

# Latency Reduction in Probabilistic Broadcast Protocols for Ad Hoc Networks

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**Abstract**—This letter introduces a strategy to reduce broadcast latency in multi-hop ad-hoc networks; this overall reduction is the result of the accumulation of a series of per-node delay reductions. Current broadcasting protocols employ uniformly distributed random delays on a per-node basis, while the proposed strategy consists of using a truncated-exponential distribution to determine these delays. Such an approach can reduce latency significantly while maintaining broadcasting reachability, yet it entails minimal additional overhead in relation to existing protocols.

**Index Terms**—Ad-hoc networks, probabilistic broadcast, random assessment delay, MAC layer.

## I. INTRODUCTION

IN AD-HOC networks employing probabilistic broadcast protocols, nodes forward broadcast packets after an interval of random duration, starting at the time of the packet arrival at the node [1].

Such probabilistic broadcast protocols decrease the forwarding probability ( $p_f$ ) as node density increases. These protocols estimate node density from neighboring nodes, either by the collection of copies of already-transmitted broadcast packets or by the reception of special control packets sent periodically for that purpose (i.e., Hello packets) [1]. A random uniformly distributed delay (defined in an interval  $[0, T]$ ) is employed in both of these two types of protocols. In the former, it defines an interval for nodes to receive copies of transmitted packets. In the latter, the delay prior to transmission decreases the probability of nearby nodes forwarding broadcast packets simultaneously.

After a certain random delay, the broadcast protocol passes the broadcast packet from the network layer to the MAC (data link) layer. This contributes to a reduction in collisions on MAC layers employing Carrier Sensing (CS) access mechanisms, since packets are more sparsely distributed in time. Reducing collisions is of paramount importance in ad-hoc networks, since collisions can lead to disconnected network

segments, which affect the percentage of nodes covered by packet propagation (reachability).

However, the use of random delays results in slow packet dissemination. There is, thus, a tradeoff between reducing collisions and reducing random delays, and this impacts on the end-to-end latency [2] (latency for short). Since most protocols employ a uniform distribution, changing the domain range of the uniform distribution has been considered as a possible way for reducing latency, although at the expense of reducing reachability [3], [4].

In this letter, it is shown that changing the distribution of random delays from uniform to truncated-exponential reduces the latency in probabilistic broadcast protocols without compromising reachability. For the same interval  $[0, T]$ , the mean duration of random delays can be reduced by using a truncated-exponential distribution for these delays rather than a uniform distribution. Given a multi-hop transmission in which successive random delays (one at each hop) occur along an end-to-end path, employing a distribution with a smaller mean duration results in lower latencies (i.e., the time elapsed between transmission and the arrival of the packet at the last node). The ability to avoid compromising reachability is the consequence of the higher coefficients of variation (CV) of truncated-exponential random delays, which reduces the synchronization of transmissions on the MAC layer [5], thus, avoiding collisions. Moreover, the computation of truncated-exponential values requires only the computation of an inverse exponential function, which is a closed-form equation. Thus, it introduces negligible overhead in existing protocols.

Reducing latency is essential for delay-sensitive applications [6]. Moreover, fast broadcasting of information compensates for the uncertainty resulting from topological changes in the analysis of Vehicular Ad Hoc Networks (VANETs), which allows making the assumption that VANETs have a static random distribution of nodes [7].

This letter is organized as follows. Related work is summarized in Section II. Section III provides an analytical assessment of the effects of using the proposed distribution. Finally, simulation results validate the proposal.

## II. RELATED WORK

In [4], a hybrid scheme for avoiding low reachability is proposed (requiring Hello packets and the collection of copies of broadcast packets); this scheme uses two different uniform random delays, with different ranges. In [2], the upperbound value of the interval ( $T$ ) is different for each node, and is defined as a function of the number of expected simultaneous forwarding nodes for each hop. The hybrid scheme in [8] uses a sequence of three random delays, thus increasing the forwarding probability ( $p_f$ ) as a function of the waiting time to receive a copy of a packet. However, all of these

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techniques rely on hello messages and are not applicable to other protocols.

Another strategy associates the duration of the random delays with the inverse of the forwarding probability [9]. This strategy produces an almost identical delay for all nodes located in a neighborhood, thus allowing the use of strong assumptions about information availability, such as the Euclidean distance between nodes or the exchange of packets that contain the IDs of two-hop neighbors.

Overall, most of the strategies based on the use of uniform random delays are specific for given protocols, which makes them unsuitable for other broadcast protocols. Moreover, changes in the type of distribution of random delays have never been proposed in the literature [1].

### III. IMPACT OF THE EMPLOYMENT OF EXPONENTIALLY DISTRIBUTED RANDOM DELAYS ON COLLISION EVENTS

This section analyzes the effect of employing truncated-exponential random delays on packet collision events. It will be shown that the reductions of the mean duration of random delays lead to little variability in the probability of collision events.

One type of collision event that reduces reachability occurs when broadcast packets reach regions of the network which have not received any copy of this packet (unseen packet), with several nodes receiving copies of this packet from the same wireless transmission [10]. The probability of collisions of nodes receiving the same unseen packet at virtually the same time can be computed by considering two simultaneous events: i) the probability of having random delays ending with a time difference smaller than the duration of the backoff window (overlapping backoff periods), and ii) the probability of two backoff timers expiring in the same transmission slot.

For the analysis in this letter, we use the IEEE 802.11b Distributed Coordination Function (DCF) MAC Layer [1]. Thus, we assume a minimum backoff window size of 32 slots. No acknowledgment mechanism is assumed [10], so that no retry counter exists to increment the size of the backoff window. We also use a value of  $T = 10\text{ms}$ , chosen for intervals of random delays (network layer) since this is the most frequent value used in a variety of studies [1], [11]; this facilitates the comparison of our results with those of other studies.

#### A. Probability of Two Overlapping Backoff Periods

Let  $[0, T]$  be the time interval in which the distribution of random delays is defined, and let  $V$  be the duration of the minimum backoff window on the MAC Layer ( $V = 32 \times 20\mu\text{s}$ ). Since differences in the time of propagation and processing of a given unseen packet that reaches several nearby forwarders are negligible, the random delays of these forwarders can be assumed to start at the same time. Dividing  $[0, T]$  into intervals of length  $V$  (hereinafter V-intervals), backoff intervals will overlap if the random delay values fall in a common V-interval. Figure 1 shows the random delays of forwarders  $FW_1$  and  $FW_2$  ending in  $[V_{i-1}, V_i]$  on the network layer. Even if  $FW_1$  passes the packet to its MAC Layer at the very beginning of  $[V_{i-1}, V_i]$ , a backoff window of length  $V$  will overlap with the beginning of the backoff window on the MAC layer of  $FW_2$ . If two random delays end in adjacent V-intervals,

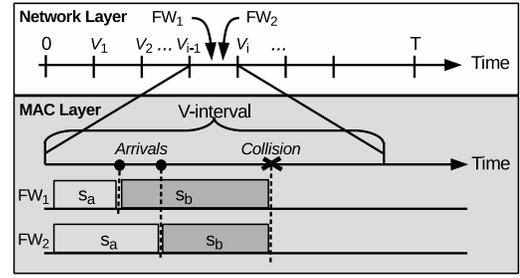


Fig. 1. Two forwarders passing broadcast packets to their MAC Layers in the same V-interval.

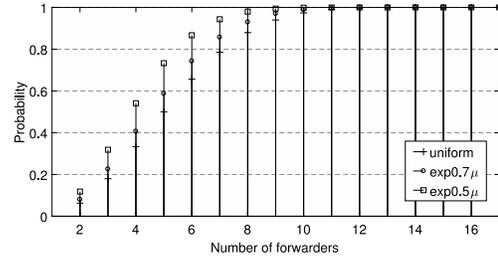


Fig. 2.  $P(R_1 \leq k)$  uniform vs. truncated exponential random delays.

their backoff windows *may* overlap. In this section, we focus on backoff windows that do overlap (Figure 1); later we will show that the random delays of additional forwarders ending in adjacent V-intervals will be negligible for the analysis.

The probability of a random delay ending in  $[V_{i-1}, V_i]$  is given by  $p_i = F(V_i) - F(V_{i-1})$ , where  $F(t)$  is the cumulative distribution function (cdf) of the random delays. For an integer number  $I$  of V-intervals,  $I = \lceil T/V \rceil$ ,  $p_i$  is the probability mass function (pmf) of having a random delay ending in the  $i$ -th V-interval.

For an arbitrary  $p_i$  over the set of V-intervals in  $[0, T]$ , the probability of two forwarders choosing the same V-interval ( $R_1$ ), with exactly  $k + 1$  forwarders, is obtained from the solution of the Birthday Paradox problem [12]:

$$P(R_1 = k) = \sum_{|A|=k} k! \Pi_A p_A, \quad (1)$$

where  $\Pi_A = \prod_{i \in A} p_i$ ,  $p_A = \sum_{i \in A} p_i$ , and the summation spans all subsets  $A$  of  $k$  V-intervals. Then, when  $k + 1$  forwarders contend, the probability of having a pair of random delays ending in a common V-interval is  $P(R_1 \leq k) = \sum_{n=1}^k P(R_1 = n)$ . Figure 2 compares the behavior of  $P(R_1 \leq k)$  for  $T = 10\text{ms}$  with different  $p_i$  obtained from uniform and truncated-exponential random delays. In Figure 2,  $\text{exp}07\mu$  and  $\text{exp}05\mu$  indicate that  $p_i$  was obtained from truncated-exponential distributions. In these distributions, the mean durations of random delays are reduced to 70% and 50% in contrast to the mean duration of delays with a uniform distribution.

Figure 2 shows two aspects of the probability of having overlapping backoff periods. First, the cumulative probability distribution,  $P(R_1 \leq k)$ , reaches almost 1 with fewer than 10 forwarders. This means that *overlapping backoff periods* are almost certain in ad hoc broadcasting since the average number of neighbors that leads to a connected network (99% of

the time) is  $> 7.5$  for nodes placed uniformly at random [13]. Second, as the number of forwarders increases, the relative increase in  $P(R_1 \leq k)$  is reduced in relation to the use of uniform random delays. For example, with 2 forwarders and  $\text{exp}05\mu$ , the probability of overlapping backoff periods almost doubles (an increase of 89%), whereas the increase is only 20% with 8 forwarders. With a larger number of forwarders, such an increase tends to disappear, leading to a 50% reduction in the mean duration for random delays. Moreover, for  $\text{exp}07\mu$  the impact is even smaller (less than a 6% increase with 8 forwarders).

When using a uniform distribution, equivalent reductions in the mean duration of random delays produce larger values of  $P(R_1 \leq k)$  than a truncated-exponential distribution, i.e., the truncated-exponential  $p_i$  always produces a smaller increase in the number of collision events for the same reductions in the mean duration of random delays.

It is possible to extend this analysis to consider two or more V-intervals [12] (e.g., multiple pairs of random delays ending with time differences smaller than  $V$ ). However,  $P(R_1 \leq k)$  is sufficient to assess the impact of the truncated-exponential distribution, since we are interested in the *relative* increase in collision events in comparison to a uniform random-delay distribution. Moreover, an extended analysis including multiple overlapping V-intervals leads to the same conclusion.

### B. Probability of Two Transmissions Beginning in the Same Slot

We have focused on the arrival of two packets in the same V-interval (Figure 1). In this section, however, the backoff timer assigned to packets by the MAC protocol is added to the arrival time of packets at the MAC Layer to determine the slot in which the transmission of a packet begins. As pointed out in [7], backoff timers should be non-persistent in ad-hoc broadcasting, therefore, our analysis adds a backoff timer to every packet.

In an interval  $[V_{i-1}, V_i]$ , the probability density function (pdf) of the arrival time of packets at the MAC Layer is  $f(t | V_{i-1} < t \leq V_i)$ , where  $f(t)$  is the pdf of random delays in  $[0, T]$ . Denoting  $g_u(t) = f(t | V_{i-1} < t \leq V_i)$  for uniform random delays, and  $g_e(t)$  for truncated exponential random delays, gives the following:

$$g_u(t) = \frac{I}{T}, \quad 0 < t \leq T/I, \quad \forall i \quad (2)$$

$$g_e(t) = \frac{\lambda e^{-\lambda t}}{1 - e^{-\lambda(T/I)}}, \quad 0 < t \leq T/I, \quad \forall i. \quad (3)$$

Notice that, for every  $i$ ,  $g_e(t)$  is the same, since a truncated exponential distribution conditioned on equal-length subsets yields the same distribution for all subsets.

Since the MAC layer uses time slots of fixed duration  $s$ ,  $g_e(t)$  can be expressed as a truncated geometric distribution  $g_e(s = s_a)$ , i.e., the probability mass function (pmf) of a broadcast packet arriving at the MAC layer at  $V_{i-1} + s_a$  is

$$g_e(s = s_a) = \frac{p(1-p)^{s_a-1}}{1 - (1-p)^S}, \quad s_a = 1, 2, \dots, S \quad (4)$$

where  $S = 32$ ,  $p = 1 - e^{-\lambda s}$ , and  $s = V/32$  is the length of a slot. For uniform random delays, the equivalent pmf is  $g_u(s = s_a) = 1/32$ .

Upon arrival at slot  $s_a$ , the packet receives a random backoff timer of  $s_b$  slots, uniformly distributed in the backoff window. Then, the probability of two transmissions beginning in the same transmission slot (collision event) is the probability of having the same result  $s_a + s_b$  for packets with overlapping backoff intervals (Section III-A). Such a collision event is illustrated in Figure 1 for the packets of forwarders  $FW_1$  and  $FW_2$ .

Denoting  $h(s = s_b)$  as the uniform pmf of backoff timers, the convolution  $w_s = g(s) \otimes h(s)$  yields the pmf of the sum  $s_a + s_b$ ;  $g(s)$  will be either  $g_e$  or  $g_u$ .

Finally, collision events have probability  $P_C = \sum_s w_s^2$ , which considers all the cases in which the sum  $s_a + s_b$  is the same for the two forwarders.

For  $T = 10\text{ms}$  and  $\text{exp}05\mu$  on the network layer, the value of  $P_C$  (at the MAC layer) is  $P_C = 0.0209$ . This value of  $P_C$  is only 0.084% higher than the value of  $P_C$  obtained when considering uniform random delays. The following proposition explains this small increase in the value of  $P_C$  considering reductions of up to 50% in the mean duration of random delays. The proposition evinces that the impact of the change in the distribution  $f(t)$  can be estimated directly from  $P(R_1 \leq k)$ , defined in Section III-A, since collision probability values on the MAC layer (which depend on  $g_u(t)$  and  $g_e(t)$ ) differ by negligible values.

*Proposition 1:* Let  $f_e(t)$  and  $f_u(t)$  be the truncated-exponential and the uniform probability distribution functions, both defined in  $[0, T]$ , with means denoted by  $\mu_e$  and  $\mu_u$ . If  $\mu_e = \alpha\mu_u$ , for  $0.5 \leq \alpha < 1$  and  $I \gg 4$ , then  $g_e(t) \rightarrow g_u(t)$ .

*Proof:* Since  $f_e(t)$  and  $f_u(t)$  correspond to distributions in  $[0, T]$ , then  $\mu_e = 1/\lambda - Te^{-\lambda T}/(1 - e^{-\lambda T})$  and  $\mu_u = T/2$ . Thus,  $\alpha\mu_u = \mu_e$  yields:

$$\frac{\alpha}{2} = \frac{1}{\lambda T} - \frac{e^{-\lambda T}}{1 - e^{-\lambda T}}, \quad (5)$$

which is a function of the product  $\lambda T$ . As  $\lambda T$  increases,  $e^{-\lambda T}/(1 - e^{-\lambda T})$  decreases much faster than  $1/\lambda T$  in (5), hence  $\alpha/2 < 1/\lambda T$ . Then, for  $0.5 \leq \alpha$ ,  $\lambda T < 4$ . Given that  $0 < t \leq T/I$  in (3),  $I \gg 4$  results in  $\lambda t \ll 1$ . Using the series expansion of the exponential function,

$$g_e(t) = \frac{\lambda[1 - \lambda t + \frac{1}{2!}(\lambda t)^2 - \dots]}{\frac{\lambda T}{T}[1 - \frac{1}{2!}\frac{\lambda T}{T} + \frac{1}{3!}(\frac{\lambda T}{T})^2 - \dots]} \quad (6)$$

$$\approx \frac{I}{T} = g_u(t), \quad 0 < t \leq T/I \quad (7)$$

Since  $g_e(t) \rightarrow g_u(t)$ , a more detailed analysis of increasing collision events on the MAC layer would be redundant. ■

## IV. SIMULATION RESULTS

This section shows the reduction in latency achieved by the employment of truncated-exponential random delays when compared to the use of uniform random delays. Such a reduction does not significantly influence reachability. Moreover, results demonstrate that a larger number of forwarders (node degree) leads to a more significant reduction in latency.

The QualNet®v.7.3 network simulator was used to illustrate the results derived theoretically. The replication method was employed to derive confidence intervals with a 95% confidence

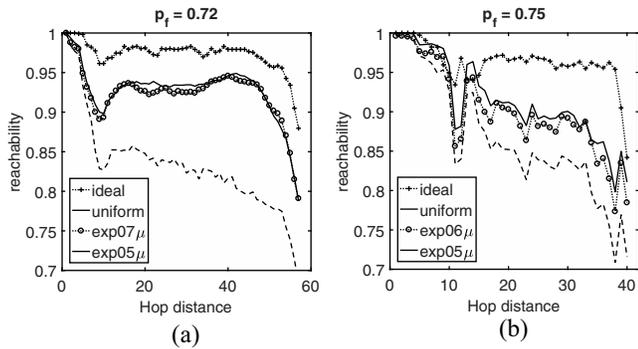


Fig. 3. Reachability for different random-delay distributions. (a) Grid scenario with a node degree of 4. (b) Random scenario with an average degree of 8.

TABLE I  
LATENCY IN TWO MULTI-HOP SCENARIOS, UNIFORM VS. TRUNCATED-EXPONENTIAL RANDOM DELAYS

Grid, $p_f = 0.72$		Random network, $p_f = 0.75$	
Rn. delay distribution	Latency (in ms) with 95% CI	Rn. delay distribution	Latency (in ms) with 95% CI
uniform	$402.9 \pm 8.4$	uniform	$265.6 \pm 5.1$
exp0.7μ	$307.9 \pm 8.0$	exp0.6μ	$182.0 \pm 4.3$
Latency reduction <b>23.6%</b>		Latency reduction <b>31.5%</b>	

level. At least 200 replications were produced for each point; replications were generated until each confidence interval was smaller than 5% of the mean value. Confidence intervals are omitted from the figures for the sake of visual legibility.

The network scenarios for comparison were those in [14], in which forwarding probability values,  $p_f$ , were set to produce reachability above 0.95, assuming no collision events (ideal conditions). Two different scenarios were simulated: a grid topology with node degree equal to 4 and a random network with an average degree of 8. These scenarios have also been used in [15] to evaluate the 802.11-MAC layer in different topologies using probabilistic broadcasting.

Figure 3 shows the reachability level of broadcast packets as a function of the distance of the nodes from the source. Figure 3a shows that when an ideal MAC layer (collision-free) is employed in the grid topology, reachability is above 0.95 even for nodes 53 hops away from the source [14]. When the 802.11 MAC layer with uniform random delays was used, reachability dropped by roughly 0.05. Such a difference occurs for nodes at a hop-distance larger than 10 for the random topology (Figure 3b).

When using truncated-exponential delays, reachability is maintained even when the mean random delays are reduced to 70% (exp0.7μ) of the original value (Figure 3a) for the grid topology and 60% (exp0.6μ) for the random topology (Figure 3b). As indicated in Section III, a larger number of forwarders influences the maintenance of reachability. Even for more substantial reductions in the mean random delay such as a 50% reduction, a larger number of forwarders contributed to maintaining the reachability up to 13 hops for the random topology (Figure 3b), while it was maintained for only 6 hops for the grid topology (Figure 3a).

Table I shows the average latency for similar reachability levels as shown in Figure 3. For both topologies,

the employment of truncated-exponential random delays reduced latency in relation to that of uniform random delays. For the grid, the value of latency was 23.6% lower; for the random topology, the value of latency was reduced by 31.5%.

## V. CONCLUSION

The present letter has shown a way of reducing latency by using probabilistic broadcasting protocols. Results show that latency can be reduced and reachability maintained for the same rebroadcasting, although this is not possible when using a wide-spread uniform distribution. This reduction is achieved by decreasing the mean duration of random delays by using a truncated-exponential distribution. Moreover, such a distribution can be applied to any probabilistic broadcasting protocol for ad hoc networks, including VANETs.

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