Image Segmentation Using Watershed and Normalized Cut

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Abstract—Several publications have studied the graph spectra (a concept extracted from linear algebra) as a partitioning tool. This work studies a method called Normalized Cut, introduced by Shi and Malik [1] and proposes an image segmentation strategy utilizing two ways to convert images into graphs: Pixel affinity and watershed transform. Both ways provide us as result a similarity matrix that is used to calculate the spectral graph properties (eigenvalues and eigenvectors) and then apply the grouping algorithm. Images of yeast cells were used to generate the preliminary results obtained and provide important knowledge for future research.

Keywords—image segmentation; watershed transform; graph partitioning; spectral graph.

I. INTRODUCTION

The relationship between some linear algebra concepts and graph theory provides an way to segment an image into meaningful regions and extract its semantic information. Spectral graph theory has evolved and encouraged numerous works on digital image processing domain[2].

Graph cuts may be explored to analyze the degree of dissimilarity between parts of an graph (or its corresponding image). In this paper, we study an approach based on graph cuts called Normalized Cut (Ncut) [1] and its association with concepts extracted from the study of eigenvalues and eigenvectors problems.

Our experiments use two approaches in order to convert images into graphs: The brightness formula showed in [1] (Pixel Affinity) and the Watershed Transform [3],[4].

II. IMAGE-GRAPH CONVERSION METHODS

Firstly, we need to convert the image into a graph. Given an image I, this work uses the following methods in order to implement this conversion.

A. Pixel affinity

The overall quality of the segmentation depends on the pairwise pixel affinity graph. The chosen properties were the intensity and position approach presented in [1]. In this formulation, each pixel corresponds to one node in a graph. Because the graph must be not cyclic, the affinity between a pixel and itself is 0. Equation (1) shows the formula to calculate the affinity matrix $W_I(i,j)$.

$$W_I(i,j) = e^{-\frac{1}{\sigma}(\|X_i - X_j\|^2 + |C_i - C_j|^2)}$$  (1)

where $X_i$ and $X_j$ are vectors containing the $xy$ positions of the pixels $i$ and $j$; $C_i$ and $C_j$ are vectors containing the RGB values (or graylevel values) of these same pixels.

B. Watershed Transform

We consider the gradient image as a topographic surface. In the watershed method, an image is segmented by constructing the catchment basins of its gradient image. The gradient image is flooded starting from selected sources (regional minima) until the whole image has been flooded. A dam is erected between lakes that meet with others lakes. At the end of the flooding process, we obtain one region for each catchment basin of the gradient image.

Hierarchical Watershed creates a set of nested partitions, i.e., a hierarchy. In this case, a partition at a coarse level is obtained by merging regions of the fine partition[4].

The watershed problem can be modeled using graphs. The gradient image is represented by a weighted neighborhood graph, where a node represents a catchment basin of the topographic surface. We use Hierarchical Watershed in order to reduce the number of nodes (supersegmentation problem) in the correspondent graph.

After the conversion, we use the area and average grayscale level of each region to set the edges weights between them.

III. GRAPH REPRESENTATION

We assume $G = (V, E)$ as an undirected graph where $V$ is the set of nodes and $E$ is the set of edges $(i, j)$. Two nodes $i, j$ are adjacent, represented by $i j$, if there exists an edge linking $i$ and $j$, and the weight associated to each edge $(i, j)$ is represented by $w(i, j)$. The mathematical representation for this graph is given as follows:

- Similarity matrix: A similarity matrix $A$ is a representation for an undirected weighted graph where each entry value $a(i, j)$ is the edge weight $w(i, j)$ linking a pair of nodes. The weights are given by a function that maximize the similarity between nodes $i$ and $j$. 
• Weighted degree matrix: Let \( d(i) = \sum w(i, j) \) be the total connection from node \( i \) to all its neighbor nodes. Then the weighted degree matrix \( D \) is the diagonal matrix with \( d \) on its diagonal.

• Laplacian matrix: The laplacian matrix of a graph \( G \) is computed from \( L(G) = (D - A) \), where \( D \) is the weighted degree matrix and \( A \) is the similarity matrix.

IV. Graph partitioning

The Ncut approach uses the algebraic properties of the laplacian matrix to separate the nodes according to the dissimilarity between them. In graph theoretic language, it is called cut:

\[
\text{Cut}(S_1, S_2) = \sum w(u, v), u \in S_1, v \in S_2
\]

where \( S_1 \) and \( S_2 \) are two disjoint sets in a graph. Instead of using the total edge weight connection, this method computes the cut cost as a fraction of the total edge connections to all the nodes in the graph:

\[
NCut(S_1, S_2) = \frac{\text{Cut}(S_1, S_2)}{\text{SumCon}(S_1, V)} + \frac{\text{Cut}(S_1, S_2)}{\text{SumCon}(S_2, V)}
\]

where \( \text{SumCon}(S, V) \) is the total connection from nodes in the set \( S \) with all nodes in \( V \). Expanding this equation the following equation can be found:

\[
\min_x NCut(x) = \min_y \frac{y^T(D - A)y}{y^TDy}
\]

This equation, called Rayleigh quotient, has a property that it can be minimized by the smallest eigenvector \( x_0 \) of the Rayleigh quotients matrix (in this case, the Laplacian matrix) and its minimum value is the corresponding eigenvalue \( \lambda_0 \). So, the Normalized cut can be minimized by solving a generalized eigenvalue system as shown below:

\[
(D - A)y = \lambda Dy
\]

Because of the first Laplacians matrix smallest eigenvalue is 0, the second smallest eigenvalue is the real valued solution to the normalized cut problem and its corresponding eigenvector can tell exactly how to partition the graph just by separating the nodes represented by the positive values in the eigenvector from the negative ones.

V. Experimental results

We applied the conversion methods and the segmentation strategy presented in yeast cells images, thanks to its importance in several chemical processes and previous related works [5]. Applying the pixel affinity method to generate a graph the results are showed in Figure 1. The original image, Figure 1(a) is converted into a graph using the pixel affinity method. After, the eigenvalues and eigenvectors are calculated. The sign of the second smallest eigenvectors values provides the partition presented on Figure 1(b).

![Figure 1. (a) Yeast original image; (b) Yeast segmentation by NCut (pixel affinity).](image)

Figure 2 shows the results of the segmentation process using the Watershed Transform (WT) and Normalized Cut. Figure 2(b) shows the cells separated from the background (first iteration); Figures 2(c) and (d) show the cells grouped by their size similarities (second iteration).

![Figure 2. (a) Yeast original image; (b) Yeast segmentation by WT and NCut; (c), (d) Yeast segmentation grouped by area.](image)

VI. Conclusions and perspectives

Our experiments were satisfactory to group the image regions that represent semantically the yest cells, when WT was used. Minimizing the Normalized Cut is an NP-complete [1], and the comparison between the two presented methods of image conversion showed that the performance is higher using the Hierarchical Watershed Transform, because it simplifies the graph in a lower number of nodes (different from [2]). The studies about the Ncut and the spectral graph theory allow numerous combinations of methods due to its versatility. Once you can represent the problem using a graph and determine the similarity matrix, the grouping algorithm will be similars. In this way, the properties chosen to compute the edge’s weights are importants for the final result. In order to improve our results, future works can combine the image conversion methods presented and consider the region’s shape as a similarity criterion.

REFERENCES


