2.1 Causal precedence—*Event e_a^α causally precedes e_b^β ($e_a^\alpha \rightarrow e_b^\beta$) if (i) $a = b$ and $\beta = \alpha+1$; (ii) $\exists m : e_a^\alpha = send(m)$ and $e_b^\beta = receive(m)$; or (iii) $\exists e_c^\gamma : e_a^\alpha \rightarrow e_c^\gamma \wedge e_c^\gamma \rightarrow e_b^\beta$.*

A cut of a distributed computation contains a prefix of each of the proceses' local histories. A consistent cut is left-closed under causal precedence and defines an instant in a distributed computation [4]. If a cut $\mathcal{C} \subset \mathcal{C}'$, we can say that $\mathcal{C}$ is in the past of $\mathcal{C}'$ (Figure 2).

Definition 2.2 Consistent Cut—*A cut $\mathcal{C}$ is consistent if, and only if,*

$$e \in \mathcal{C} \land e' \to e \Rightarrow e' \in \mathcal{C}$$

A consistent global state is formed by the states of each process in the fronteir of a consistent cut [4]. The set of consistent global checkpoints is a subset of the set of consistent global states. In Figure 2, $\mathcal{C}$ is related to a consistent global checkpoint, but $\mathcal{C}'$ is not.

2.2 Zigzag paths

Netzer and Xu have determined that checkpoints that are part of the same consistent global checkpoint cannot be related by sequences of messages called *zigzag paths* [10].

Definition 2.3 Zigzag path—*A sequence of messages $\mu = [m_1, \ldots, m_k]$ is a zigzag path from I_a^α to I_b^β if (i) p_a sends m_1 after $c_a^{\alpha-1}$; (ii) if m_i, $1 \leq i < k$, is received by p_c, then m_{i+1} is sent by p_c in the same or a later checkpoint interval; (iii) m_k is received by p_b before c_b^β.*

Two types of zigzag paths can be identified: (i) causal paths (C-paths) and (ii) non-causal paths (Z-paths). A zigzag path is causal if the reception of m_i, $1 \leq i < k$, causally precedes the send event of m_{i+1}. In Figure 2, $[m_1, m_2]$ is a C-path from I_0^1 to I_2^1 and $[m_3, m_4]$ is a Z-path from I_0^2 to I_2^2. A Z-path that starts in a checkpoint interval and finishes in a previous checkpoint interval of the same processes is a Z-cycle and identifies a useless checkpoint, that is, a checkpoint that cannot be part of any consistent global checkpoint [10]. Figure 3 presents a Z-cycle $[m_1, m_2, m_3]$ and a useless checkpoint c_i^γ.

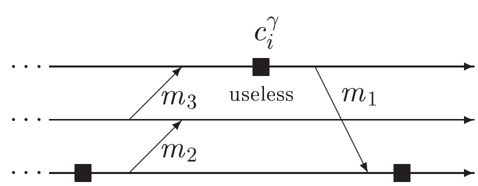


Figure 3: A Z-cycle

3 Rollback Dependency Trackability

The literature presents two approaches to define RDT. The first one is based on the study of on-line trackable dependencies, implemented through the use of dependency vectors [12]; the other one is based on the study of sequence of messages [1, 2, 3, 5].

3.1 Dependency vectors

A transitive dependency tracking mechanism can be used to capture causal dependencies among checkpoints. Each process maintains and propagates a size-n dependency vector. Let dv_i be the dependency vector of p_i, $m.dv$ be the dependency vector piggybacked on a message m, and $dv(c)$ be the dependency vector associated to a checkpoint c. All entries of dv_i are initialized to 0. The entry $dv_i[i]$ represents the current interval of p_i and it is incremented immediately after a checkpoint (including the initial one). Every other entry $dv_i[j]$, $j \neq i$, represents the highest interval index of p_j that p_i has knowledge about and it is updated using a component-wise maximum every time a message m with a greater value of $m.\,dv[j]$ arrives to p_i. Figure 4 depicts the dependency vectors established during a distributed computation.

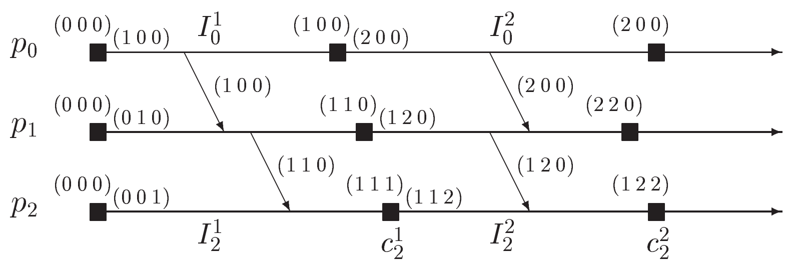


Figure 4: A distributed computation with dependency vectors

Note in Figure 4 that $dv(c_2^1)$ is (1, 1, 1) and it correctly captures all zigzag paths that reach I_2^1. Unfortunately, not all dependencies can be tracked on-line. For example, $dv(c_2^2)$ is $(1, 2, 2)$ and it does not capture the zigzag path from I_0^2 to I_2^2.

Definition 3.1 On-line trackability—*A zigzag path from I_a^α to I_b^β is on-line trackable through the use of dependency vectors if $dv(c_b^\beta)[a] \geq \alpha$.*

Definition 3.2 Dependency vector characterization of RDT—*A checkpoint pattern satisfies RDT if all zigzag paths are on-line trackable.*

RDT is a desirable property because efficient algorithms can be used to construct consistent global checkpoints if all zigzag paths are on-line trackable. Also, an RDT checkpoint pattern does not admit useless checkpoints [12].

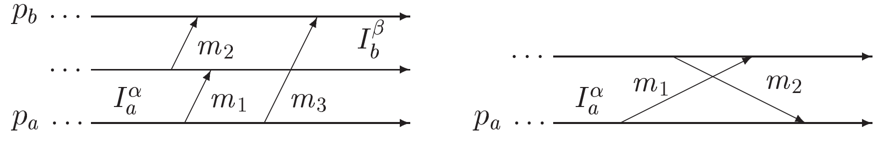
3.2 Causal doubling

A CCP may present Z-paths and satisfy RDT if all Z-paths are *doubled* by a C-path [2, 3].

Definition 3.3 Causal doubling—*A Z-path from I_a^α to I_b^β is* causally doubled *if there is a C-path μ from I_a^α to I_b^β or $a = b$ and $\alpha \leq \beta$.*

Definition 3.4 Message-based characterization of RDT—*A checkpoint pattern satisfies RDT if all Z-paths are causally doubled.*

A Z-path can be doubled by a causal one if the pair of checkpoints related by that Z-path is also related by a C-path [2, 3]. Another possibility for a Z-path from I_a^α to I_b^β to be doubled is if it starts and finishes in the same process and I_b^β does not precede I_a^α. In Figure 5 (a), $[m_1, m_2]$ is causally doubled by m_3 and in Figure 5 (b), $[m_1, m_2]$ is trivially doubled due to the execution of p_a.



(a) $[m_1, m_2]$ is doubled by m_3

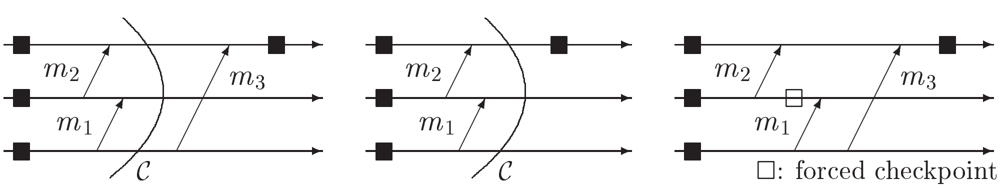
(b) $[m_1, m_2]$ is trivially doubled

Figure 5: Causal doubling

3.3 RDT protocols

A communication-induced checkpointing protocol that enforces RDT allows processes to take checkpoints asynchronously, but they may be induced by the protocol to take forced checkpoints in order to break non-trackable dependencies [1, 12]. Forced checkpoints must be taken upon the arrival of a message, but before this message is processed by the computation. The decision to take a forced checkpoint must be based only on the local knowledge of a process; there are no control messages, no global knowledge or knowledge about the future of the computation. These assumptions impose some restrictions on the set of CCPs that can be produced by RDT protocols.

The CCP depicted in Figure 6 (a), for example, would never have been produced by an RDT procotol. This pattern has a Z-path $[m_1, m_2]$ that is doubled by message m_3 in the future of a consistent cut $\mathcal{C}$. At $\mathcal{C}$, the processes of the computation cannot rely on the existence of m_3, since a scenario such as the one depicted in Figure 6 (b) could have happened, producing a CCP that does not satisfy RDT. Under an RDT protocol, the CCP presented in Figure 6 (a) should present at least one forced checkpoint (Figure 6 (c)).



(a) (b) (c)

Figure 6: The behavior of RDT protocols

The RDT property must be enforced in every consistent cut of a computation that runs an RDT protocol. This observation has lead us to introduce the concept of left-doubling [5].

3.4 Left-doubling

A C-path μ belongs to a consistent cut $\mathcal{C}$ if the reception of the last message of μ belongs to $\mathcal{C}$. A Z-path ζ can be seen as a concatenation of ℓ C-paths $\mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_\ell$ and ζ belongs to a consistent cut $\mathcal{C}$ if all causal components of ζ belong to $\mathcal{C}$.

Definition 3.5 Left-doubling—*A Z-path ζ is left-doubled in relation to a consistent cut $\mathcal{C}$ if (i) ζ belongs to $\mathcal{C}$ and (ii) ζ is doubled by a C-path μ that also belongs to $\mathcal{C}$.*

A consistent cut $\mathcal{C}$ satisfies the RDT property if, and only if, all Z-paths that belong to $\mathcal{C}$ are left-doubled. Using the concept of left-doubling, we have proved that a protocol that breaks all "non-visibly doubled PMM-paths" must enforce RDT [5].

3.5 The minimal characterization of RDT

Definition 3.6 PMM-path—*A PMM-path is a Z-path composed of a **p**rime **m**essage m_1 and a **m**essage m_2.*

A message m from I_k^κ to p_i is *prime* if m is the first message received by p_i that brings information about I_k^κ. Figure 7 presents a PMM-path $[m_1] \cdot [m_2]$ from I_k^κ to I_j^γ.

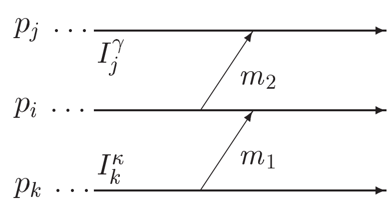


Figure 7: A PMM-path

Definition 3.7 Visibly Doubled PMM-path—*A PMM-path $[m_1]\cdot[m_2]$ is visibly doubled if (i) is causally doubled by a C-path μ and (ii) the reception of the last message of μ causally precedes the sending of m_1.*

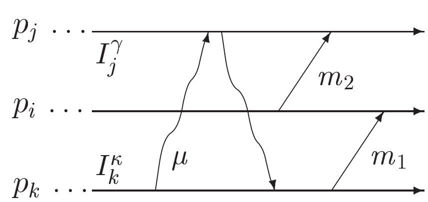


Figure 8: A visibly doubled PMM-path

Figure 8 presents a visibly doubled PMM-path $[m_1] \cdot [m_2]$ from I_k^κ to I_j^γ. We should note that $[m_1] \cdot [m_2]$ is left-doubled in relation to any consistent cut of the computation, since any consistent cut that contains m_1 should also contain μ. The set of "non-visibly-doubled PMM-paths" characterizes the minimal set of Z-paths that be must tested for breaking by an RDT protocol [5].

Definition 3.8 The minimal characterization of RDT—*A CCP satisfies the RDT property if all PMM-paths are visibly doubled.*

In the following sections, we are going to focus on the problem of implementing an RDT protocol that enforces this minimal characterization.

4 A quadratic approach

The core of a protocol that enforces the minimal characterization of RDT lies on the detection of non-visibly doubled PMM-paths by a process p_i. Let us consider a PMM-path $[m_1] \cdot [m_2]$ from I_k^κ to I_j^γ such that m_1 is received by p_i after the sending of m_2 (Figure 7). Before processing m_1, p_i must detect the establishment of this PMM-path and verify whether it is visibly doubled (Figure 8). If $[m_1] \cdot [m_2]$ is visibly doubled, m_1 can be processed immediately. Otherwise, p_i must take a forced checkpoint before processing m_1.

4.1 Detecting PMM-paths

In order to detect all PMM-paths formed upon the reception of a message, process p_i must record for what processes it has sent messages during the current interval. To do this, p_i maintains a vector of booleans $sent\_to_i$, such that all entries of $sent\_to_i$ are set to `false` when p_i takes a checkpoint, and an entry $sent\_to_i[j]$ is set to `true` when p_i sends a message to p_j. A PMM-path is detected by p_i upon the reception of a message m from p_k when the following condition holds:

$$\exists\, j : sent\_to_i[j] \wedge m.\,dv[k] > dv_i[k]$$

4.2 Detecting non-visibly doubled PMM-paths

The detection of whether $[m_1] \cdot [m_2]$ from I_k^κ to I_j^γ is visibly doubled by a C-path μ can be divided into two cases:

I. From the point of view of p_i, the interval I_j^γ is in the past of p_j
Figure 9 shows a scenario in which p_i receives knowledge that m_2 was received during I_j^γ, but μ was received during $I_j^{\gamma+1}$. Since a C-path μ from I_k^κ to $I_j^{\gamma+1}$ does not double a PMM-path from I_k^κ to I_j^γ, process p_i must take a forced checkpoint before processing m_1.

To detect visibly doubled PMM-paths, p_i must evaluate (i) whether m_2 was received by p_j and in which checkpoint interval, and (ii) whether p_j has received knowledge about I_k^κ and in which checkpoint interval. For p_i to be able to answer these questions, the processes of the computation would have to maintain and propagate an **unbounded** amount of control information, proportional to the number of messages and checkpoint intervals.

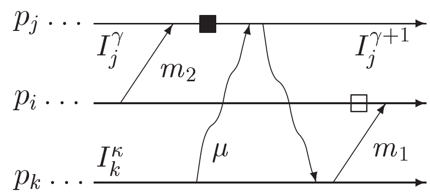


Figure 9: I_j^γ is in the past of p_j

II. From the point of view of p_i, the interval I_j^γ is not in the past of p_j
Figure 10 presents three scenarios to show that the problem of detecting visibly doubled paths is much easier when in p_i's view I_j^γ is not in the past of p_j. In Figure 10 (a), p_i receives knowledge that both m_2 and μ were received during I_j^γ. In Figures 10 (b, c), p_i receives knowledge that μ was received by p_j, but p_i does not receive knowledge about the reception of m_2. In these cases, the existence of a C-path μ from I_k^κ to p_j guarantees to p_i that $[m_1] \cdot [m_2]$ is causally doubled.

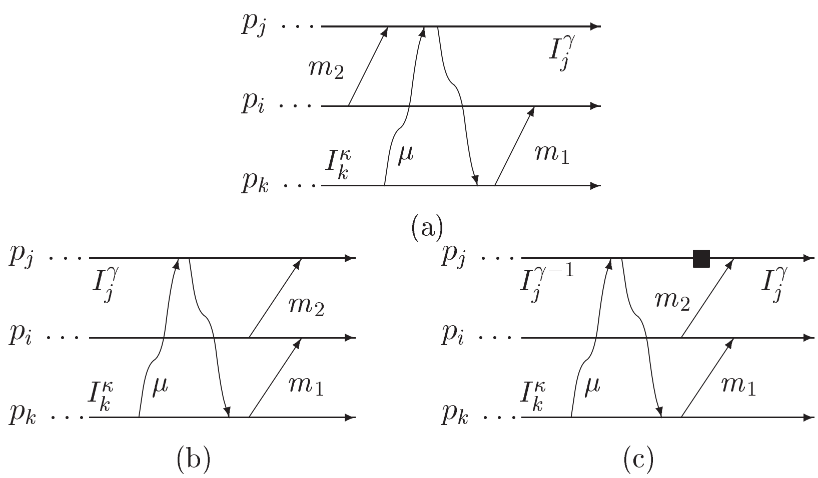


Figure 10: I_j^γ is not in the past of p_j

Fortunately, there is an approach to handle case **I** that requires only $O(n)$ control information, as explained in next Section. Section 4.4 shows an $O(n^2)$ approach to handle case **II**. In Section 5, we are going to prove that case **II** can also be handled in $O(n)$.

4.3 Process p_i knows that I_j^γ is in the past

Let us assume that, upon the reception of m_1, p_i knows that m_2 was received in an interval that is on the past of p_j. In Figure 11 (a), p_i receives knowledge about $I_j^{\gamma+1}$ due a C-path ν that arrives to p_i before m_1. The concatenation of ν and m_2 forms a Z-cycle. Since the RDT property does not allow Z-cycles, p_i should have taken a forced checkpoint before processing the last message of ν. The information about $I_j^{\gamma+1}$ could have arrived with m_1. In Figure 11 (b), p_k receives knowledge about $I_j^{\gamma+1}$ due a C-path ν that arrives to p_k before the sending of m_1. The concatenation of ν, m_1 and m_2 also forms a Z-cycle and p_i should have taken a forced checkpoint before processing m_1.

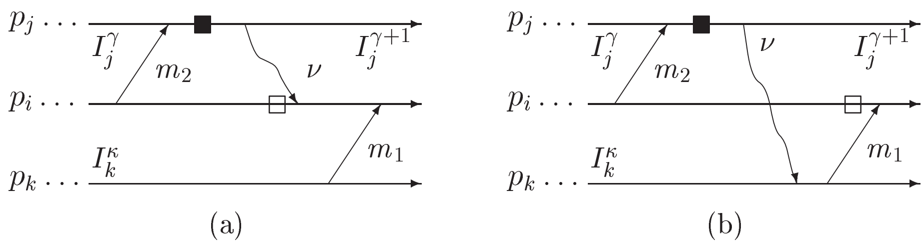


Figure 11: Process p_i knows that I_j^γ is in the past

The Z-cycle $[\nu]\cdot[m_2]$ of Figure 11 (a) and the Z-cycle $[\nu\cdot m_1]\cdot[m_2]$ of Figure 11 have only two causal components. Z-cycles formed by two causal components $[\nu_1]\cdot[\nu_2]$ and are called CC-cycles (Figure 12). CC-cycles can be easily detected and breaked on-line if p_i takes a forced checkpoint before processing the last message of ν_1. Process p_i must take the forced checkpoint only if $[\nu_2]\cdot[\nu_1]$ "contains" a checkpoint, that is, it is not *simple* [1, 3].

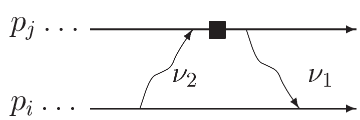


Figure 12: A CC-cycle $[\nu_1]\cdot[\nu_2]$

To keep track of simple paths, each process maintains and propagates a size-n boolean vector *simple*. Let $simple_i$ be the vector maintained by p_i, and $m.simple$ be the vector piggybacked on a message m. The entry $simple_i[i]$ is always `true`, and the entries $simple_i[k]$, $k \neq i$, are reset to `false` when p_i takes a checkpoint. When p_i receives a message m, each entry $simple_i[k]$ is updated as follows:

if $m.dv[k] > dv_i[k]$ **then** $simple_i[k] \leftarrow m.simple[k]$

if $m.dv[k] = dv_i[k]$ **then** $simple_i[k] \leftarrow simple[k] \wedge m.simple[k]$

Process p_i detects a CC-cycle upon the reception of m using the following condition:

$$m.dv[i] = dv_i[i] \wedge m.simple[i] = \texttt{false}$$

In the scenarios of Figure 11, the second causal component of the CC-cycle is represented by a single message m_2. However, keeping track of only CC-cycles of the form $[\nu][m]$ would increase the complexity of the required control information due to the propagation of knowledge about single messages. Also, as an RDT protocol must break all CC-cycles, using the above condition does not increase the number of forced checkpoints.

4.4 Tracking C-paths from I_k^κ to p_j

Even if a process p_i breaks CC-cycles, PMM-paths such as the ones described in case **II** of Section 4.2 (Figure 10) needed to be tested for breaking. Thus, if p_i has sent a message m_2 to p_j and receives a prime message m_1 from I_k^κ, p_i must verify whether there is a C-path μ from I_k^κ to p_j. We should note that before the reception of m_1, process p_i cannot have information about μ, otherwise m_1 would not be prime (Figure 13). Thus, the information about μ can only arrive on the control information piggybacked on m_1.

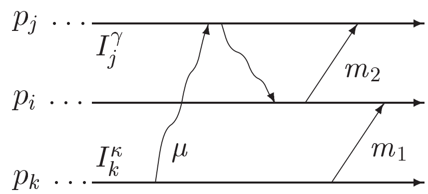


Figure 13: Message m_1 is not prime

According to the definition of dependency vectors, $dv_i[k]$ is the greatest interval index of p_k that p_i has knowledge about. Let us consider a matrix of booleans $causal_i$ such that each entry $causal_i[k][j]$ indicates whether, up to the knowledge of p_i, there is a C-path from $I_k^{dv_i[k]}$ to p_j. The entries on the diagonal of $causal_i$ are always `true`, since there is a trivial causal flow from a process to itself. Every other entry $causal_i[k][j], k \neq j$ is initialized to `false`. When p_i takes a checkpoint, all entries $causal_i[i][j], i \neq j$ are reset to `false`, indicating that there is no C-path from this new interval. Let $m.\,causal$ be a matrix piggybacked on a message m. When p_i receives m from p_s, $causal_i$ is updated as follows:

$$\forall k, \text{ if } m.\,dv[k] > dv_i[k] \text{ then } \forall l : causal_i[k][l] \leftarrow m.\,causal[k][l]$$

$$\forall k, \text{ if } m.\,dv[k] = dv_i[k] \text{ then } \forall l : causal_i[k][l] \leftarrow causal_i[k][l] \vee m.\,causal[k][l]$$

$$causal_i[s][i] \leftarrow \texttt{true}$$

$$\forall l : causal_i[l][i] \leftarrow causal_i[l][i] \vee causal_i[l][s]$$

4.5 Checkpoint induction condition

A forced checkpoint is induced by p_i upon the reception of a prime message m from I_k^κ if (i) there is a CC-cycle or (ii) there is a PMM-path from p_k to p_j, but there is no C-path from I_k^κ to p_j:

$$\begin{array}{ll} \text{(i)} & (m.\,dv[i] = dv_i[i] \wedge m.\,simple[i] = \texttt{false}) \vee \\ \text{(ii)} & (\exists j : sent\_to_i[j] \wedge m.\,dv[k] > dv_i[k] \wedge \neg m.\,causal_i[k][j]) \end{array}$$

The approach presented in this Section is similar to the one presented by Baldoni, Helary, Mostefaoui and Raynal, although their protocols break more complex Z-paths [1, 3]. Their approach requires $O(n^2)$ control information since it tracks the existence of C-paths from every process p_k to every process p_j of the computation. Baldoni, Helary, and Raynal claim that $O(n^2)$ is optimal with respect to the size of the control information [3]. In the next Section, we are going to show an $O(n)$ RDT protocol that breaks visibly-doubled Z-paths.

5 A linear approach

Linearity of the control information comes as a result of two observations: (i) we do not need to keep track of C-paths from I_k^κ to p_j, instead, we are going to take advantage of dependency vector restrictions imposed by an RDT protocol and (ii) we can perform comparison operations on dependency vectors as a whole instead of keeping track of single entries, as in Definition 3.1. This alternative approach is the key to the complexity reduction. Thus, let us define the following comparison operations:

$$\begin{array}{lcl} dv(c_j^\gamma) \geq dv(c_k^\kappa) & \Leftrightarrow & \forall i, 0 \leq i < N, \quad dv(c_j^\gamma)[i] \geq dv(c_k^\kappa)[i] \\ dv(c_j^\gamma) = dv(c_k^\kappa) & \Leftrightarrow & \forall i, 0 \leq i < N, \quad dv(c_j^\gamma)[i] = dv(c_k^\kappa)[i] \end{array}$$

5.1 Dependency vector restrictions under RDT

Let us begin with dependency vector restrictions that must hold for all CCPs, not only on CCPs produced by RDT protocols.

Theorem 5.1 *Under RDT, the existence of a C-path μ from I_k^κ to I_j^γ guarantees that $dv(c_j^\gamma) \geq dv(c_k^\kappa)$.*

Proof: For the sake of contradiction, let us assume the existence of an entry l of $dv(c_j^\gamma)$ such that $dv(c_j^\gamma)[l] < dv(c_k^\kappa)[l] = \lambda$ (Figure 14). The information about I_l^λ must have arrived at p_k due to a C-path μ' and the last message of μ' must have been received after the sending of the first message of μ. The concatenation of μ and μ' forms a Z-path from I_l^λ to I_j^γ that is not on-line trackable (a non-causally doubled Z-path). □

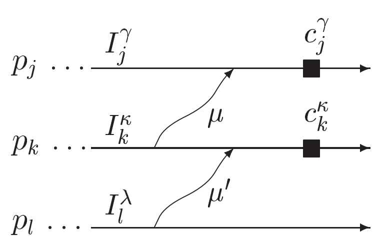


Figure 14: Contradiction hypothesis of Theorem 5.1

Corollary 5.2 *Under RDT, the existence of a C-path μ from I_k^κ to I_j^γ and of a C-path μ' from I_j^γ to I_k^κ guarantees that $dv(c_j^\gamma) = dv(c_k^\kappa)$.*

Proof: Due to μ, $dv(c_j^\gamma) \geq dv(c_k^\kappa)$. Due to μ', $dv(c_k^\kappa) \geq dv(c_j^\gamma)$. Thus, $dv(c_j^\gamma) = dv(c_k^\kappa)$. □

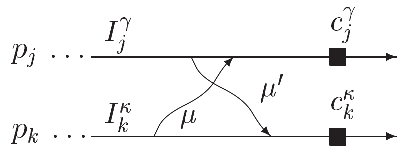


Figure 15: The existence of μ and μ' guarantees that $dv(c_j^\gamma) = dv(c_k^\kappa)$

5.2 Dependency vector restrictions under an RDT protocol

Since an RDT protocol must enforce RDT in every consistent cut of the computation, let us explore dependency vector restrictions during the progress of checkpoint intervals.

Theorem 5.3 *Let μ be a C-path μ from I_k^κ to I_j^γ and let $\mathcal{C}$ be a consistent cut that contains μ. Let $e_j^{\gamma'} \in I_j^\gamma$ and $e_k^{\kappa'} \in I_k^\kappa$ be the events of p_j and p_k that belong to the fronteir of $\mathcal{C}$. Under an RDT protocol, the following restriction should hold: $dv(e_j^{\gamma'}) \geq dv(e_k^{\kappa'})$.*

Proof: For the sake of contradiction, let us assume the existence of an entry l of $dv(e_j^{\gamma'})$ such that $dv(e_j^{\gamma'})[l] < dv(e_k^{\kappa'})[l] = \lambda$ (Figure 16). The information about I_l^λ must have arrived at p_k due to a C-path μ' and the last message of μ' must have been received after the sending of the first message of μ. The concatenation of μ and μ' forms a Z-path from I_l^λ to I_j^γ that is not left-doubled in relation to $\mathcal{C}$. □

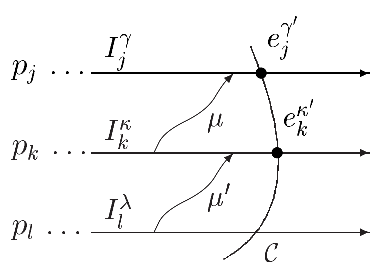


Figure 16: Contradiction hypothesis of Theorem 5.3

Corollary 5.4 *Let μ be a C-path μ from I_k^κ to I_j^γ and μ' be a C-path from I_j^γ to I_k^κ. Let $\mathcal{C}$ be a consistent cut that contains μ and μ'. Let $e_j^{\gamma'} \in I_j^\gamma$ and $e_k^{\kappa'} \in I_k^\kappa$ be the events of p_j and p_k that belong to the frontier of $\mathcal{C}$. Under an RDT protocol, the following restriction should hold: $dv(e_j^{\gamma'}) = dv(e_k^{\kappa'})$.*

Proof: Due to μ, $dv(e_j^{\gamma'}) \geq dv(e_k^{\kappa'})$. Due to μ', $dv(e_k^{\kappa'}) \geq dv(e_j^{\gamma'})$. Thus, $dv(e_j^{\gamma'}) = dv(e_k^{\kappa'})$. □

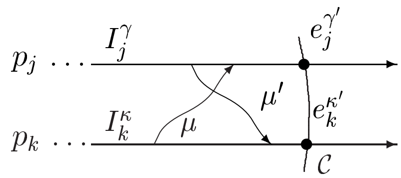


Figure 17: The existence of μ and μ' guarantees that $dv(e_j^{\gamma'}) = dv(e_k^{\kappa'})$

5.3 Process p_k knows that $dv_j = dv_k$

Let μ be a C-path from I_k^κ to I_j^γ and μ' be a C-path from I_j^γ to I_k^κ such that the last message of μ is received before the first message of μ' is sent (Figure 18). Upon the arrival of μ', p_k receives knowledge about μ and, according to Corollary 5.4, it is also able to conclude that $dv_j = dv_k$.

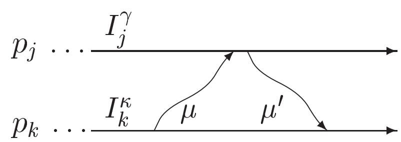


Figure 18: Process p_k knows that $dv_j = dv_k$

The following theorem shows that the verification of equal dependency vectors can replace the verification of the existence of C-paths.

Theorem 5.5 *Let I_k^κ be the current interval of a process p_k. Under an RDT protocol, p_k knows the existence of a C-path μ from I_k^κ to p_j if, and only if, to the knowledge of p_k, $dv_j = dv_k$.*

Proof:

(i) $dv_j = dv_k \Rightarrow$ a C-path μ from I_k^κ to p_j

Since $dv_j = dv_k$, $dv_j[k] = dv_k[k] = \kappa$ and there must be a C-path μ from I_k^κ to p_j.

(ii) a C-path μ from I_k^κ to $p_j \Rightarrow dv_j = dv_k$

Process p_k must have received knowledge about μ due to a C-path μ' from p_j to p_k. The first message of μ' must have been sent after the reception of the last of μ. Since an RDT protocol does not allow CC-cycles, there cannot be a checkpoint between the reception of the last of μ and the sending of first message of μ'. Thus, these two events occured in the same checkpoint interval and, according to Corollary 5.4, upon the reception of the last message of μ', p_k knows that $dv_j = dv_k$ (Figure 18).

Also, p_k will not be able to increase its dependency vector till the end of I_k^κ. For the sake of contradiction, let us assume that p_k receives information about I_l^λ through a C-path

ν after the reception of the last message of μ'. According to Corollary 5.2, $dv(c_j^\gamma)$ should be equal to $dv(c_k^\kappa)$, and p_j must also receive information about I_l^λ through a C-path ν'. Let $\mathcal{C}$ be the minimum consistent cut that cointains μ', that is, the cut formed by the the reception of the last message of μ' and all the events that causally precede this reception (Figure 19). Thus, at $\mathcal{C}$, neither p_j nor p_k have knowledge about I_l^λ. From $\mathcal{C}$ is possible to construct a sequence of consistent cuts that reflect the progress of the computation, adding one event at a time. Either ν or ν' is going to be included first during the sequence. If ν is included first, we would have a consistent cut, say $\mathcal{C}'$, such that $[\nu] \cdot [\mu]$ is not left-doubled in relation to $\mathcal{C}'$ (Figure 19). Analogously, if ν' is included first, we would have a consistent cut such that $[\nu'] \cdot [\mu']$ is not left-doubled in relation to it. □

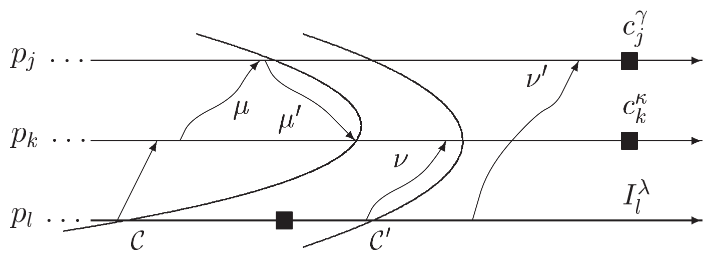


Figure 19: Contradiction hypothesis of Theorem 5.5

Corollary 5.6 *Under an RDT protocol, p_k knows that $dv_k = dv_j$ if $dv_k[k] = dv_j[k]$.*

Proof: Since $dv_k[k] = dv_j[k]$, there must exist a C-path from the current interval of p_k to p_j and, according to Theorem 5.5, $dv_k = dv_j$. □

5.4 Keeping track of equal dependency vectors

Each process maintains and propagates a size-n boolean vector *equal*. Let $equal_i$ be the vector maintained by p_i, and $m.\,equal$ be the vector piggybacked on a message m. The entry $equal_i[i]$ is always `true`, and the entries $equal_i[k]$, $k \neq i$, are reset to `false` when p_i takes a checkpoint. When p_i receives a message m from p_k without taking a forced checkpoint, it must update $equal_i$. If $m.\,dv[i] = dv_i[i]$, p_i learns that $dv_i = dv_k$ (Figure 20 (a)). If up to the knowledge of p_k when it sent m there is a process p_j such that $dv_k = dv_j$, p_i also learns that $dv_i = dv_j$ (Figure 20 (b)). This behavior can be summarized as follows:

`if` $m.\,dv[i] = dv_i[i]$ `then` $\forall j : equal_i[j] \leftarrow equal_i[j] \vee m.\,equal[j]$

There is no need for p_k to propagate additional information about dependency vectors, because if, up to the knowledge of p_k, $dv_j \neq dv_k$, p_i cannot derive from any information contained in m that $dv_i = dv_j$. According to Corollary 5.6, to the knowledge of p_k, $dv_j[k] \neq dv_k[k]$. When p_i receives m, $dv_i[k] = dv_k[k] \neq dv_j[k]$ and $dv_i \neq dv_j$ (Figure 20 (c)). Thus, keeping track of equal dependency vectors requires only $O(n)$ control information.

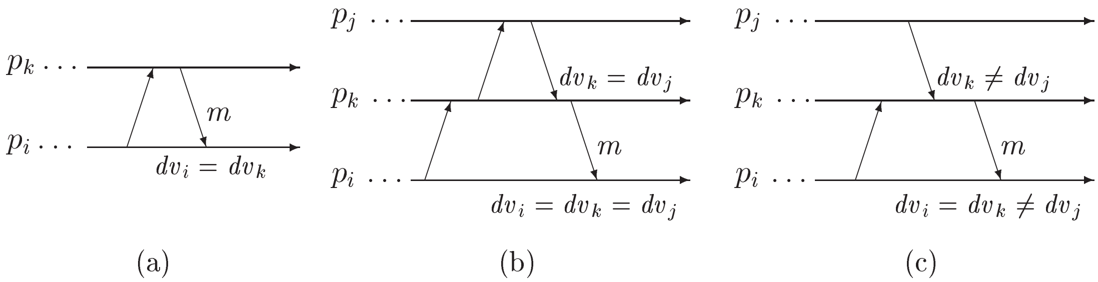


Figure 20: Updating $equal_i$

5.5 Checkpoint induction condition

A forced checkpoint is induced by p_i upon the reception of a prime message m from p_k if (i) there is a CC-cycle or (ii) there is a PMM-path from p_k to p_j, but $dv_k \neq dv_j$:

$$\begin{array}{ll} \text{(i)} & (m.\,dv[i] = dv_i[i] \wedge m.\,simple[i] = \texttt{false}) \vee \\ \text{(ii)} & (\exists j : sent\_to_i[j] \wedge m.\,dv[k] > dv_i[k] \wedge \neg m.\,equal[j]) \end{array}$$

We should note the above checkpoint induction condition is analogous to the one presented in Section 4.5. The only difference is the replacement of the test $\neg m.\,causal[k][j]$ for $\neg m.\,equal[j]$.

5.6 Optimizations

Let us continue to explore properties of the processes' behavior under an RDT protocol to simplify an implementation of the minimal characterization of RDT.

Theorem 5.7 *If p_i receives a non-prime message m, all entries of $m.\,dv$ are known by p_i.*

Proof: Assume that m was sent by p_k during I_k^κ, and there is a C-path μ from I_k^κ to p_i such that μ arrived to p_i before m. Assume that $m.\,dv[l] = \lambda > dv_i[l]$ and let μ' be a C-path from p_l to p_k that arrived to p_k after the sending of the first message of μ and before the sending of m. It is possible to construct a consistent cut $\mathcal{C}$ such that $\mu'.\mu$ is not left-doubled in relation to $\mathcal{C}$ (Figure 21). □

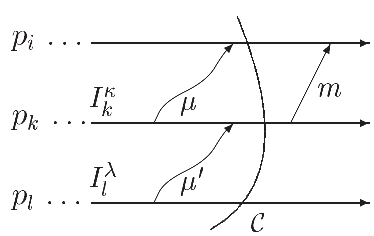


Figure 21: Contradiction hypothesis of Theorem 5.7

Due to Theorem 5.7, upon the reception of a non-prime message m, there is no need to check and update dv_i. Also, since an entry of $simple_i$ can change only if at least one entry of dv_i has changed, the updating of $simple_i$ can be skipped.

According to the second part of the proof of Theorem 5.5, when p_k knows that p_j knows its current interval, say I_k^κ, p_k cannot increase dv_k till the end of I_k^κ. Thus, we can divide a checkpoint interval of any process p_i into three phases:

Phase 0: while no message has been sent, no PMM-path can be formed, and dv_i can incorporate new dependencies without restrictions.

Phase 1: after at least one message has been sent, dv_i can change according to the induction condition presented in Section 5.5.

Phase 2: after p_i has received knowledge about other process with an equal dependency vector, no new dependency can be incorporated into dv_i.

Unfortunately, the updating of vector *equal* cannot benefit from the described optimizations. Figure 22 illustrates that vector *equal* must be updated even if no new dependency is established. When p_1 receives the second message from p_0, it does not change dv_1. However, p_1 receives knowledge that $dv_1 = dv_0$. Using this information, p_2 will be able to save a forced checkpoint when it receives a message from p_1.

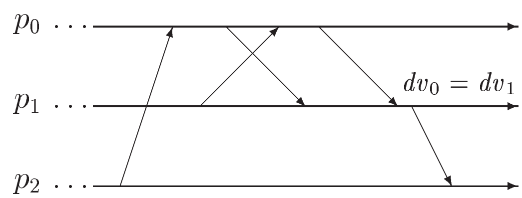


Figure 22: Vector *equal* must be updated even if no new dependency is established

Figure 23 illustrates that vector *equal* must be updated during phase 2. Process p_2 starts phase 2 when it receives a message from p_1 and it learns that $dv_1 = dv_2$. When p_0 receives a message from p_1, it learns that $dv_0 = dv_1$. Also, when p_0 sends a message to p_2, p_2 learns that $dv_0 = dv_2$. Using this information p_3 will be able to save a forced checkpoint when it receives a message from p_2. Thus, even if a process is in the phase 2 of the algorithm, it must continue to update vector *equal* because the collected information may help other processes to save checkpoints.

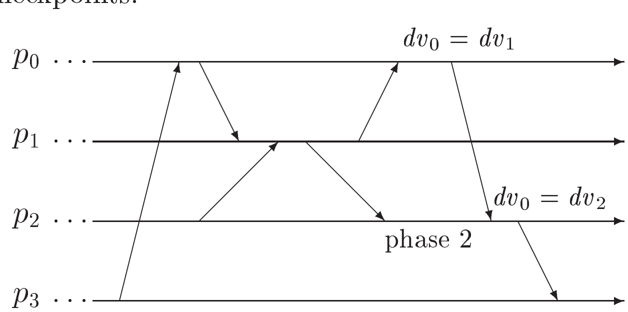


Figure 23: Vector *equal* must be updated during phase 2

An implementation of the minimal characterization of RDT including all optimizations is described in Class `RDT_Minimal`, using Java[1] [7].

Class RDT_Minimal.java

```
public class RDT_Minimal {
    public static int N = 100;                        // Number of processes in the application
    public int pid;                                   // Unique process' identifier
    protected int [ ] dv = new int[N];     // Dependency vector, automatically initialezed to (0,...,0)
    protected boolean [ ] equal = new boolean [N];    // Keeps track of equal dependency vectors
    protected boolean [ ] simple = new boolean [N];   // Keeps track of simple paths
    protected boolean [ ] sent_to = new boolean [N];  // Keeps track of sent messages
    public int phase;                                 // Keeps track of interval phase

    public class Message {
        public int sender, receiver;
        public int [ ] dv;
        public boolean [ ] equal;
        public boolean [ ] simple;
    }

    public void takeCheckpoint() {
      // Write state into stable memory
        for (int i=0; i < N; i++) {                   // Reset control vectors
            equal[i] = false;
            simple[i] = false;
            sent_to[i] = false;
        }
      equal[pid] = true;
      simple[pid] = true;
      dv[pid]++;                                      // Increment dependency vector
      phase = 0;                                      // Reset phase counter
    }

    public RDT_Minimal(int pid) { this.pid=pid; }   // Constructor

    public void run() { takeCheckpoint(); }   // Start execution

    public void finalize() { takeCheckpoint(); }   // Finish execution

    public void sendMessage(Message m) {
      m.dv = (int [ ]) dv.clone();                    // Piggybacks control information
      m.equal = (boolean [ ]) equal.clone();
      m.simple = (boolean [ ]) simple.clone();
      sent_to[m.receiver] = true;
      if (phase == 0) phase = 1;
      // Send message
    }
```

[1] Java is a trademark of Sun Microsystems, Inc.

Class RDT_Minimal.java

```
    private boolean mustTakeForcedCheckpoint(Message m) {
        if (phase == 0) return false;                    // Every new dependency can be accepted
        if (phase == 2) return true;                      // No new dependency can be accepted
        if (m.dv[pid] == dv[pid] && !m.simple[pid]) return true;    // Non causally doubled CC-cycle
        for (int i=0; i < N; i++)                  // Verify whether all PMM-paths are visibly doubled
            if (sent_to[i] && !m.equal[i]) return true;
      return false;
    }

    public void receiveMessage(Message m) {
      if (m.dv[m.sender] > dv[m.sender]) {                                     // New dependency
        if (mustTakeForcedCheckpoint(m))
            takeCheckpoint();
        for (int i=0; i < N; i++)                                      // Dependency vector update
            if (m.dv[i] > dv[i]) {
                dv[i] = m.dv[i];
                simple[i] = m.simple[i];
            } else if (m.dv[i] == dv[i])
                simple[i] = simple[i] && m.simple[i];
      }
      if (m.dv[pid] == dv[pid]) {                                                // m.dv == dv
          for (int i=0; i < N; i++)
              equal[i] = equal[i] || m.equal[i];
          phase = 2;
      }
      // Message is processed by the application
    }
}
```

6 Conclusion

The simplest RDT protocols are based only on checkpoints, message-send, and message-receive events: No-Receive-After-Send, Checkpoint-After-Send, Checkpoint-Before Receive, and Checkpoint-After-Send-Before-Receive [12]. Clearly, these protocols are prone to induce a very large number of forced checkpoints. Fixed-Dependency-Interval (FDI) [8, 12] and Fixed-Dependency-After-Send (FDAS) [12] maintain and propagate dependency vectors. They force the dependency vector of a process to remain unchanged during an entire checkpoint interval (FDI) or after the first message-send event of an interval (FDAS).

The RDT protocol proposed by Baldoni, Helary, Mostefaoui, and Raynal (BHMR) was the first protocol to consider visibly doubled Z-paths [1]. Afterwards, Baldoni, Helary, and Raynal have presented a family of RDT protocols, including a refined version of BHMR [3]. Tsai, Kuo, and Wang have proved that BHMR never takes more forced checkpoints than FDAS [11]. However, the more elaborated condition used by BHMR requires the propagation of an $O(n^2)$ matrix of booleans, as described in Section 4.

Recently, we have proposed an RDT protocol, called RDT-partner, that breaks only non-trivially doubled PMM-paths [6]. RDT-partner requires only $O(n)$ control information and our simulation results have shown that it takes virtually the same number of forced checkpoints as BHMR. However, given the results presented in this article, it is know also possible to break all non-visibly doubled PMM-paths in $O(n)$.

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