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Smart Energy Pricing for Demand-Side Management in Renewable Energy Smart Grids

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Abstract

The Smart Grids are expected to provide various benefits to society by integrating advances in power engineering with recent developments in the field of Information and Communications Technology. One of the advantages is the support to efficient demand-side management (DSM), e.g. changes in consumer demands for energy based on using incentives. Indeed, DSM is expected to help grid operators balance time-varying generation by wind and solar units, and the optimization of their usage. This paper focuses on DSM considering renewable energy generation and proposes an auction, in which consumers submit bids to renewable energy usage plans. An additional model is introduced to allow consumers to compute their bid for a given usage plan. Both models have been extended to include energy storage devices. The proposed model is compared to a system with time-varying pricing for energy, where it is shown to allow consumers to use more appliances, to lead to a larger profit and to reduce the peak-to-average ratio of energy consumption. Finally, the impact of the use of energy storage in households and in the energy provider is also considered.

Keywords: Demand-side management; smart grid; smart pricing; auction; energy storage devices

1. Introduction

The European Technology & Innovation Platform – Smart Networks for Energy Transition (ETIP-SNET) defines smart grids as “electricity networks that can intelligently integrate the actions of all users connected to it — generators, consumers, and those that do both — in order to efficiently deliver sustainable, economic and secure electricity supplies.” (European Technology & Innovation Platforms, 2018). In a usual smart grid scenario, each consumer is equipped with *energy consumption scheduling* (ECS) de-

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vices that are able to automatically start, stop, increase or decrease the energy consumption of appliances, as well as smart metering equipment that allows the grid to gather advanced information on energy consumption on the end users. Having such infrastructure, energy providers can obtain more information on how and when the energy is consumed, thus charging accordingly and leading the system to a better use overall instead of coping with peak demands. Consumers also benefit from the infrastructure by making better scheduling for their appliance usage while taking into consideration the changes in the energy cost set by the providers at a certain time. However, this comes at a high cost of investments in Information and Communication Technology (ICT), such as Cognitive Radio Network infrastructures, smart sensing equipment (Khan et al., 2016; Shah et al., 2013) and research on the efficient algorithmic implementation of such systems.

Initiatives such as the Europe 2020 Climate and Energy Package — which aims to cut greenhouse gas emissions by 20%, establish 20% of the EU's energy from renewable sources, and show an improvement of 20% in energy efficiency by 2020 — have pushed for the implementation of smart grids through legislation and incentives, indicating that they may, in fact, play a key role in the transition into a more sustainable energy production, distribution, and consumption (European Commission, 2008). As opposed to the current grid, smart grids take advantage of advanced metering infrastructure, and supervisory control and data acquisition, while also being capable of self-healing. There is a vast body of literature regarding smart grids, however this paper focuses on the connection between game theory and demand-side management in smart grids. For an overview of the applications of game theory in smart grids, please refer to Saad et al. (2012).

Demand-Side Management (DSM) is a set of techniques implemented by utility companies designed to influence the energy consumption of their end users in order to achieve a more efficient grid operation in relation to the available power plant capacity (Ng and Sheble, 1998). The main DSM techniques include *load management* and *demand response*. Load management is usually implemented as direct load control where based on an agreement between the utility company and consumers, the utility company can remotely control the operation of certain appliances of their consumers in order, for instance, to avoid global peaks of usage. On the other hand, demand response is based on implementing financial or other incentives to influence consumers' demand for energy. One common way to implement demand response is to use *smart pricing*, in which the utility company sets the price of the energy according to the aggregated load of the consumers, encouraging their end-users to shift their load to off-peak hours.

In this paper, a smart grid is considered in which traditional power plants (carbon, nuclear, etc.) are integrated with renewable power plants (solar panels, wind turbines, etc.) to provide power to small communities of consumers. A novel smart pricing scheme is proposed to allocate energy to consumers (households) on the basis of constraints imposed by energy production. The proposed scheme charges consumers for an energy usage plan. As usual in Auction Theory, consumers' satisfaction with a usage plan is modeled as a monetary gain, which translates to an auction bid for the respective usage plan. A separate model is also proposed to help consumers to compute their value for a given energy usage plan based on a discrete model for describing appliance usage. Plans are assigned to consumers in such a way that the renewable energy capacity is not exceeded, yet maximizes the overall value of these assigned energy plans. The Vickrey–Clarke–Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973) is used to incentivize consumers to be truthful by charging them in a way that it is in their best interest (financially speaking) to report their true values for the usage plans.

Since consumers only report their auction bids for usage plans, this smart pricing scheme allows for more freedom in modeling of household appliances while also maintaining full privacy for consumers since their demands are never explicitly reported to the utility company (or the other consumers).

The proposed model is able to shift energy consumption from peak hours to off-peak hours. As a consequence, it can better exploit the energy generated by renewable sources. When compared to a time-varying pricing model, it also allows consumers to use more appliances, which in turn, leads to better social welfare (the satisfaction in society). Additionally, it also leads to a larger overall profit for the utility company and it reduces the peak-to-average ratio of energy consumption.

The contributions of this paper are the following:

- Introduction of a model for handling demand-side management in which power is allocated according to consumers bids for a set of possible usage plans, with the advantages of being appropriate for renewable power plants and ensuring consumers' privacy. The energy usage plans can be designed by the power provider or developed by considering historical values.
- A discrete model to describe consumers' value for a given energy usage plan which is appropriate for modeling a wide range of operating scenarios in terms of power demand. For example, it is possible to model *must-run appliances* (appliances that must run within a specific time window), *non-interruptible appliances* (appliances that cannot be paused in order to save energy, e.g., an electric stove), *interruptible appliances* (appliances that can be paused in order to save energy, e.g., charging a Plug-in Hybrid Electrical Vehicle – PHEV) and various combinations of these. This discrete model provides a way to describe customers' utility gained from executing each appliance activity, thus incorporating the discomfort that comes from the user not executing or delaying the execution of such appliances.
- Both models are extended to consider the use of energy storage devices on both the premises of the consumers and the energy provider. The introduction of energy storage is particularly significant in scenarios involving renewable power plants since it allows the partial reshaping of the power production pattern, e.g. providing energy at times when the production is low but demand is high (for example at night, when there is no solar energy production). In general, household energy storage devices allow consumers to utilize an energy usage plan better, possibly increasing the value that can be gained from it. Moreover, power plant energy storage devices improve energy availability, and as a consequence, increase overall social welfare by allowing the assignment of more valuable plans to consumers.

The remainder of this paper is organized as follows. An overview of the related work is shown in Section 2. Section 3 describes the overall system model including: an optimization model to allocate providers' energy usage plans while maximizing the social welfare; a model that can be used by consumers to compute their utility for a usage plan, given the utility values of using their appliances; an extension of the proposed models to incorporate energy storage devices. In Section 4, the practical performance of the proposed models is evaluated with numerical experiments. Finally, in Section 5, the final remarks are presented.

2. Related Work

Due to the intricate nature of energy providing, many models of DSM have been proposed in the literature, mostly focusing on achieving a smaller peak-to-average ratio (PAR, the ratio between the highest load and the average load) by shifting load to off-peak hours.

Fahrioglu and Alvarado (2000) presented the first article in the literature to consider smart pricing and game theory in this context. In it, the use of incentives is considered that encourage consumers to sign up for the desired demand management contract with the (non-renewable) energy supplier while revealing their actual value for energy usage. Afterward, several articles utilized game theory in order to price energy consumption.

Mohsenian-Rad et al. (2010) consider a demand-side management model consisting of an incentive-based consumption schedule scheme by using ECS devices for each consumer. In their model, the price of energy at each time is computed as a quadratic function of the aggregated load of the consumers, and the consumers react by letting their ECS devices change the starting or stopping time of their appliances. They prove that the proposed game converges to the unique Nash Equilibrium, i.e. a point in which no single player can improve the price paid by shifting any of their appliances if all players always choose their best strategy selfishly. The game is solved with both a centralized algorithm on the provider's side and a distributed algorithm to be executed individually in each consumer's ECS, thus propagating only the total load of each consumer and partially preserving their privacy.

Zhu et al. (2011) expand on the model proposed by Mohsenian-Rad et al. (2010) allowing it to consider a broader amount of appliances, including power-shiftable appliances which are allowed to shift the amount of power provided to them. The authors use integer linear programming and integer quadratic programming to solve both centralized and distributed versions of the proposed game, although they do not provide any mathematical guarantees that the proposed models admit a unique Nash Equilibrium. Afterward, Liu et al. (2014) propose a PAR constrained model while taking into consideration the consumers' preferences such as minimizing the operation delay (considering shifts). The article proposes a distributed algorithm to minimize the amount of information exchanged and compare their results empirically with the model proposed by Zhu et al. (2011).

Logenthiran et al. (2012) propose a DSM model based on load shifting in which the objective is to minimize the difference between an objective load profile, defined by the utility company, and the actual consumption profile. A heuristic-based Evolutionary Algorithm is proposed to solve the problem.

Many DSM models rely on shifting the use of some appliances to off-peak hours by means of monetary incentives. However, an argument can be made that these shifts are not always desired by the consumers, thus the price paid by energy does not necessarily reflect the actual utility of the consumers. Li et al. (2017) propose a remodeling of DSM as a sparse load shifting problem that minimizes the number of interruptions and restarts for consumers' appliances. This model admits multiple Nash equilibria, and therefore the algorithm proposed searches for an equilibrium that minimizes PAR. Wang et al. (2018) use a discomfort function (along with a cost of energy generation and cost of energy from the provider) to compute the utility of each consumer while planning the use of energy storage. Yang et al. (2013) propose a time of use pricing scheme for DSM in which the utility of the consumers is modeled as the price paid for their load plus a user satisfaction function based on the difference between their demand and their load. A backward induction technique is used to find an equilibrium, but no proof of convergence of the algorithm is provided. Srinivasan et al. (2017) present an evaluation of different dynamic pricing techniques, including the one proposed by Yang et al. (2013), using real data from the Singapore electricity market.

From the perspective of the utility company, a real-time pricing scheme can be implemented considering a scenario where DSM is used. Bu and Yu (2013) proposes a decision-making framework for the utility companies modeled as a four-stage Stackelberg game, in which during the three first stages the utility companies decide what kind of energy source to use, how much energy to procure and then the retail price to offer to the consumers. In the fourth stage, the consumers adjust their demands according to the prices offered. Backward induction is used to determine a sub-game perfect equilibrium. Belhaiza and Baroudi (2015) propose a model for demand management in neighborhood area networks that involves strategies from both the side of the consumers and the side of the utility companies.

Most DSM models rely on repeatedly exchanging information between consumers and the utility company (sometimes even between consumers) until an equilibrium is reached. This raises concerns about revealing private information about consumers' energy usage. Because of this, it is preferable to do all computations in a distributed way so that unnecessary data is not transmitted to other parties. Even then, most DSM models require consumers to report their true demand for energy. Furthermore, due to the cyclic nature of best-response games, convergence is not always achievable in viable computational time. Because of these difficulties, auctions as a pricing model for DSM in smart grids have been studied recently in the literature, where consumers' usually bid once for each energy offering and based on these bids the utility company chooses how much energy every consumer should receive. In this context, Atzeni et al. (2014) use auctions in the day-ahead phase of its DSM model. Samadi et al. (2012) apply the VCG mechanism for dynamic pricing in such a way to maximize the aggregation of the utility functions of all consumers subtracted by the (non-renewable) energy production cost. Since a VCG mechanism is used, there is no incentive for consumers to lie about their true preferences regarding energy usage. Finally, Ma et al. (2014) propose an enhanced Arrow-d'Aspremont-Gerard-Varet (AGV) mechanism (Arrow, 1977; d'. Aspremont and Gérard-Varet, 1979) for this model, which also ensures truthfulness.

Differently from the works of Samadi et al. (2012) and Ma et al. (2014), the proposed model does not restrict consumers on how they can obtain value from energy usage and on how they can use the energy provided. The proposed model for bid computation can consider several different types of household appliances. Moreover, as in the proposed system, the energy allocation is independent of how bids are generated in the auction; the bid computation model can be exchanged for more suitable ones, especially in the case of commercial or industrial consumers. Additionally, since the bids only represent how much a plan is valued for a consumer, the consumer never has to explicitly state their actual load demands which preserve their privacy.

The inclusion of renewable energy sources into the grid adds more complexity to it. For a comprehensive view of the difficulties of integrating renewable energy resources and some of the recent efforts in doing so, please refer to the work of Rehmani et al. (2018).

Due to the unpredictable nature of renewable energy sources, the actual energy produced on a certain day can vary with factors such as wind speed and solar irradiance. Because of this, methods of forecasting these factors have been developed in order to provide accurate predictions for the amount of renewable energy that can be generated by a plant in a given time period. Thus, forecasting of wind and solar power have been extensively studied in the literature. However, evaluating these predictions can be challenging since their performances can also vary with many factors, such as forecast time horizons, quality of the data used, distribution of wind speeds, and topography of the area studied (Ssekulima et al., 2016). One common metric used to evaluate these models is the Root Mean Squared Error (RMSE). This metric is simply the square root of the quadratic mean of the differences between predicted and observed values (standard deviation of the prediction errors), and it gives some insight into how far the observed values are from the predicted line.

Soman et al. (2010) present an overview and comparative analysis of various forecasting techniques for wind power and speed. They separate the techniques in the following time horizons: very short term (few seconds to 30 seconds ahead); short term (30 minutes to 6 hours ahead); medium term (6 hours to 1 day ahead); and long term (1 day to 1 week ahead). Ssekulima et al. (2016) present a review of both solar and wind forecasting in the context of integration of renewable energy to the grid and its challenges. More recently, Ahmed and Khalid (2019) review several renewable energy forecast techniques in the literature with an application-oriented approach. One of the applications presented by the authors is the impact of using forecasting of renewable energy generation on energy markets.

In order to achieve higher accuracy, a trend has developed of combining several different models to mitigate the individual weaknesses of each of them (Ssekulima et al., 2016). Ren et al. (2015) review several ensemble methods based on existing models for accurately forecasting wind and solar power. For example, one can notice in the authors' experiments that for time horizons of 24 hours, many techniques achieved an RMSE of around 0.10. Because of this, it is reasonable to assume that a DSM model can use a state-of-the-art forecasting method with relatively low errors to predict the amount of renewable energy available in a day-ahead fashion.

Regarding models for renewable energy usage in demand-side management, some works (Atzeni et al., 2013b; Soliman and Leon-Garcia, 2014; Chen et al., 2014; Ye et al., 2016; Wang et al., 2018; Zazo et al., 2017) consider the integration of decentralized renewable energy sources, as well as energy storage devices. The model proposed in this paper is different from theirs since it considers that the energy supplier is the one supplying renewable energy, not the consumers.

Works, such as the ones by Atzeni et al. (2013b) and Zazo et al. (2017), consider the intermittent and stochastic nature of renewable energy generation and the unreliability of the consumers' load prediction when it comes to real-time consumption. Since the model proposed in this paper considers that the energy is provided by selling energy usage plans which are complemented with non-renewable energy, the consumers can modify their load according to their needs. Furthermore, regarding the uncertain nature of renewable energy generation, the model proposed in this paper does not address this problem directly, but it can be slightly modified to consider the case where the energy supplier utilizes batteries and non-renewable energy to reduce its impact on the system. In the experiments, it is shown that this does not significantly impact the quality of the proposed model. Another possibility would be to be conservative when predicting the energy generation in such a way that any excedent renewable energy could be sold as non-renewable energy (as the model already does).

3. System Model

In this section, a system model for smart pricing in DSM in the context of smart grids is presented. First, a general model of smart pricing is described, then in the subsequent subsections, two optimization problems that compose the proposed model are formally defined: the allocation of usage plans to consumers who have given their bids; and the evaluation of energy usage plans for bidding on the side of the consumers. Both problems are modeled as integer linear programming formulations that can be solved to optimality by the commercial solver Gurobi (Gurobi Optimization Inc., 2015) in a reasonable time.

3.1. General Description

In the scenario considered in this paper, the utility company controls a centralized renewable energy source and access to supplementary non-renewable energy. The company aims at allocating the clean energy produced to consumers in a way that maximizes the social welfare of the community while attending to the constraints imposed by the capacity for production of renewable energy. This scenario can be encountered, for instance, in communities that have access to renewable energy sources and want to avoid using non-renewable energy as much as possible or in governments which want to provide social welfare using renewable energy instead of relying on non-renewable sources.

In the model, a set T of discrete time slots $\{1, 2, \ldots, t_{\max}\}$ with $t_{\max}$ being a positive integer is considered. For each time slot, consumers receive a constant amount of energy from the utility company. For instance, one possibility would be to consider each time slot as an hour in a day, therefore $t_{\max}$ would be 24.

The utility company has a set P of *energy usage plans*. An energy usage plan is a non-negative vector that represents the maximum energy (in kWh) a consumer is allowed to utilize during each time slot from this source (the user can buy supplementary energy which will possibly be non-renewable). The consumer will, however, only receive the amount of energy actually used. The energy usage plans can be developed by the utility company considering historical data, such as energy production and consumption. Alternatively, they can consist of simple usage plans such as constant plans in which the same amount of energy is provided for every hour or plans which provide low levels of energy for some hours and higher levels for others. In fact, the power provider can adjust the energy usage plans as time passes in order to control the energy demand. In Section 4, some sample usage plans are presented.

Energy allocation is modeled as a sealed-bid auction using the VCG mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973). That is, energy usage plans are provided to a set C of consumers who will bid for them. Each consumer $c \in C$ bids a value $b_{c,p}$ for the energy usage plan $p \in P$. Bids can be computed in any way desired by the consumer, but the VCG mechanism guarantees that the best strategy for the consumers is to report their values truthfully, i.e. consumers will not benefit from giving a bid that does not correspond to their true intentions. As the auction is a sealed-bid, each consumer submits their bids only once and the auction is executed after all bids are submitted. The utility company has a limited amount of energy available for each time slot limited by their capacity to produce renewable energy, therefore if w_t is the available energy (in kWh) at time t, consumers compete against each other for this (limited) energy resource.

The objective is to assign at most a single usage plan to every consumer, constrained by the power plant capacity w in order to maximize *social welfare*, which is the sum of $b_{c,p}$, for each consumer $c \in C$ and plan p. Since consumers could strategically misrepresent their actual value in order to profit, the VCG mechanism is utilized to guarantee that it is in the best interest of consumers to declare values truthfully. Thus, in this paper it is considered that $b_{c,p}$ represents the real value of usage plan p for consumer c, i.e. consumers do not provide wrong or forged information. In Section 3.3, one way for consumer c to compute a bid $b_{c,p}$ for a given usage plan p is introduced. Figure 1 presents a flowchart of the proposed system model.

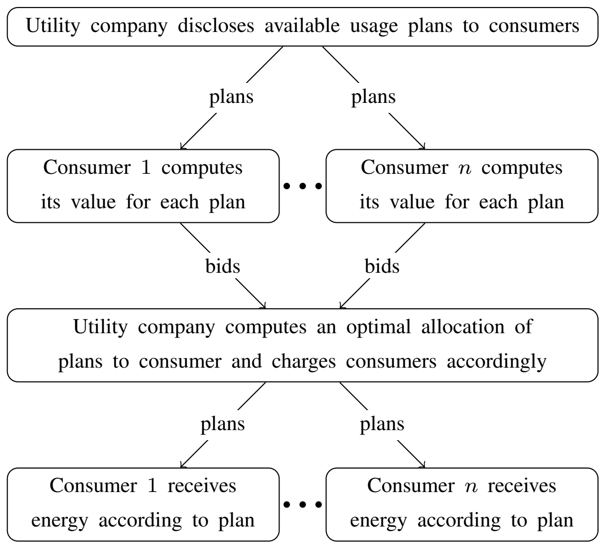

Fig. 1. A flowchart representing the system model.

In the proposed model, the cost of generating energy is not explicitly considered, since the model is designed for renewable energy sources such as wind and solar energy, where the output is not controlled by how much resources are consumed. Thus, the pricing strategy focuses on guaranteeing fair resource usage instead of covering energy production expenses or maximizing profit. Nonetheless, profits can be reverted to system maintenance or improvement as a consequence of using renewable energy.

3.2. Energy Allocation

In the proposed model, the utility company needs to define how to assign energy plans to consumers with the aim of maximizing social welfare based on consumers' bids but constrained by the power plant capacity. That is, the company must allocate usage plans to consumers so that the consumer that values the most each usage plan obtains it as long as there is enough renewable energy to accommodate it in the aggregated load.

This problem is modeled as an integer linear program. Consider that $T = \{1, 2, \ldots, t_{\max}\}$ is a set of discrete time slots, P is a set of usage plans, C is the set of consumers, $b_{c,p}$ is the bid of consumer $c \in C$ for plan $p \in P$, p_t is the amount of energy (in kWh) of usage plan p at time slot t and w_t is the power plant capacity (in kWh) at time t. Let x be a binary vector indexed by $C \times P$, where $x_{c,p}$ indicates if usage plan p is assigned to consumer c. The following integer program formulation consists of finding the values of x that

$$
\begin{aligned}
\max & \sum_{c \in C} \sum_{p \in P} b_{c,p}\, x_{c,p} \\
\text{s. t.} \quad & \sum_{p \in P} x_{c,p} \le 1, && \forall c \in C, \\
& \sum_{c \in C} \sum_{p \in P} p_t\, x_{c,p} \le w_t, && \forall t \in T, \\
& x_{c,p} \in \{0, 1\}, && \forall c \in C, \forall p \in P.
\end{aligned}
$$

This integer linear programming formulation can be solved using a commercial solver such as Gurobi (Gurobi Optimization Inc., 2015) to obtain an optimal allocation of energy usage plans to consumers at a reasonable time.

Consumer c will be charged the value $\pi_c(b, x)$, which depends on the bids b reported by the consumers as well as on the optimal solution x of the integer linear program computed for b.

In order to compute prices, the VCG mechanism is used in conjunction with the Clarke pivot rule. This guarantees that every consumer will pay a non-negative price for energy consumption, that no consumer will pay more for an energy usage plan than their bid for it, and that consumers will be compelled to report their bids truthfully. Thus, applying the VCG mechanism with the Clarke pivot rule, the value of $\pi_c(b, x)$ is defined as:

$$
\pi_c(b, x) = \max_{x' \in \mathcal{S}} \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} - \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x_{c',p}, \tag{1}
$$

where $\mathcal{S}$ is the set of feasible solutions for the integer linear program. That is, every consumer is charged their *externality*, the difference between the maximum social welfare obtained by other consumers when consumer c is not in the market and the social welfare obtained by the other consumers when consumer c is in the market.

Even though the model can have multiple optimal solutions, any of these solutions would be fair to the consumers, since, for any two optimal solutions x^* and $\tilde{x}$ in which a consumer c receives plan p^* in x^* and plan $\tilde{p}$ in $\tilde{x}$, the value obtained from the usage plan minus the price paid is the same, that is,

$b_{c,p^*} - \pi_c(b, x^*) = b_{c,\tilde{p}} - \pi_c(b, \tilde{x})$. In fact,

$$b_{c,p^*} - \pi_c(b, x^*) = b_{c,p^*} - \max_{x' \in \mathcal{S}} \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} + \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x^*_{c',p} \tag{2}$$

$$= \sum_{c' \in C} \sum_{p \in P} b_{c',p} x^*_{c',p} - \max_{x' \in \mathcal{S}} \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} \tag{3}$$

$$= \max_{x \in \mathcal{S}} \left\{ \sum_{c' \in C} \sum_{p \in P} b_{c',p} x_{c',p} \right\} - \max \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} \tag{4}$$

$$= \sum_{c' \in C} \sum_{p \in P} b_{c',p} \tilde{x}_{c',p} - \max_{x' \in \mathcal{S}} \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} \tag{5}$$

$$= b_{c,\tilde{p}} - \max_{x' \in \mathcal{S}} \left\{ \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x'_{c',p} \right\} + \sum_{c' \in C \setminus \{c\}} \sum_{p \in P} b_{c',p} x_{c',p} \tag{6}$$

$$= b_{c,\tilde{p}} - \pi_c(b, \tilde{x}) \tag{7}$$

where the equality between (3) and (4) follows from the optimality of x^* and the equality between (4) and (5) follows from the optimality of $\tilde{x}$.

3.3. Bid Computation

In order to consume energy according to the scheduled energy in a usage plan, a consumer must decide which appliances to turn on or off and at what specific time. A consumer has many alternatives to choose from when deciding this appliance usage. Let an *alternative* be a combination of the energy usage of one appliance in kWh for every time slot, with a value, representing consumer satisfaction associated with using such appliances in the times described by the alternative. For example, the consumer can choose to watch TV from 20:00 to 22:00, consuming a constant amount of energy for each time unit, or use a washing machine from 09:00 to 12:00, which consumes a variable amount of energy dependent on the washing cycle. Some alternatives, however, are mutually exclusive. For example, the consumer can charge their Plug-in Hybrid Electric Vehicle (PHEV) from 21:00 to 06:00 using a certain amount of energy during all of those hours or charge the PHEV from 00:00 to 05:00 but using a larger amount of energy per hour. The consumer can, however, choose only one of the alternatives since the PHEV is only charged once.

In the case of renewable energy, it may be unrealistic to expect consumers to be able to satisfy their demands using only this type of energy in the fulfillment of their usage plan. It is thus assumed that consumers can also buy supplementary energy at a fixed hourly rate, possibly from another energy provider, in order to supplement the energy which a usage plan provides. This system is well suited for renewable energy, especially in the case of small generators in a community, and is actually optional, since the price of the supplementary energy can be set to infinity (or, in practice, a relatively large value), which will forbid the usage of non-renewable energy.

More formally, this scenario can be described in the following way:

- a set $T = \{1, 2, \dots, t_{\max}\}$ of discrete time slots,
- a usage plan p, where p_t is the capacity of the usage plan at time t,
- a set A of alternatives where, for every $a \in A$, v_a is the value gained from choosing alternative a and $u_{a,t}$ is the amount of energy consumed by alternative a at time t,
- a family $\mathcal{A} = \{A_1, A_2, \dots, A_k\}$ where $A_i \subseteq A$ and exactly one alternative must be chosen from A_i,
- a cost r_t for every supplementary kWh consumed at time t.

Let y be a binary vector indexed by A where y_a indicates if alternative a is chosen and λ be a vector indexed by T where λ_t is the amount of supplementary energy (in kWh) consumed at time t. The following integer linear program formulation computes the optimal bid $b_{c,p}$ for a consumer c with alternative set A for an energy plan p.

$$
\begin{aligned}
b_{c,p} = \max \sum_{a \in A} v_a y_a &- \sum_{t \in T} r_t \lambda_t \\
\text{s. t.} \qquad \sum_{a \in A_i} y_a &= 1, && \forall A_i \in \mathcal{A}, \\
\sum_{a \in A} u_{a,t} y_a &\leq p_t + \lambda_t, && \forall t \in T, \\
y_a &\in \{0, 1\}, && \forall a \in A, \\
\lambda_t &\geq 0, && \forall t \in T.
\end{aligned}
$$

Appliance i can be modeled by defining a set A_i of alternatives for it. Since exactly one alternative for every set A_i is chosen, a must-run appliance is represented by a set of alternatives, as exactly one of them will be chosen. Optional appliances can be represented by adding an alternative with zero energy consumption and zero value.

Non-interruptible appliances can be modeled considering only alternatives where the appliance runs from start to finish, that is, alternatives a where $u_{a,t} > 0$ if and only if t is in a pre-specified time interval. Interruptible appliances can be modeled by considering all possible interruptions that can occur. For example, an interruptible appliance which should run for 4 hours, but it can be interrupted only once in the middle of the cycle and that consumes $1kW$ per hour can be modeled by considering as alternatives every vector u_a so that there are k and ℓ in T with $u_{a,k} = u_{a,k+1} = u_{a,\ell} = u_{a,\ell+1} = 1$ and $u_{a,t} = 0$ for every $t \notin \{k, k+1, \ell, \ell+1\}$.

The model can also represent the arbitrary mutual exclusion of alternatives for different appliances (e.g. a consumer might decide either to watch TV or to use a computer in the evening, but not both). In Sect. 4.1, more concrete examples of appliances are given.

3.4. Incorporating Energy Storage

Extensions of the previous optimization problems considering energy storage devices (Atzeni et al., 2013b; Chen et al., 2014; Wang et al., 2018) are presented in this section. Energy storage devices help consumers to better utilize their energy usage plans so that unused energy can be consumed at times of greater need. Energy storage devices are also useful to utility companies. By using energy storage devices, for example, utility companies can store energy during days when solar energy production is high and use it at night, when no solar energy is generated.

Let y be a binary vector indexed by A where y_a indicates if alternative a is chosen, λ be a vector indexed by T where λ_t is the amount of supplementary energy (in kWh) consumed at time t, κ, z^+ and z^- be vectors indexed by T such that κ_t represents the amount of energy (in kWh) stored at the beginning of time slot t, z_t^- represents how much energy (in kWh) is discharged at time slot t and z_t^+ represents how much energy (in kWh) is charged during time slot t. The following integer linear program formulation consists of finding y, λ, κ, z^+ and z^- that

$$
\begin{aligned}
b_{c,p} = \max \sum_{a \in A} v_a y_a - \sum_{t \in T} r_t \lambda_t & \\
\text{s. t.} \qquad \sum_{a \in A_i} y_a = 1, & \qquad \forall A_i \in \mathcal{A}, \\
\sum_{a \in A} u_{a,t} y_a \leq \beta z_t^- - \alpha z_t^+ + p_t + \lambda_t, & \qquad \forall t \in T, \\
z_t^+ - z_t^- \leq \delta^{\max}, & \qquad \forall t \in T, \\
\kappa_t \leq K, & \qquad \forall t \in T, \\
\kappa_{t+1} = \gamma \kappa_t + z_t^+ - z_t^-, & \qquad \forall t \in T, \\
y_a \in \{0,1\}, & \qquad \forall a \in A, \\
\lambda_t, \kappa_t, z_t^-, z_t^+ \geq 0, & \qquad \forall t \in T,
\end{aligned}
$$

where $\kappa_{t_{\max}+1} = \kappa_1$, K is the capacity of the energy storage device in kWh, $\alpha \geq 1$ and $\beta \leq 1$ are parameters associated with charging and discharging efficiencies, respectively, $\gamma \leq 1$ is the leakage rate,
Mark 1.3 and $\delta^{\max}$ is the maximum charging rate in kWh. Even though this model allows the battery to be charged and discharged at the same time, notice that this would waste more energy. In fact, any feasible solution where the battery is both charged and discharged at time t can be modified to either charge $z_t^+ - z_t^-$ kWh if $z_t^+ - z_t^- \geq 0$ or discharge $z_t^- - z_t^+$ kWh if $z_t^+ - z_t^- \geq 0$. This integer linear programming model is a generalization of the model previously described in Section 3.3.

The model for energy allocation to consumers can be extended to include energy storage in a similar way and will not be shown in this paper. Moreover, one can compute the prices, as shown before in Section 3.2 using Equation (1).

4. Simulation and Experimental Results

In this section, the performance of the proposed model using a randomly generated instance is assessed. Results comparing the proposed model with a system in which consumers do not have ECS are omitted since it is well established in the literature that the usage of ECS devices can improve the social welfare (Samadi et al., 2012). In the evaluation, the impact of adding energy storage devices both on the consumers' and energy providers' side is considered. First, the way in which instances are generated is described in detail and, later, the results are discussed.

4.1. Data generation and assumptions

Consumers are assumed to have a set of appliances in which the alternatives represent different usage patterns for those appliances. Thus, every A_i of $\mathcal{A}$ represents the alternatives for a specific appliance i. For many of those appliances, all alternatives are equally good for the consumer regardless of the starting and ending times and, all have the same satisfaction value. For other appliances, the value depends on the starting and ending times. For one of the appliances, there is a basic alternative a which brings the consumer the most satisfaction when using the appliance, and this is the basis for setting the value of the other alternatives.

The level of satisfaction obtained by using the appliance at time t, according to the basic alternative a^*, is computed by using the following function (as done by Samadi et al. (2012)):

$$v(\omega, u_{a^*,t}) = \begin{cases} \omega\, u_{a^*,t} - \dfrac{u_{a^*,t}^2}{4}, & \text{if } 0 \le u_{a^*,t} < 2\omega \\ \omega^2, & \text{if } u_{a^*,t} \ge 2\omega \end{cases}$$

where ω is a random parameter chosen uniformly from a list of possibilities. That is, the value v_{a^*} of alternative a^* is computed as $\sum_{t \in T} v(\omega, u_{a^*,t})$. Thus, v_{a^*} is the maximum value for that appliance. This function is used only to generate an instance for the model and, in fact, a consumer could give any non-negative value for any alternative.

Table 1 presents the parameters considered for the appliances for which any alternative is equally good. It is also possible to define the time relationships between appliances, such as the following: the clothes dryer (if used) starts only immediately after the washing machine is finished; if the clothes dryer is going to be activated, the corresponding washing machine activation is free to start at any time.

For air conditioning, lighting and entertainment, the consumer has an ideal time interval, I, of usage and obtains the value v_{a^*} if they can be used in this time interval. Moreover, all the discrete non-empty subintervals of I are alternatives, each with value $\rho_a^k v_{a^*}$, where $\rho_a \in [0, 1]$ is the value discount rate of appliance a and k is the number of hours missing in the corresponding time interval. Table 2 shows the associated parameters for these appliances. The ideal time interval for air conditioning usage is randomly chosen for each consumer: it is given by r – 22:00 where the same r is chosen uniformly in the interval from 09:00 to 17:00.

It is also possible to consider alternatives, such as using a stove in the morning and in the evening, with 1 kWh being consumed for 2 hours after being activated. The value of ω, in this case, is chosen from the set $\{10, 12, 14, 16, 18\}$. In the morning version, the starting times are chosen from the set

Table 1
Parameters of appliances whose alternatives have the maximum value.

Appliance	ω	**Starting times**	**Duration**	**Power (kW)**
Clothes iron	$4, 6, 8, 10$	08:00–16:00	2 h	1.00
Dishwasher	$4, 6, 8, 10$	Any time	1 h	1.44
Generic ($\times 4$)	$2, 4, 6$	06:00–23:00	1 h	1.50
Pool pump	$2, 4, 6, 8$	Any time	2 h	2.00
Vaccum cleaner	$4, 6, 8, 10$	08:00–16:00	2 h	1.50
Washing machine	$4, 6, 8, 10$	08:00–16:00 or any time (with clothes dryer)	2 h	1.70
Clothes dryer	$4, 6, 8, 10$	Optionally after washing machine	2 h	1.25
Water heaters (x2)	$2, 4, 6, 8, 10, 12$	06:00–07:00 or 18:00–22:00	1 h	1.00

Table 2
Parameters of appliances based on a target usage time interval.

Appliance	ω	**Target int.**	ρ	**Power (kW)**
Air Conditioning	$2, 4, 6, 8, 10, 12$ or 14	See description	0.8	1.00
Entertainment	$6, 8, 10, 12, 14, 16, 18$ or 22	18:00–00:00	0.9	1.50
Lighting (evening)	$2, 4, 6, 8, 10$ or 12	18:00–00:00	0.9	0.25
Lighting (morning)	$2, 4, 6, 8, 10$ or 12	06:00–08:00	0.9	0.25

$\{11{:}00, 12{:}00, 13{:}00\}$ in which 12:00 is the preferred one and, for the evening time slot, the starting time is chosen from $\{18{:}00, 19{:}00, 20{:}00, 21{:}00, 22{:}00\}$, with 20:00 being the preferred time. Let p be the preferred starting time of the appliance, an alternative starting at time t has a value of $0.9^{|t-p|}v_{a^*}$.

In relation to charging a PHEV, ω is chosen from the set $\{2, 4, 6, 8, 10\}$, and, for every subinterval I between 18:00–08:00, there is an alternative A that consumes 9.9 kWh evenly distributed in I. The value of A is thus given by $0.9^k v_{a^*}$, where k is the number of hours not in the interval of 20:00–06:00, and a^* is the alternative which charges the PHEV from 20:00 to 06:00.

Three types of energy usage plans are considered:

- $\mathrm{F{\scriptstyle LAT}}_k$: a usage plan where every position has value k,
- $\mathrm{U}_{k,i}$: a usage plan where the value is k for time slots $1, 2, \ldots, i-1$ and $k+1$ elsewhere,
- $\mathrm{D}_{k,i}$: a usage plan where the value is k for time slots $i+1, i+2, \ldots, t_{\max}$ and $k+1$ elsewhere.

where $k \in \{0, 0.25, \ldots, 3.0\}$ and $i \in \{0, \ldots, 23\}$.

The instance considered has $n = 100$ consumers and, if some of the consumers have energy storage devices, only 30% are considered to do so.

The renewable energy production capacity was obtained by scaling the mean solar and wind energy produced in the CSUD region of Italy for each hour in a five day period (from 06–11–2016 to 10–11–2016). Fig. 2 shows the mean energy production in the period. Let W'_t be the mean renewable energy produced at time t. In order to obtain the energy production capacity, W'_t is scaled by $2n|T|/\sum_{t\in T} W'_t$, and, thus, the mean energy capacity per hour and per consumer is 2 kWh; this is sufficient to induce some competition among clients while meeting the clients' basic energy needs.

The consumers' energy storage device is a lithium-ion battery with parameters $\alpha = 0.9^{-1}$, $\beta = 1.1^{-1}$,

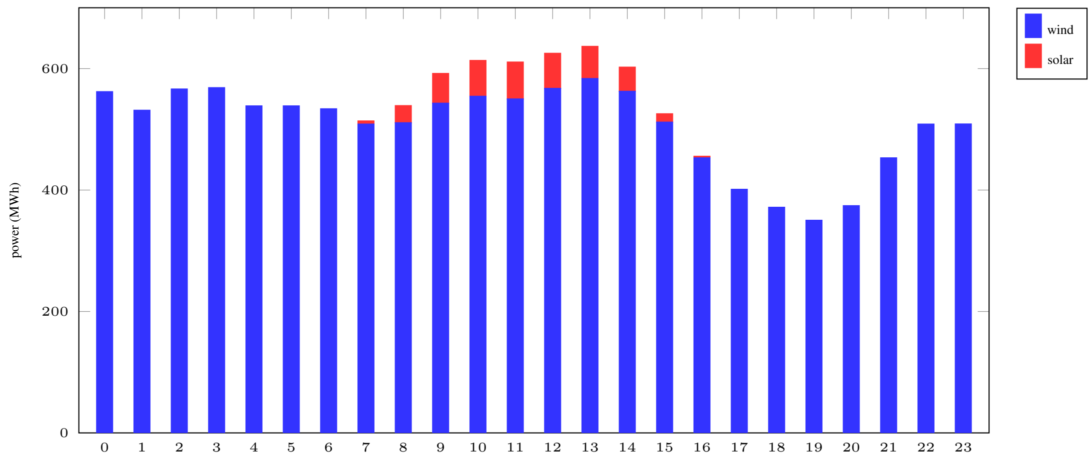

Fig. 2. Mean solar and wind energy production in Italy (CSUD region) from 06-11-2016 to 10-11-2016 (Terna S.p.A. - Rete Elettrica Nazionale, 2006).

$\delta^{\max} = 0.5\,\mathrm{kWh}$, $\gamma = \sqrt[24]{0.9}$ and $K = 4\,\mathrm{kWh}$ as specified by Atzeni et al. (2013a), and the energy provider's battery has a value of 40 for the parameter K and 5 for the parameter $\delta^{\max}$, and the same values for parameters α, β and γ.

In order to establish a baseline for the proposed model, we compare it to a time-varying pricing model, where consumers can buy energy at the price r_t at time t. In this model, renewable energy is considered to have priority over non-renewable energy, that is, non-renewable energy will only be sold after selling all the renewable energy at time t. Thus, this model only wastes renewable energy when there is no demand at a specific time. This scenario describes the situation where, for instance, the consumers are not willing to participate in the auction for acquiring renewable energy, so they pay the same fixed price that they would pay if they only used the supplemental non-renewable energy (although some of the renewable energy may be delivered to them to diminish the environmental impact of the community).

A similar assumption is made for the proposed system. Any unallocated renewable energy can be sold as supplementary energy at a time-varying price with renewable energy sold first and non-renewable energy afterward. Thus, in this model, there are two types of renewable energy loss: one because consumers may not consume all the energy allocated to them in their given usage plan; and the other because the demand at a given time is less than the renewable energy produced.

In the evaluation, three different *scenarios* are considered; one where supplementary energy is "inexpensive" ($r_t = 6$ for every $t \in T$), one in which supplementary energy is "expensive" ($r_t = 9$ for every $t \in T$) and, finally, one "mixed" scenario where supplementary energy is expensive at peak demand hours and inexpensive otherwise ($r_t = 6$ for $1 \le t \le 18$ and $r_t = 9$ for $19 \le t \le 24$).

4.2. Social welfare and revenue

Table 3 presents the number of appliances used on average by a consumer, the percentage of the maximum possible social welfare, and the percentage of the maximum possible revenue obtained by the two models.

Table 3
The number of appliances used on average and percentage of the maximum social welfare and revenue obtained by the two models.

	Proposed			Time-varying		
Scenario	Appliances	Welfare	Revenue	Appliances	Welfare	Revenue
Inexpensive	15.99	90.50%	36.55%	12.07	79.78%	37.25%
Mixed	15.67	88.81%	37.07%	11.87	74.85%	32.66%
Expensive	14.55	85.33%	41.19%	10.24	63.09%	18.89%

The proposed model was able to increase the average number of appliances used by a consumer and the social welfare by 32.48% and 13.44% in the inexpensive scenario, by 32.01% and 18.66% in the mixed scenario, and by 42.09% and 35.24% in the expensive scenario, respectively. From this, it can be concluded that the consumers have a strong incentive to participate in the auction, since by doing so they will greatly increase their welfare and the number of appliances used.

This result was expected since in the time-varying pricing model alternatives are chosen according to the difference between value and price, which leads consumers to choose only alternatives with high value and low energy consumption. On the other hand, in the proposed model a consumer considers all appliances at once when bidding for an energy usage plan. Moreover, it is the objective of the proposed model to maximize social welfare (even though this is done indirectly by considering energy usage plans), while the time-varying model is not concerned with this.

Finally, the revenue in the proposed model decreased by 1.89% in the inexpensive scenario, but increased by 13.50% in the mixed scenario and 118.02% in the expensive scenario. This means that since the proposed model allows consumers to use more energy, it is able to earn more money for energy.

4.3. Energy allocation

Figures 3, 5 and 7 present renewable and non-renewable energy consumption and the renewable energy waste in the proposed model for the three scenarios considered, while Figures 4, 6 and 8 present those statistics for the time-varying model of pricing.

As the figures show, the proposed model wastes less renewable energy than the time-varying model for all three scenarios. In fact, renewable energy waste with the proposed model was 5.86% for the inexpensive model, 5.67% for the mixed model, and 10.77% for the expensive model, while, for the time-varying pricing model, it was 31.09% for the inexpensive model, 30.24% for the mixed model, and 46.60% for the expensive model.

Interestingly, the proposed model suggested using less non-renewable energy than the time-varying model of pricing in the inexpensive scenario but suggested using more energy than that suggested in

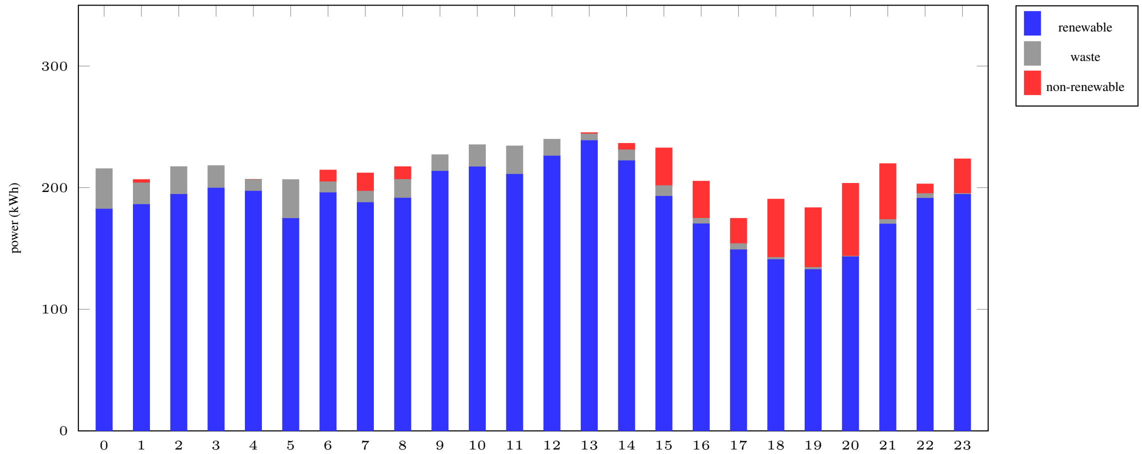

Fig. 3. Energy consumption using the proposed model in the inexpensive scenario.

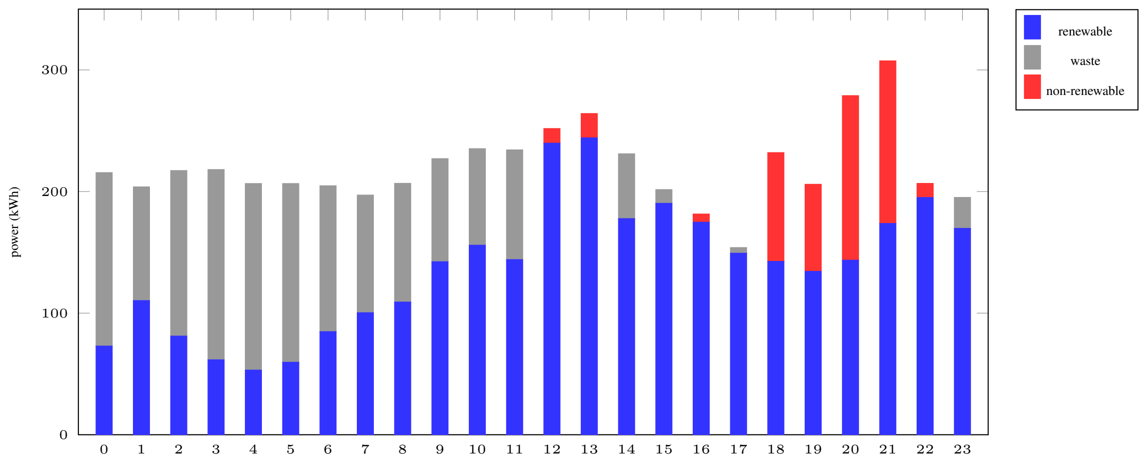

Fig. 4. Energy consumption using the time-varying model of pricing in the inexpensive scenario.

the mixed and expensive scenarios. The proposed model suggested using 365.81 kWh, 225.69 kWh and 137.17 kWh for the inexpensive, mixed, and expensive scenarios, respectively, with the time-varying model suggesting 480.40 kWh, 104.78 kWh and 61.49 kWh for the same scenarios. This happens because when energy is expensive, the consumers tend to use fewer appliances in the time-varying model.

Fortunately, it is possible to decrease the suggested non-renewable energy use by increasing the price of supplementary energy. In preliminary experiments, it was noticed that with higher values of r_t (such as 12 or 15) the proposed model led to a smaller consumption of non-renewable energy while still keeping better social welfare, revenue, number of appliances used and PAR when compared with the time-varying model.

Finally, as shown in Figures 3, 4, 5, 6, 7 and 8 and summarized in Table 4, the proposed model has a

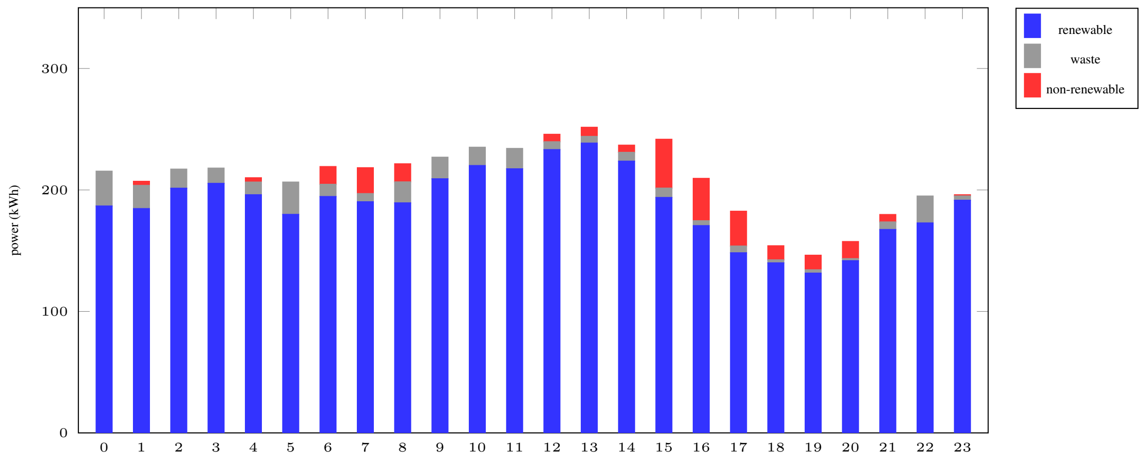

Fig. 5. Energy consumption using the proposed model in the mixed scenario.

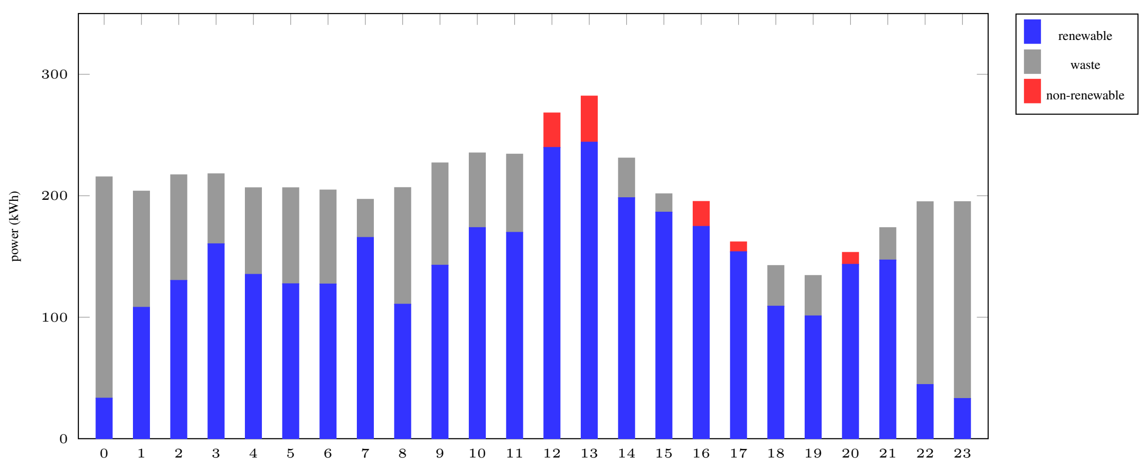

Fig. 6. Energy consumption using the time-varying model of pricing in the mixed scenario.

smaller peak energy consumption and a higher mean energy consumption, and consequently a smaller PAR.

4.4. Energy storage devices usage

Figures 9, 10 and 11 present the renewable and non-renewable energy consumption and the renewable energy waste for the proposed model when energy storage devices are available.

As the figures show, the renewable energy waste decreased to the level of 27.99%, 32.80%, and 45.87% of that generated by the proposed model without energy storage devices, for the inexpensive,

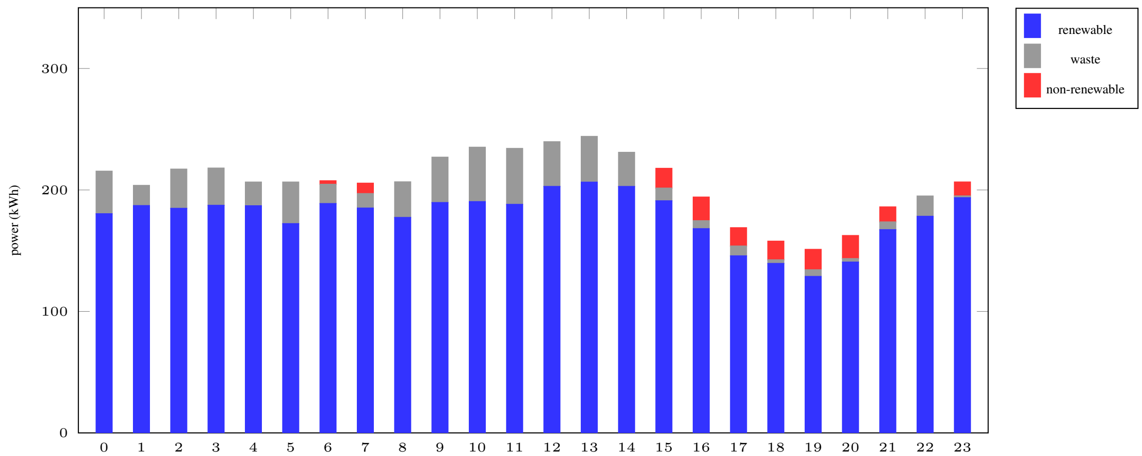

Fig. 7. Energy consumption using the proposed model in the expensive scenario.

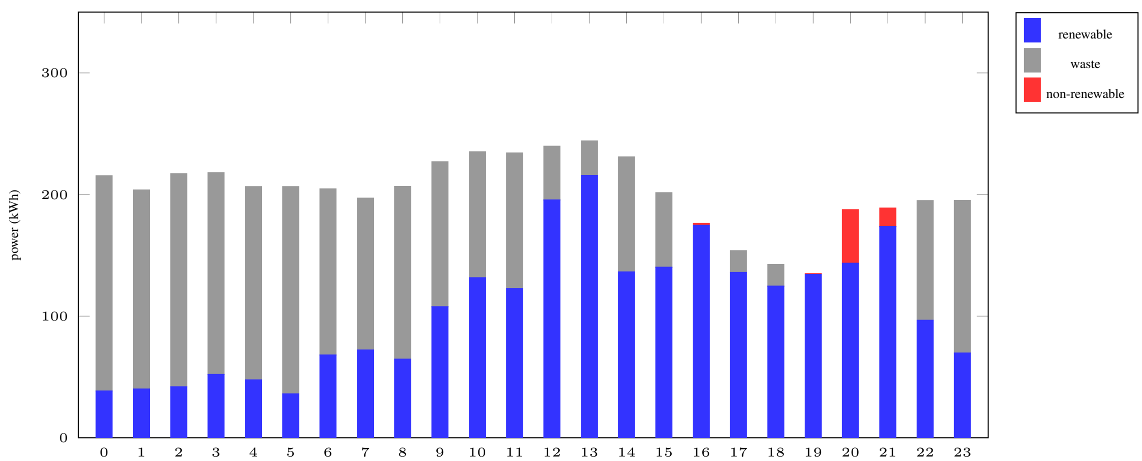

Fig. 8. Energy consumption using the time-varying model of pricing in the expensive scenario.

mixed and expensive models, respectively.

Moreover, the proposed model with energy storage devices increased social welfare by 0.76%, 1.26% and 2.06% in the inexpensive, mixed and expensive scenarios, respectively, when compared with the proposed model without energy storage devices.

The amount of non-renewable energy used by the proposed model with energy storage devices decreased 1.15% for the inexpensive model, but increased 16.46% for the mixed scenario and 13.68% for the expensive scenario. This was expected since there is an increase in social welfare (the consumers use more energy), a decrease in renewable energy waste (there is less renewable energy available) and the possibility of modifying the energy usage plan by storing energy when inexpensive and using it when expensive.

Table 4
Mean, peak and average to peak energy consumption for both models.

	Proposed			Time-varying		
Scenario	peak	average	peak/average	peak	average	peak/average
Inexpensive	239.58	203.51	1.18	307.35	157.82	1.95
Mixed	246.07	198.05	1.24	281.95	144.31	1.95
Expensive	207.27	184.15	1.13	215.62	109.35	1.97

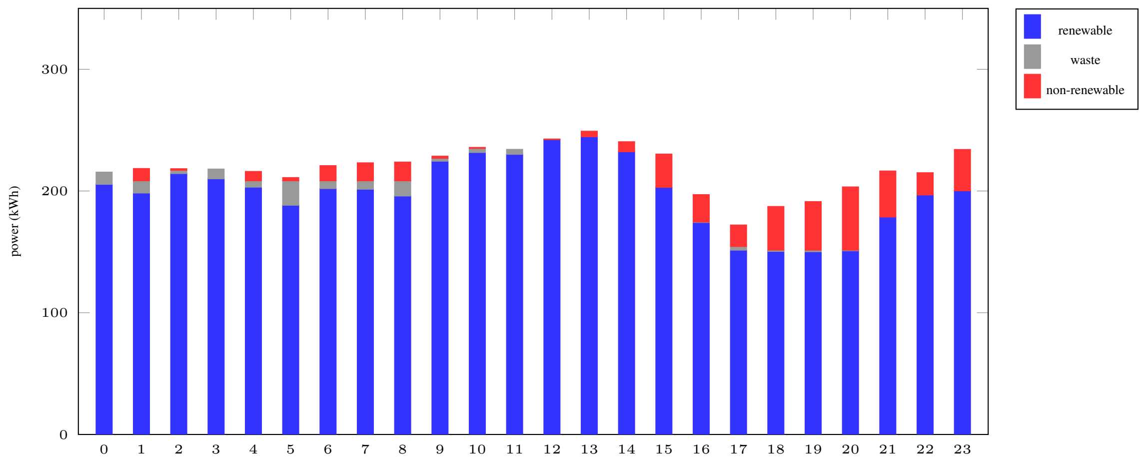

Fig. 9. Energy consumption using the proposed model with energy storage devices in the inexpensive scenario.

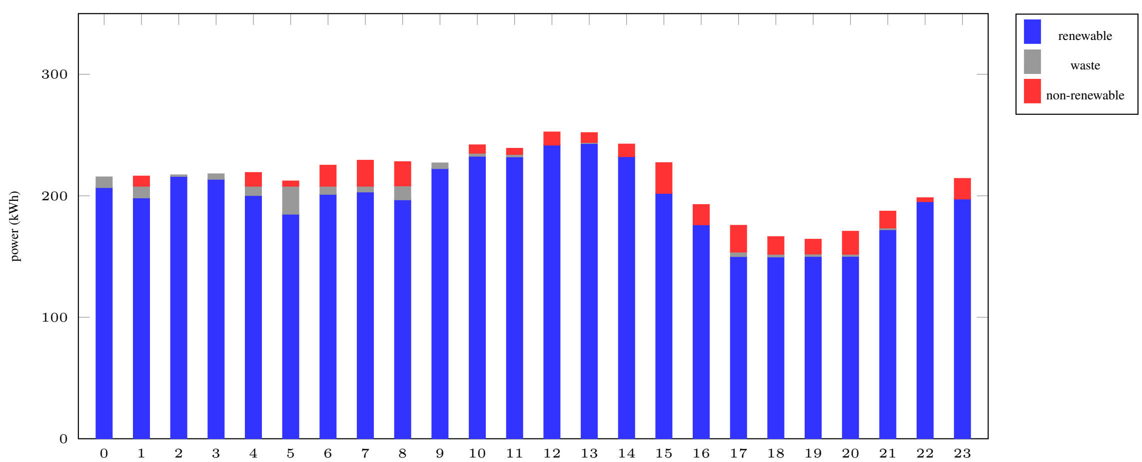

Fig. 10. Energy consumption using the proposed model with energy storage devices in the mixed scenario.

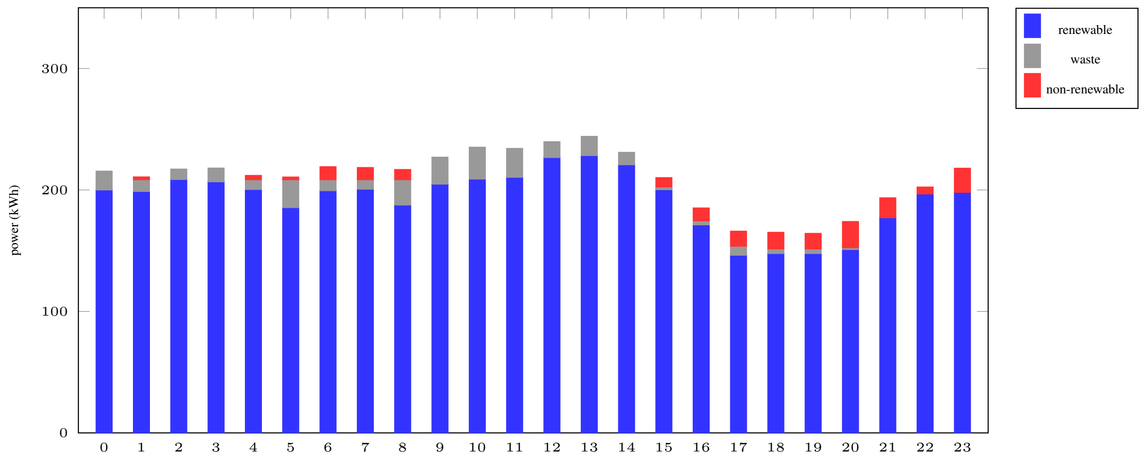

Fig. 11. Energy consumption using the proposed model with energy storage devices in the expensive scenario.

As for revenue, when the proposed model was used with energy storage devices, it increased by 4.44%, 4.79%, and 4.20% for the inexpensive, mixed and expensive scenarios, respectively.

Finally, the ratio of peak-to-average consumption for the proposed model with energy storage devices was 1.16, 1.20, and 1.15 for the inexpensive, mixed and expensive scenarios, respectively, a result similar to that obtained without energy storage devices.

4.5. Uncertainty of Renewable Energy Generation

Since the presented model does not directly handle the unpredictability of renewable energy sources, it must use a day-ahead prediction technique in order to approximate the capacity for the plant. As one of the techniques discussed in the literature to mitigate the impact of renewable energy generation prediction errors is to use storage units on the provider's side (Ssekulima et al., 2016), it is proposed that the presented model should be extended in the following way: if the predicted value is greater than that observed, then the system will store renewable energy in batteries (or waste it if they are at full capacity); and, if the predicted value is smaller than that observed, then the system will first try to use the energy stored in the batteries. If they are empty, it will acquire non-renewable energy in order to fulfill the auction contracts with consumers.

In the following section, a comparison between the cases with certainty and uncertainty in renewable energy production considering the proposed model (for simplicity without using energy storage devices as described in Section 3.4) is presented. It was considered that there are 2 additional provider-side batteries (with the same parameter considered in Section 4.1) used specifically for mitigating the prediction errors.

In order to simulate uncertainty, for every period of time t, the actual amount of energy produced at time t was chosen from a normal distribution with mean w_t and standard deviation $w_t/10$ independently at random. The analysis was made considering 100 trials randomly generated.

On average, for the inexpensive scenario, the renewable energy used decreased by 0.31% (from 4518.54 kWh to 4504.43 kWh, with a standard deviation of 22.26 kWh), the non-renewable energy used increased by 3.86% (from 365.81 kWh to 379.92 kWh, with a standard deviation of 22.26 kWh) and the waste decreased by 15.32% (from 281.45 kWh to 238.34 kWh, with a standard deviation of 64.92 kWh). For the mixed scenario, on average, the renewable energy used decreased by 0.38% (from 4527.68 kWh to 4510.44 kWh, with a standard deviation of 24.30 kWh), the non-renewable energy used increased by 7.64% (from 225.69 kWh to 242.93 kWh, with a standard deviation of 24.30 kWh), and the waste decreased by 16.57% (from 272.31 kWh to 227.18 kWh, with a standard deviation of 64.78 kWh). Finally, for the expensive scenario, on average, the renewable energy used decreased by 0.05%, (from 4282.57 kWh to 4280.29 kWh, with a standard deviation of 24.30 kWh), the non-renewable energy used increased by 1.66% (from 137.17 kWh to 139.45 kWh, with a standard deviation of 6.60 kWh), and the waste decreased by 16.95% (from 517.42 kWh to 429.71 kWh, with a standard deviation of 64.78 kWh). Thus, as long as there is a reasonably accurate forecast model and storage units on the provider's side to help mitigate the errors, the presented model remains effective despite the uncertainty of the energy sources. See Figure 12 (in contrast with Figure 5) for an example of energy consumption in the mixed scenario when there is uncertainty, including the usage of storage devices for prediction error mitigation.

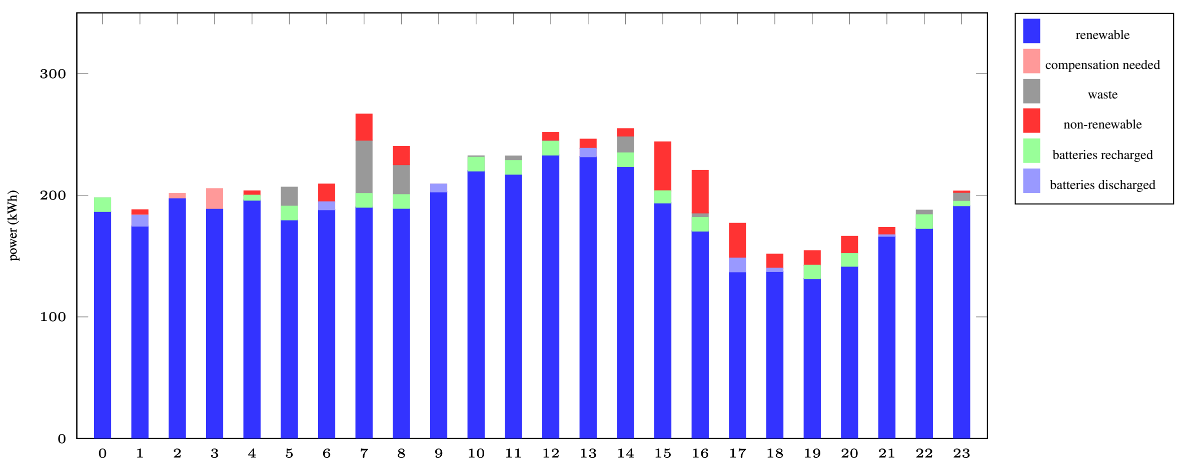

Fig. 12. Energy consumption using the proposed model with uncertainty mitigation for a test case of the mixed scenario. The amount of external non-renewable energy needed to compensate for the prediction error is shown in light red, the amount of energy used to charge the mitigation storage units is shown in light green, and the amount of energy taken from the storage units to mitigate the prediction errors is shown in light blue.

4.6. Performance Evaluation

All the numerical experiments were executed in a computer with the following configuration: Intel (R) Xeon (R) CPU X3430 with 2.40GHz of clock rate, 4 cores and 8GB of RAM with Linux 64bits (Ubuntu). Both optimization problems were modeled as integer linear programs that were solved using the python2 API of Gurobi (Gurobi Optimization Inc., 2015) version 8.1.1. All the simulations and analysis were coded in Python 2.7.

Although the problems that appear in the proposed model are NP-hard, these problems could be solved reasonably fast with a commercial ILP solver when considering an instance of a size that is common in the literature (for example, Samadi et al. (2012) considers an instance with 50 consumers). For the energy allocation problem, which is solved in a centralized way, the execution time was 853, 583 and 710 seconds for the inexpensive, mixed and expensive scenarios of the proposed model (without batteries) respectively. When considering energy storage devices, the extended model was solved in 295, 214 and 456 seconds for the inexpensive, mixed and expensive scenarios, respectively.

For the bid computations, several ILP problems were solved, i.e. one for each consumer and usage plan. The total time to solve all the bid computation problems was 1494, 1785 and 2193 seconds for the inexpensive, mixed and expensive scenarios respectively. The average computing time per consumer was about 14, 17 and 21 seconds for the inexpensive, mixed and expensive scenarios respectively. The computation times for the model with energy storage devices was close to its counterpart, in which it was 14, 16 and 20 seconds for the inexpensive, mixed and expensive scenarios, respectively.

5. Conclusions

This paper has presented a new model for smart pricing for demand-side management by using energy consumption scheduling devices for renewable energy. The proposed model is based on an auction where consumers submit bids for a set of possible energy usage plans designed by the utility company and then allocates energy according to the power plant production capacity. A model for generating consumers' bids which can consider must-run, non-interruptible, interruptible appliances and other variations is presented. Both models are extended to consider the usage of energy storage devices.

The main advantage of the proposed model is that it does not make any assumptions about how consumers will derive value from energy consumption nor does it depend on how bids are computed by consumers. That is, the model presented for generating consumers bids is only a suggestion, and can be replaced by any other model with the same purpose. This simplifies the perspective of the consumer about the system since a bid represents how much a consumer is willing to pay for a specific usage plan. Moreover, this allows for the creation of models for other consumers such as universities, hospitals or industries, as long as one can define how those consumers allocate energy from a given energy usage plan and how they obtain value from energy usage.

Numerical examples show that the proposed model offers great improvement when compared with the time-varying model of pricing. In fact, it was able to increase social welfare, as well as the mean number of appliances used by a consumer and revenue, while reducing the renewable energy waste and peak-to- average ratio. Even though more non-renewable energy is consumed, this can be reduced by increasing the price of supplementary energy, so that the values are lower than those obtained using the time-varying pricing model while still obtaining better social welfare and high revenue. It has also been shown that energy storage devices can improve social welfare while reducing energy waste. Finally, the numerical simulations give us confidence that the models could be implemented efficiently with existing ILP solvers both for the energy allocation and bid computation problems.

As future work, the proposed model could be extended to better consider the unpredictable nature of renewable energy generation. In the experiments, it is shown that this unpredictability does not significantly affect the proposed model and that these effects can be mitigated using additional batteries. Nonetheless, a solution that specifically considers the unpredictability could obtain better results.

Acknowledgements

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