Inscribed Rectangles Algorithm for Routing, Core and Spectrum Assignment for SDM Optical Networks

Pedro M. Moura
State University of Campinas
Institute of Computing
Brazil
email: pedrom@lrc.ic.unicamp.br

Nelson L. S. da Fonseca
State University of Campinas
Institute of Computing
Brazil
email: nfonseca@ic.unicamp.br

***Abstract*—This paper proposes a Routing, Core and Spectrum Assignment (RCSA) algorithm based on the image processing Inscribed rectangle algorithm. The solution aims at discovering portions of spectrum capable of accommodating requests with low computational complexity. Advanced fitting policies are proposed to chose which portion of the spectrum to choose in order to reduce blocking and crosstalk. Results show that the proposed algorithm can reduce the blocking ratio under low loads and crosstalk under all loads, when compared to other RCSA algorithms in the literature.**

I. Introduction

In the past decade, there has been a growth of heterogeneous applications over the Internet. While some applications such as e-mail have low bandwidth requirements, others have high demands, such as IPTV [1]. Elastic (flexgrid) optical network technology can cope with such heterogeneous demands since they employ a fine-grained multiplexing of the spectrum, capable of allocating different bandwidths demands. These networks employ Orthogonal Frequency Division Multiplexing (OFDM), which divides the spectrum into several slots (also called subcarriers) with low data rates and overlapping orthogonal frequencies. Such multiplexing technology achieves high spectrum allocation efficiency and wastes less resources when compared to traditional Wavelength Division Multiplexing [2] [3] [4].

However, optical network technology is expected to reach its physical limitation in a near future as a consequence of the increased demand of bandwidth from Internet applications. To expand the transmission capacity of optical fibers, space division multiplexing (SDM) has been proposed [5]. Employing multicore fibers (MCF) is one of the promising choice for SDM, since it employs several independent cores in a fiber cable, allowing them to be treated as single core fibers (SCF) [6] [7] [8].

The Routing and Spectrum Assignment (RSA) problem in elastic optical networks consists in finding routes and available spectrum to establish a connection. Such allocation is subject to the continuity and contiguity constraints. The former imposes that the set of slots assigned by an algorithm must be the same in all links along a route to avoid costy opto-electrical conversion of the optical signal, while the latter imposes that the slots allocated must be contiguous, to achieve high spectrum efficiency. In SDM optical networks, this problem is extended to the Routing, Core and Spectrum Allocation (RCSA) problem. RCSA algorithms allocate routes and spectrum as do the RSA algorithms, but they also allocate cores for the establishment of a connection. The addition of the space dimension in multiplexing increases the complexity of spectrum assignment, since the spectrum of all cores have to be taken into consideration during the computation of routes.

The spectrum of various network cores can be represented as a matrix, as proposed in [9], each line of this matrix representing a core, and each position of the lines representing a slot. This is similar to the representation of a binary image, and some already existing algorithms used in image processing can be applied in RCSA algorithms to find available portions of the spectrum with low complexity.

One of the image processing algorithms that can be used is the Inscribed Rectangle search (IR), that aims at finding rectangles of connected pixels, i.e. same value neighbor pixels, with the shape of a rectangle. Figure 1 illustrates the process by selecting two different rectangles inside the connected shapes. Several rectangles can be found inside a shape, depending on its format, and, one needs to be chosen to establish a connection.

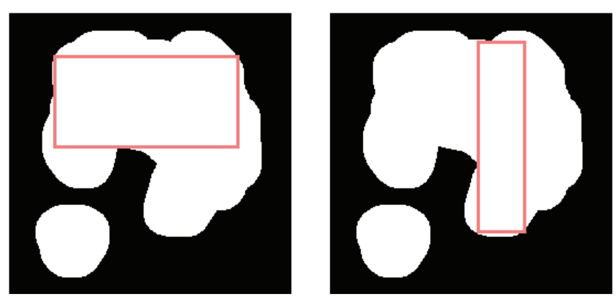

Fig. 1. Inscribed rectangles inside a shape.

The inscribed rectangles algorithm outputs a set of all possible distinct rectangles inside the image, which is useful to the RCSA problem as their sizes and shapes can be explored to develop solutions that can ameliorate problems caused by the usage of the space dimension, such as crosstalk and fragmentation. In Figure 1, the second rectangle would potentially cause more crosstalk, since it is larger, demanding a higher number of cores to allocate the connection.

By allocating a higher number of contiguous slots on the same core, it is possible to have less guard bands to divide the connections, which can lead to higher spectrum efficiency. However, the algorithm also needs to allocate connections in situations with no available slots on the same core. The usage of the inscribed rectangles fulfills this need by favoring large assignments. Besides, the usage of rectangles can lead to a better usage of the spectrum, less fragmentation and consequently lower blocking ratio.

II. Related Work

In [6], an introduction to the spectrally and spatially flexible optical networks was provided, presenting the fundamental concepts, benefits and challenges. Some examples of implementation were described, proposing network components capable of transmitting data in SDM optical networks.

In [7], an overview of the possible SDM approaches, focusing on network planning was given. The paper introduced the RCSA problem (called RSMLSA by the authors) and listed the challenges due to the use of the space dimension. Network control frameworks were also presented to endorse the usage of centralized solutions based on Software Defined Networks and Path Computation Elements (PCE).

In [10] an fragmentation aware RSA algorithm was proposed to decrease the number of connection allocated causing fragments on the optical spectrum. The idea is expanded to the SDM dimension in [8], proposing an RCSA algorithm that mitigates the spectrum fragmentation was introduced. Spectrum fragmentation occurs due to the dynamic establishment and tear-down of connection of different bandwidth, leaving portions of contiguous spectrum unable to accommodate new requests. Simulation results showed that the proposed algorithm can reduce the blocking ratio when compared to approaches that do not take fragmentation into consideration. The paper also shows that the crosstalk is generated by transmissions, and proposes a technique to monitor in-band crosstalk.

In [11], an integer linear programming (ILP) formulation for the RCSA problem was introduced to minimize the maximum number of slots required by any core of a link. A heuristic is also proposed for a scalable solution. Results indicate that the heuristic approximates the optimal solution given by the ILP.

In [5], a dynamic RCSA algorithm was introduced to mitigate crosstalk. The algorithm considers the spectrum occupancy state in order to allocate a new connection, avoiding portions that would have caused unnecessary crosstalk. Results show that this approach can reduce blocking.

In [9], it was proposed the first algorithm applying image processing to the RCSA problem, with the Connected Component Labeling (CCL) algorithm used to find available portions of the spectrum. Although the CCL algorithm can efficiently allocate portions of the spectrum with low computational complexity, however, it cannot be employed to make decisions based on the shape of the available portions of the spectrum.

The proposed algorithm takes into account the shape of the portions of the spectrum, selected by the inscribed rectangles algorithm, as they can be easily measured by their height and width and use this information to efficiently allocate the connections with with spectrum fitting policies.

III. Inscribed Rectangles

This section describes the Inscribed Rectangles algorithm, based on the Largest Rectangle Algorithm [12]. It consists in dividing the problem into smaller sub-problems by first finding smaller squares and then combining them into larger ones, as commonly done in dynamic programming solutions.

The problem of finding the rectangles is approached by creating a mask matrix M, with the same size as the image, and filling it bottom up and right left, based on the neighbors of the pixels. The pixels presenting neighbors previously marked with value 0, are marked as 1 in M. For pixels that have only neighbors with value larger than 0 the following equations is used to define the value of M:

$$M_{i,j} = min(M_{i+1,j}, M_{i,j+1}, M_{i+1,j}) + 1 \quad (1)$$

This procedure will result in a mask that represents the depth of the squares, resulting in multiple squares of different indexes.

The IR algorithm is described in the three following steps:

1) Compute squares array containing inscribed square size for each pixel (using the algorithm described).
2) Scan rows of the mask; for each row, scan columns from right to left and update the widths, heights and sizes arrays with the largest inscribed rectangle with width greater or equal than the height for each pixel.
3) Scan columns of the mask; for each column scan rows from bottom to top. Update widths, heights and sizes arrays with the largest inscribed rectangle with width shorter than the height for each pixel.

At the end of the process, an array with all possible rectangles will be available. To turn this array into a set, it is needed to eliminate the rectangles contained by others, this is done by adding into the set only rectangles that are not contained by others, i.e. rectangles smaller than others with greater starting pixels and lower ending pixels.

The described algorithm is the base of the proposed RCSA algorithm described in the following section.

IV. Inscribed Rectangles RCSA

The RCSA algorithm was designed to operate in networks with dynamic arrival of requests for the establishment of lightpaths. It is assumed that it is implemented in ideal Path Computation Elements and that information about the status of spectrum availability is stored in PCE databases.

Since the complexity of routing algorithms for optical networks increases with the addition of the spatial dimension in multiplexing, new algorithms need to reduce this computational complexity. The proposed algorithm aims at keeping the computational complexity low by representing the optical spectrum as a binary matrix of slots. Such representation is the same as used in binary images, and therefore, it is possible to apply developed methods in image processing to reduce the complexity of the task of finding available spectrum.

In the proposed algorithm, the optical network is represented by a graph, in which the vertices represent SDM Optical Cross Connectors (OXC), and the edges represents multi-core fibers connecting two vertices. Each edge has an associated matrix that represent its optical spectrum. Free slots of a link are represented by the value 0 in the matrix element and the value 1 is used otherwise.

A K Shortest Paths (KSP) algorithm [13] is employed to calculate routes. The spectrum of the candidate paths are then analysed to accommodate bandwidth requests.

The Inscribed Rectangle Algorithm is used to find rectangles in the spectrum with size higher than the demand of the bandwidth requested. The set of rectangles is used as input for a fitting policy, that can choose the most appropriated rectangle in the list, considering different parameter. In this paper, two advanced fitting policies are proposed in the following section.

The algorithm consists of the three following steps:

1) The candidate paths are calculated by using the KSP algorithm;
2) To guarantee the spectrum continuity, the discovery of available slots in all links along each path is needed. This is done by calculating a matrix that stores the result of a binary AND operation between the spectra of the path.
3) The IR is calculated resulting in a set of rectangles, and in case there is available spectrum, a fitting policy is used to decide in which one the connection is established.

The following notation will be used to describe the algorithm:

s: source node;

d: destination node;

b: bandwidth demand in slots;

$r(s,d,b)$: request from the node s to the node d with bandwidth demand b;

$G = (V, E, S)$: network graph composed of a set of nodes V, a set of edges E and a set of matrices S representing the optical spectrum availability in the links, which are associated with the edges of the graph;

$E = \{e_{u,v}\}$: set of edges $e_{u,v}$ connecting u and v in G;

$S = \{s_{u,v}\}$: set of matrices $s_{u,v}$ representing the spectrum of the link between u and v in G;

$P = \{p_i\}$: set of paths, used to store the result of the KSP algorithm;

$L_i = \{l_{i,j}\}$: set of rectangles of the spectrum, output of the inscribed rectangles algorithm, in which each $l_{i,j}$ is a set of slots;

m_i: auxiliary matrix that stores the contiguous slots of the path i;

Algorithm 1 IR-RCSA

1: $P = KShortestPaths(G, s, d)$
2: $\forall j,k \mid \exists e_{j,k} \in p_i \; m_i = m_i$ **and** $s_{j,k}$
3: $\forall i \mid L_i = InscribedRectangles(m_i)$
4: **if** $\forall i,j|l_{i,j}| < d$ **then**
5: $\quad block(r(s,d,b))$
6: **else**
7: $\quad H = fit(r(s,d,b), p_i, L_i)$
8: $\quad \forall h_{j,k} \; s_{j,k} = 1$
9: **end if**

$H = \{h_{j,k}\}$: set of slots allocated by the fitting function.

Algorithm 1 shows the IR-RCSA algorithm. In Line 1, the candidate paths are calculated using the KSP algorithm, and the results are stored in a set of paths P. In Line 2, for each link of the path p_i, the spectra are summed, using the binary operation and the result is stored in m_i. This operation guarantees the spectrum continuity of the connection since the free slots of m_i are free in all links along the path. In Line 3, the Inscribed Rectangle algorithm runs for each potential path and the sets of rectangles available for each path i is stored in L_i. In Line 4, if there is no rectangle with size larger than the demand d, in number of slots, the request is blocked. Otherwise, in Line 7, a rectangle from the list given by the IR algorithm is chosen by the fitting policy to accommodate the bandwidth demand. The allocated slots are set as occupied in Line 8 in the spectrum matrices.

The computational complexity of the algorithm depends on the complexity of the IR algorithm used, which is $O(hw)$, where h and w are the height and the width of the spectrum respectively. This can be translated to the number of slots in all cores of the network, or $c \times s$, where c is the number of cores and s the number of slots in each core. The IR algorithm runs k times in the RCSA algorithm, resulting in a complexity of $O(kcs)$. The values of c and s can be considered as constants for a given topology, and k is a parameter of the algorithm that can also be fixed. In a static network scenario, the algorithm will always perform the same number of operations with a fixed k, therefore, the complexity is $O(1)$.

V. Fitting Policies

This section introduces the fitting policies used to choose in which rectangle found by the inscribed rectangles algorithm the requests will be allocated. One of the policies focuses on reducing the blocking ratio as much as possible and the other focuses on avoiding crosstalk between the connections.

A. *Minimal Blocking*

The Minimal Blocking (MB) policy is based on the best fit policy from [9], accounting the width of the rectangle. The aim is to allocate the connections using one core only whenever possible. In this way, it uses less guard bands and can potentially diminish the crosstalk since the spectrum is not used in adjacent cores.

The MB policy chooses the best fitting rectangle, i.e. the rectangle that has the smallest size that can accommodate the demand, and has the greatest width of all the rectangles that can fit the request.

The MB policy can be described by the equation:

$$o = r_i \mid h_i \times w_i \geq d, \; \forall r_j \in R \; w_i > w_j \tag{2}$$

where:

$R = \{r_i\}$: set if rectangles found by the inscribed rectangles algorithm;

h_i: height of the i_{th} rectangle;

w_i: width of the i_{th} rectangle;

d: demand in slots;

o: output rectangle.

B. Minimal Crosstalk

The Minimal Crosstalk (MXT) policy is based on the Random Fit policy, that achieved the lowest crosstalk values in [9]. However, it also produced relatively high blocking ratios under low loads, which makes the policy unattractive to service providers. The MXT policy aims at minimizing the crosstalk without affecting the blocking ratio.

The MXT policy uses a random weighted function to allocate the request, in which the width corresponds to the weight. This method avoids the abnormal behaviour of the Random Fit policy, that leads to high blocking ratios, but it diminishes the crosstalk.

Th MXT policy can be described by the equation:

$$o = r_i \mid A \times W < w_i, \forall w_j w_i > w_j \tag{3}$$

where:

W: sum of all rectangles widths in the set;

A: random value between 0 and 1.

VI. Numerical Evaluation

To assess the performance of the proposed algorithm, simulation experiments were employed and the three fitting policies were compared. The parameter $k = 5$ was used in the KSP algorithm since no significant gain were observed with higher values. The FlexGridSim [14] simulator was employed in the simulations. Each replication simulated 100,000 requests. Confidence intervals with 95% confidence level were generated. The NSF (Figure 3) and the USA (Figure 2) topologies were used in the simulations. The NSF topology has 16 nodes and 25 links whereas the USA topology has 24 nodes and 43 links. The links were composed by MCFs with 7 core and each core was divided in 264 slots of 12,5GHz [15]. Connections were dynamically generated by randomly selecting source and destination pairs. The traffic load is given by the utilization of the bottleneck link, which has the current highest number of optical lighpaths assigned [5].

Three metrics will be assessed in this paper, Bandwidth Blocking Ratio (BBR), Crosstalk per Slot (CpS) and Average Number of Hops. The results obtained for the IR-RCSA algorithm will be compared by that obtained by the Best Fit

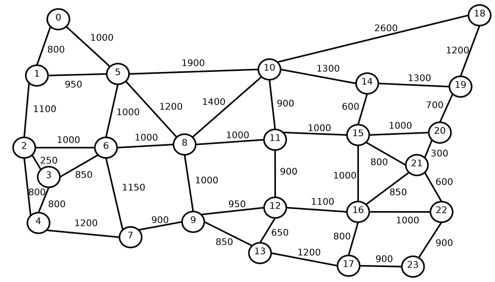

Fig. 2. The USA topology.

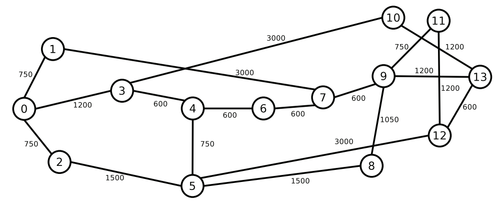

Fig. 3. The NSF topology.

(BF) and Random Fit (RA) jointly used with the CCL-RCSA algorithm proposed in [9].

Figure 4 shows the BBR as a function of the load for the USA topology. The results produced by the proposed policies are similar to the BF, with MB producing lower BBR under loads higher than 0.7, and one order of magnitude lower under loads higher than 0.9. When compared to the RA policy, the results are different, since it produces high BBR values under loads lower than 0.6. The proposed policies do not block requests under loads lower than 0.45, and even under 0.5 they produce BBR values one order of magnitude lower than those given by the RA policy.Under loads higher than 0.8, the proposed algorithm produces BBR values one order of magnitude higher than those produced by the RA policy.

When compared, the proposed algorithms produce similar results, with the MB policy blocking less connections, as a consequence of its goal of minimizing the BBR. The difference between the two is negligible under loads lower than 0.7, and the maximum difference occurs under loads higher than 0.9, being one order of magnitude. The MB achieves higher efficiency in spectrum usage due to the low usage of guard bands and its wise use of the spectrum, as a consequence of not employing on a random choices.

Figure 5 shows the average CpS for the USA topology. The crosstalk per slot is the average ratio between the number of slots being affected by crosstalk and the total number of slots used in a link. The proposed policies allocate less connections affected by crosstalk when compared to the RA policy; the difference is up to 360% under loads of 0.4, with a minimum difference of 54% under loads of 0.55. The MXT policy achieves lower CpS due to its capacity of spreading the connections randomly across the spectrum. The difference between the CpS produced by the proposed policies is up to 87% under loads of 0.4.

Figure 6 shows the average number of hops for the established lightpaths for the USA topology. The variation of average number of hops for those algorithms is small, due to

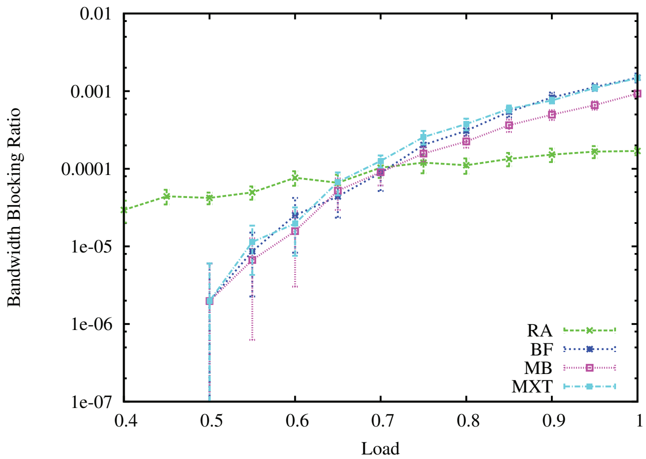

Fig. 4. Bandwidth Blocking Ratio as a function of the load for the USA topology.

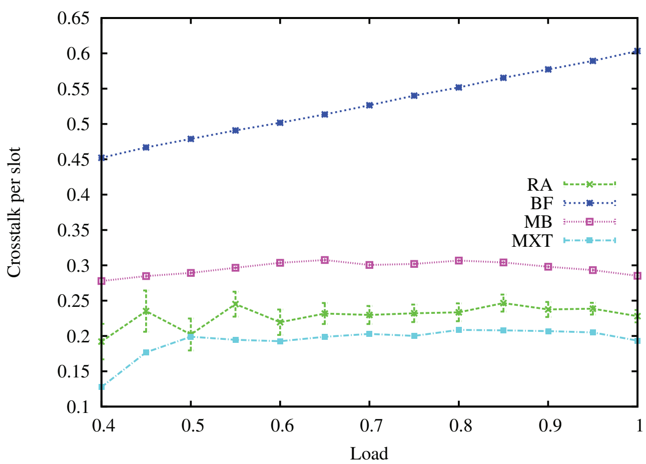

Fig. 5. Crosstalk per Slot as a function of the load for the USA topology.

the usage of KSP. The maximum variation is 0.05 hops, which equals to one hop more in each twenty lightpaths.

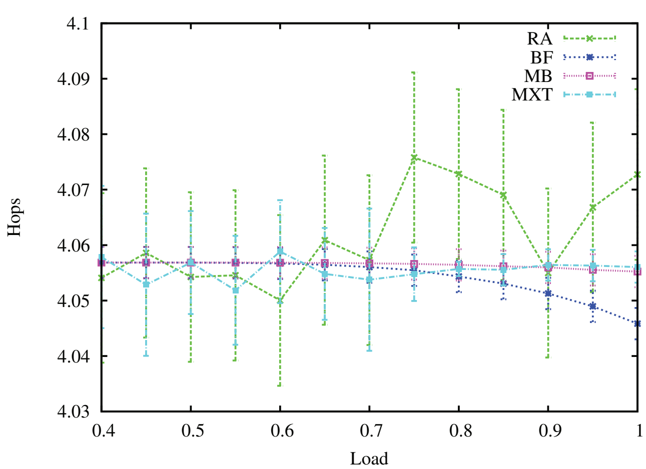

Fig. 6. Number of average hops of the lighpaths for the USA topology.

Figure 7 shows the BBR as a function of the load for the NSF topology. The node connectivity is lower than that in the USA topology, meaning that there is a lower number of available routes and spectrum to allocate. The difference in blocking is not significant when comparing BF to the proposed policies. The RA exhibits the same behavior found in the USA network, with BBR values two order of magnitude higher under loads lower than 0.45, and one order of magnitude lower under loads higher than 0.75.

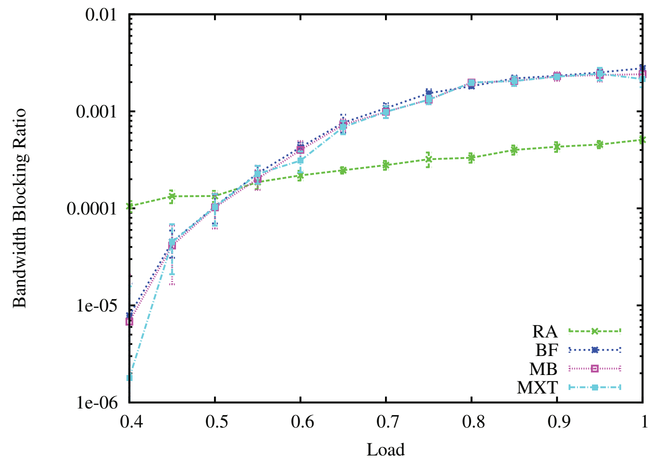

Fig. 7. Bandwidth Blocking Ratio as a function of the load for the NSF topology.

Figure 8 shows the average Crosstalk per Slot for the NSF topology. For this topology, the policy MB does not outperforms the RA policy, presenting similar values under all loads. When comparing RA to MXT, the latter achieves 370% less CpS under low loads and 170% under high loads. For this topology, the difference in BBR is negligible, but since the crosstalk is lower, the MXT policy presents more potential to perform better.

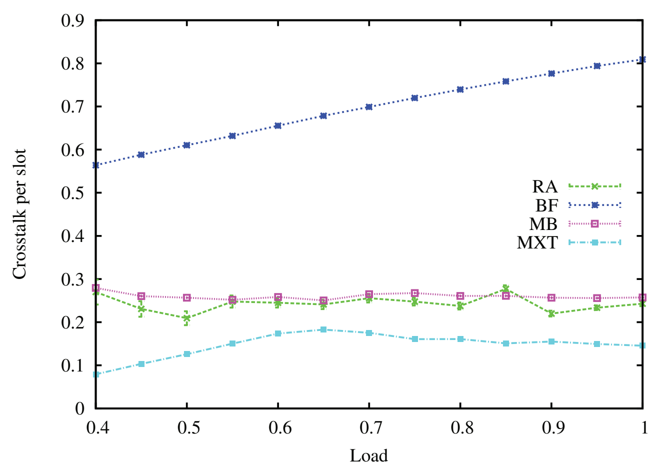

Fig. 8. Crosstalk per Slot as a function of the load for the NSF topology.

Figure 9 shows the average number of hops in the established lightpaths for the NSF topology. The average number of hops vary only 0.1, producing results similar to those found for the USA topology.

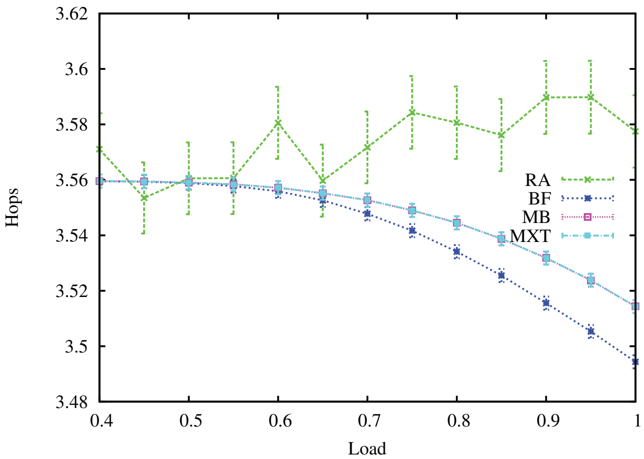

Fig. 9. Number of average hops of the lightpaths for the NSF topology.

VII. Conclusion

This paper introduced the IR-RCSA algorithm which uses an Inscribed Rectangle algorithm to discover contiguous regions of the spectrum for the establishment of connection in SDM optical networks with low computational complexity. Two spectrum fitting policies were evaluated. Results showed that the IR algorithm that can employ advanced fitting policies capable of lowering blocking ratio and crosstalk under different conditions.

The Minimal Blocking and Minimal Crosstalk policies were proposed, with the first achieving lower BBR values, mainly under high loads, and the latter allocating connections less affected by crosstalk. Results show that better performance is achieved when using the proposed policies.

VIII. Acknowledgements

This work was sponsored by Coordenação de Aperfeicoamento de Pessoal de Nível Superior (CAPES), Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and INCT Fotônica para Comunicações Ópticas (FOTONICOM).

References

[1] X. Chen, A. Jukan, A. C. Drummond, and N. L. S. da Fonseca, "A multipath routing mechanism in optical networks with extremely high bandwidth requests," in *Global Telecommunications Conference, 2009. GLOBECOM 2009. IEEE*, pp. 1–6, 2009.

[2] K. Christodoulopoulos, I. Tomkos, and E. Varvarigos, "Elastic bandwidth allocation in flexible ofdm-based optical networks," *Lightwave Technology, Journal of*, vol. 29, pp. 1354 –1366, may1, 2011.

[3] P. M. Moura, R. A. Scaraficci, and N. L. S. da Fonseca, "Algorithm for energy efficient routing, modulation and spectrum assignment," in *2015 IEEE International Conference on Communications (ICC)*, pp. 5961–5966, June 2015.

[4] P. M. Moura, N. L. S. da Fonseca, and R. A. Scaraficci, "Traffic grooming of batches of deadline-driven requests in elastic optical networks," in *2014 IEEE Global Communications Conference*, pp. 1284–1289, Dec 2014.

[5] S. Fujii, Y. Hirota, H. Tode, and K. Murakami, "On-demand spectrum and core allocation for reducing crosstalk in multicore fibers in elastic optical networks," *Optical Communications and Networking, IEEE/OSA Journal of*, vol. 6, pp. 1059–1071, Dec 2014.

[6] T. Xia, H. Fevrier, T. Wang, and T. Morioka, "Introduction of spectrally and spatially flexible optical networks," *Communications Magazine, IEEE*, vol. 53, pp. 24–33, Feb 2015.

[7] D. Klonidis, F. Cugini, O. Gerstel, M. Jinno, V. Lopez, E. Palkopoulou, M. Sekiya, D. Siracusa, G. Thouenon, and C. Betoule, "Spectrally and spatially flexible optical network planning and operations," *Communications Magazine, IEEE*, vol. 53, pp. 69–78, Feb 2015.

[8] R. Proietti, L. Liu, R. Scott, B. Guan, C. Qin, T. Su, F. Giannone, and S. Yoo, "3d elastic optical networking in the temporal, spectral, and spatial domains," *Communications Magazine, IEEE*, vol. 53, pp. 79–87, Feb 2015.

[9] P. M. Moura and N. L. S. da Fonseca, "Routing, core and spectrum assignment based on connected component labelling for sdm optical networks," in *2016 IEEE International Conference on Communications (ICC)*, pp. 1–6, May 2016.

[10] P. M. Moura, N. L. S. da Fonseca, and R. A. Scaraficci, "Fragmentation aware routing and spectrum assignment algorithm," in *Communications (ICC), 2014 IEEE International Conference on*, pp. 1137–1142, June 2014.

[11] A. Muhammad, G. Zervas, D. Simeonidou, and R. Forchheimer, "Routing, spectrum and core allocation in flexgrid sdm networks with multi-core fibers," in *Optical Network Design and Modeling, 2014 International Conference on*, pp. 192–197, May 2014.

[12] "Automatic cropping of non rectangular images," Mar 2013.

[13] J. Y. Yen, "Finding the k shortest loopless paths in a network," *Management Science*, vol. 17, no. 11, pp. 712–716, 1971.

[14] P. M. Moura and A. C. Drummond, "FlexGridSim: Flexible Grid Optical Network Simulator." http://www.lrc.ic.unicamp.br/FlexGridSim/.

[15] K. Igarashi, K. Takeshima, T. Tsuritani, H. Takahashi, S. Sumita, I. Morita, Y. Tsuchida, M. Tadakuma, K. Maeda, T. Saito, K. Watanabe, K. Imamura, R. Sugizaki, and M. Suzuki, "110.9-tbit/s sdm transmission over 6,370 km using a full c-band seven-core edfa," *Opt. Express*, vol. 21, pp. 18053–18060, Jul 2013.