Fractal Traffic Models for Internet Simulation

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Abstract

We construct five new fractal traffic models based on the unified framework of Fractal Point processes (FPPs) [20], and analyze their asymptotic statistical properties. Two of them are On-Off types, suitable for characterizing an aggregate fractal traffic source. The other three provide a structure suitable for flow modeling, allowing for gaining a quantitative understanding of how application- or flow-level fractal dynamics (such as user activity, session/flow arrivals, duration, and volume distributions) affect packet-level fractal dynamics. Such a structure, together with analytic results obtained in this work, opens an entirely new horizon that enables us to explain the causes and origins of complex multiple-time-scale behaviors of packet-level dynamics in a quantitative manner, i.e., multi-fractal patterns over small time scales and mono-fractal behavior over large time scales. Consequently, they provide network traffic engineering researchers and practitioners with practical and flexible tools for analyzing, devising and performing various traffic engineering studies. All of the FPPs are implemented in OPNET as IP-routable traffic generators.

1 Introduction

Fractal geometry, frequently portrayed as the opposite to mathematically elegant and tractable Euclidean geometry, has revealed an entirely new horizon on the way our minds observe and interpret the various shapes of nature we see every day. This is attributed to its simplicity in explaining the extreme variability of natural shapes which would be otherwise difficult, if not impossible, using its Euclidean counterpart. The recent discovery of fractal characteristics of Internet traffic, to which we contribute every day in all forms of our work, has perhaps created a similar impact on the way teletraffic researchers and practitioners observe and interpret Internet traffic. Consequently, it is not surprising to witness tremendous interest focused on a wide variety issues dealing with this fractal nature of Internet traffic. These issues range from searching for new ways of characterizing this seemingly universal statistical property to understanding its implications for better engineering the Internet. Broadly speaking, these efforts can be summarized into the following five inter-related areas:

- *Traffic Analysis Tools:*
  This area pertains to statistical estimation tools that detect whether a given Internet traffic trace exhibits fractal properties, and if so, over what range of time scales they are manifested. These tools also provide a theoretical foundation for accurately estimating parameters unique to fractal traces such as the Hurst parameter and Fractal Onset Time Scale, a quantity that marks the beginning of mono-fractal behavior in time domain. [20].
- *Modeling and Characterization:*
  The so called *native* fractal models include, among many others, Fractional Gaussian Noise process, Fractional ARIMA processes, Chaotic Maps, Cox's $M/G/\infty$ process, and Fractal Point Processes described in this paper. These fractal models not only help to characterize fractal traffic in a compact way and generate its synthetic version, but also provide invaluable insights into explaining the possible origins of the fractal dynamics of Internet traffic [5].
- *Fast and Accurate Generation of Synthetic Fractal Traffic:*
  With the size of the Internet and the diversity of current and future Internet user applications increasing at an astonishing rate, simulation seems the only viable approach for meaningful and accurate Internet traffic engineering. This requires algorithms that can generate a large amount of synthetic fractal traffic in an accurate and computationally efficient manner [9, 15, 23]. Another challenge in this area is that since most fractal traffic models employ heavy-tailed distributions, there exists an inherent bias in the synthetically generated fractal traffic caused by the finite length of the trace under study and simulation. While this is an open issue, papers [10, 12, 25] provide analytical results that may be employed for compensating such bias.
- *Understanding the Cause(s) for the Fractal Nature of Internet Traffic:*
  One of the vexing questions that Internet traffic researchers have been attempting to address is to determine the source(s) of the fractal nature of Internet traffic. Crovella and Bestavros [5] attribute the self-similarity of the World Wide Web traffic to the observed heavy-tailed distributions of the size of Web files and idle times (i.e., user think times). Willinger and his colleagues [27] report that the source of the fractal Ethernet traffic is the transmission and idle times of each LAN host that are also heavy-tailed. Both works base their conclusions on connecting their data analysis findings to the self-similar process constructed by the superposition of many renewal reward processes defined in [24] and refined in

[27]. Aracil *et al.* [3] report a similar result on FTP file sizes that was found to be heavy-tailed as well. On the other hand, both [3] and [19] have found that session arrivals (FTP data sessions in [3] and Web requests in [19]) exhibit fractal behavior. All of these findings suggest that there exist multiple sources that contribute to the fractal behavior of Internet traffic.

- *Implications for Internet Traffic Engineering:*
  After the fractal property of Internet traffic was uncovered, there has been a considerable body of research devoted to understanding the impact of the self-similarity on traffic engineering in the context of ATM and B-IDSN [6, 7, 13, 22]. These studies examine the practical implications for buffer dimensioning and/or capacity planning for traffic sources with long-range dependence. While this effort concerns how to *deal with* fractal traffic *sources* for network design, there has been a growing interest in *utilizing* the fractal *dynamics* of Internet traffic. For example, [26] proposes to deploy thousands of probes engaging in both independent and orchestrated measurement of network paths in an attempt to characterize the network's end-to-end behavior and to predict and locate troubled spots before they occur. This requires a sound understanding of Internet traffic dynamics via a mathematically rigorous framework due to the need to correlate large-scale measurements reported by those probes in a coherent manner. Recent Internet traffic measurements that report strong evidence of fractal properties [5, 16] naturally make the notion of fractals a promising candidate as the foundation for such a modeling framework.

Fractal Point Processes (FPPs) have served as useful tools for addressing these issues. Compared to other modeling approaches, FPPs achieve this with the parameterization method for controlling the time scales over which (mono-)fractal behavior occurs. Because fractal properties appear over a wide but finite range of time scales, a model must be able to provide mathematical relations between model parameters and the resulting time scales over which fractal characteristics are manifested. The new models presented in this paper further extends their ability to *quantitatively* characterize the impact of session-level workload dynamics on the fractal dynamics at the packet level.

This paper is organized as follows. In section 2, we first present a taxonomy of FPPs that describes how each model is constructed based on the two methods [20]: *renewal* and *doubly-stochastic Poisson process* (DSPP) methods. This serves as the tool for readily recognizing the structural aspects of different FPP models. Section 3 introduces a new wavelet-based statistic called Allan Variance Index of Dispersion for Counts (AV-IDC). While it takes the same power-law form as the original IDC for FPPs, the AV-IDC is superior since it is wavelet-based and does not suffer from the bias that the IDC does. Section 4 characterize a total of eight FPPs, including the five new models, in terms of the Three Fundamental Parameters (TFPs), namely average arrival rate (λ), Hurst parameter (H), and Fractal Onset Time Scale (T_0 from IDC or T_1 from AV-IDC). For each model, the mathematical relations between its model parameters and the TFPs are established. Section 5 summarizes the paper with potential applications, ranging from background/aggregate traffic generators to user activity modeling.

2 Fractal Point Processes

Fractal stochastic process (not necessarily point processes) exhibit scaling in their statistics. This scaling property mathematically leads to power-law forms [25]. As a result, several statistics of a fractal stochastic process assume power-law forms. In the context of studying Internet traffic dynamics, fractal concerns the second-order statistics of the wide-sense stationary *rate* process $\{X_n^T\}, n = 1, 2, \ldots$ where X_n^T denotes the number of arrivals (or workload arrived) during the n-th interval of duration T. The correlation, power spectral density, and IDC of $\{X_n^T\}$ exhibits scaling behavior over a significant range of time and frequency scales. Strictly speaking, the point process $dN(t)$ giving rise to such a rate process $\{X_n^T\}$ is called fractal-*rate* point processes [25]. But for simplicity, we call them Fractal Point Processes (FPPs) throughout this paper, much the way the Internet traffic self-similarity actually means the second-order self-similarity of the rate process.

2.1 A Taxonomy

The earlier work [20], which laid out the foundation of the FPP framework, describes two methods that yield FPPs: renewal method and DSPP method. Two FPPs based on the renewal method were analyzed: Fractal Renewal Point Processes (FRP), and the Superposition of the FRPs (Sup-FRPs). The DSPP method yields a variety of Fractal-Modulated Poisson Point Processes (FMPPs). Let $I(t)$ denote a wide-sense stationary, continuous-time stochastic process with an autocorrelation function $R_I(\tau)$. A FMPP $dN(t)$ is defined as a DSPP whose rate process $I(t)$ has a power-law decaying autocorrelation $R_I(\tau) \sim \tau^{-\alpha}$, with $0 < \alpha < 1$ over a wide range of time scales [20]. This simple definition yields a surprisingly rich set of traffic modeling tools which prove quite effective for generating synthetic fractal traffic, characterizing and reproducing a variety of Internet traffic exhibiting second-order fractal properties (second-order self-similarity), revealing the causes of Internet traffic second-order self-similarity, and characterizing flows in the Internet [4]. The FMPPs constructed this way are further divided into two types depending on how the rate process $I(t)$ is constructed: On-Off type and Shot-Noise type.

The rate process $I(t)$ of an on-off FMPP comprises the superposition of M i.i.d. on-off processes, where both the on and off periods are i.i.d. and at least one of the two states has power-law distributed dwell times with infinite variance. We define and analyze three different forms of on-off FMPPs: (i) both the on and off periods follow an identical power-law distribution (PowON-PowOFF, also known as the FBNDP process [20]); (ii) the on-period distribution decays as a power-law, while the off period follows an exponential distribution (PowON-ExpOFF); (iii) the converse of the last case (ExpON-PowOFF). While related processes have been analyzed in the literature [27], our new contribution lies in developing relationships between the model parameters and the TFPs for PowON-ExpOFF and ExpON-PowOFF.

Figure 1 shows a block diagram of a shot-noise FMPP. The shot-noise process $I(t)$ is obtained as an output of a linear filter $h(t)$, where $h(t)$ can be either random or deterministic, driven by an input point process $dN_1(t)$. This rate process drives the Poison point process generator that yields the output point process $dN_2(t)$. When $dN_1(t)$ is a *homogeneous* Poisson point process (HPP) and $h(t)$ is a *deterministic* linear filter, a *classical* shot-noise process results. This well-known process has long been studied in the context of communications theory [14]. In this paper, we generalize this classical stochastic process by allowing other well-defined point processes for $dN_1(t)$ and randomness in $h(t)$.

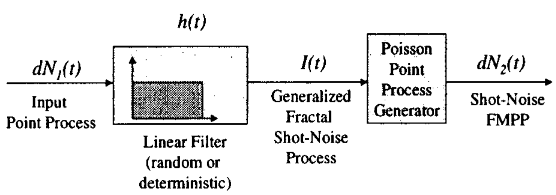

Figure 1: Block diagram for constructing shot-noise FMPPs, also known as the Fractal-Shot-Noise-Driven Poisson Point Processes (FSNDPs).

As mentioned earlier, $dN_2(t)$ becomes a shot-noise FMPP, or equivalently, the Fractal-Shot-Noise Driven Poisson Point Process (FSNDP) when the shot-noise process $I(t)$ has the power-law decaying autocorrelation behavior over a broad range of time scales. Later, we prove that this fractal shot noise $I(t)$ is achieved under either or both of the following conditions: (i) the input point process $dN_1(t)$ is fractal (i.e., Sup-FRP); (ii) the linear filter $h(t)$ is fractal.

The family of FSNDPs can be further classified depending on the types of $dN_1(t)$ and $h(t)$. The first type, denoted as H-FSNDP, has an HPP as the input point process $dN_1(t)$. In this case, the linear filter $h(t)$ must be fractal[1] filter is defined as in order to yield an FMPP. In this paper, we consider an H-FSNDP with a random, rectangle-shaped fractal filter. We note that in the previous work [20], an H-FSNDP with a deterministic, power-law decaying fractal filter was analyzed.

The second type, denoted as F-FSNDP, has a FPP (as opposed to HPP) as the input point process $dN_1(t)$. In this case, the rate process $I(t)$ is fractal regardless of whether $h(t)$ is fractal. While any FPP may be employed for $dN_1(t)$ in theory, we choose the Sup-FRP model [23] for our study based on the reports that some session-level arrivals such as FTP data sessions and Web requests are faithfully characterized by this model [3, 19]. In this paper, two F-FSNDPs are analyzed. The first F-FSNDP employs a random, rectangle-shaped non-fractal (exponential) filter. Consequently, the fractal nature of this model is exclusively determined by the input FPP (Sup-FRP in this case). The second F-FSNDP we consider in this paper employs the same filter as that of the H-FSNDP: a random, rectangle-shaped fractal filter. In this case, both the input FPP and the filter *independently* contribute to the fractal nature of the resulting FSNDP. Later, we show that this results in a FPP that exhibits up to three distinct fractal behaviors (tri-fractal).

The last type, denoted as O-FSNDP, can have any well-defined point process as its $dN_1(t)$ (excluding HPP and FPP) as long as $h(t)$ is fractal. This type is included in order to represent cases that an empirical point process (i.e., a time series derived from an actual Internet trace) can be used for $dN_1(t)$ and still yield a valid FSNDP (insofar as the filter is fractal). Figure 2 presents all of the FPPs discussed so far, organized by their types and filters.

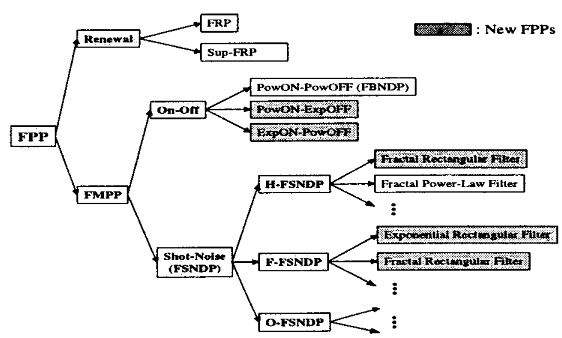

Figure 2: Taxonomy of FPPs.

2.2 Fractal Linear Filters

Fractal linear filters are an integral part of the FSNDPs as they provide a source of fractal characteristics for them. Suppose $H(\omega)$ is the Fourier transform of the linear filter $h(t)$, i.e., $F(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$. Then, a fractal linear filter results when the power spectral density (PSD), $|H(\omega)|^2$ (or its expected form for random filters), follows a power-law form. In this section, we present two examples of fractal linear filters: power-law decaying filter, and rectangular-shape filter.

The previous work [20] has considered the following deterministic, power-law decaying fractal filter:

$$h(t) = \begin{cases} Kt^{-(1-\alpha/2)} & \text{for } A < t < B, \\ 0 & \text{otherwise,} \end{cases} \qquad (1)$$

where the filter height $K > 0$ is a constant. We note that the fractal characteristics of this filter remains intact even when the constant height (K) is replaced with

[1] A linear filter is *fractal* when its power spectral density exhibits $1/f$-noise characteristic.

i.i.d. random heights $\{K_i\}$ as long as the first and second moments of this random variable are finite [11]. It is easy to see that the random heights are likely to produce traces with higher dynamics in the rate process than with the constant height.

Another important linear filter for our study is the following rectangle-shape random filter [19]:

$$h_i(t) = \begin{cases} K_i & 0 < t < B_i \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

with both $\{K_i\}$ and $\{B_i\}$ being i.i.d. random variables. We consider two different distributions for $\{B_i\}$ for the F-FSNDP. When $\{B_i\}$ is heavy-tailed with infinite variance, a fractal filter results. On the other hand, when $\{B_i\}$ is exponentially distributed, a non-fractal (exponential) filter results.

We note that this rectangular filter provides an easy-to-understand structure for flow characterization [4]: K_i and B_i represent the observed throughput and duration of i-th flow, respectively. Equivalently, one may use flow volume W_i instead of throughput by the simple relation $W_i = K_i B_i$. Figure 3 illustrates a shot-noise FMPP with $h(t)$ in Eq. (2).

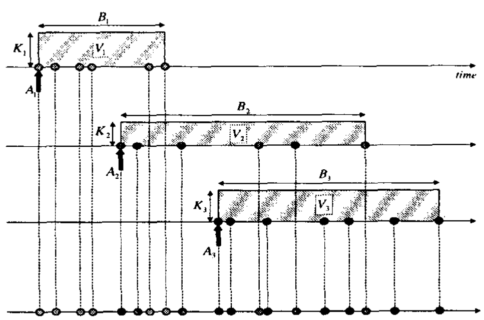

Figure 3: Shot-Noise FMPP with rectangular filter. $\{A_i\}$ characterizes flow arrival process, which may be mapped into arrival times of flows detected.

3 Allan Variance IDC

For the FPPs, the IDC and AV-IDC take the same power-law form:

$$\begin{array}{lrcl} \text{IDC:} & F(T) & \approx & 1 + (T/T_0)^{\alpha} \\ \text{AV-IDC:} & A(T) & \approx & 1 + (T/T_1)^{\alpha} \end{array} \tag{3}$$

with

$$T_0^{\alpha} T_1^{-\alpha} = 2 - 2^{\alpha}. \tag{4}$$

and $0 < \alpha < 1$. The parameters T_0 and T_1 mark the lower time limits from which point the scaling behavior begins to appear. For this reason, we call them *Fractal Onset Time Scales* (FOTS).

The AV-IDC, which is based on the Allen variance (as opposed to the ordinary variance), is defined in terms of the variability of *successive counts* [25]. In analogy with the original IDC, the AV-IDC $A(T)$ for the rate process $\{X_n^T\}$ is defined as

$$A(T) \equiv \frac{\mathrm{E}[(X_{n+1}^T - X_n^T)^2]}{2\mathrm{E}[X_n^T]} \tag{5}$$

Comparing this with the original IDC $F(T)$ which employs the ordinary variance and is defined as

$$F(T) \equiv \frac{\mathrm{E}[(X_n^T)^2] - \mathrm{E}^2[X_n^T]}{\mathrm{E}[X_n^T]}, \tag{6}$$

we have

$$\begin{aligned} A(T) &= 2F(T) - F(2T) \\ &= 2 + 2(T/T_0)^{\alpha} - [1 + (2T/T_0)^{2}] \\ &= 1 + (2 - 2^{\alpha})(T/T_0)^{\alpha} \\ &\equiv 1 + (T/T_1)^{\alpha}, \end{aligned}$$

As a traffic analysis tool, the AV-IDC proves superior to the original IDC for the study of Internet traffic dynamics. First, the standard estimation for the ordinary variance of the IDC tends to be biased because of the data being strongly dependent on each other over very large scales of time [12, 1], which is the case for LRD traffic. The AV-IDC does not suffer from this limitation. Second, estimates of α obtained from the AV-IDC can range up to a value of three instead of saturating at unity for the IDC. In this case, the Hurst parameter is related to α as $\alpha = 2H + 1$. Although we have not found any Internet packet trace that exhibits the power-law exponent greater than unity, this property enables us to detect such cases if they occur. Finally, the AV-IDC is closely related to wavelet analysis since the Allen variance, $\mathrm{E}[(X_{n+1} - X_n)^2]$, can be recast as the variance of the integral of the point process under study multiplied by the following Haar wavelet

$$\psi(t) = \begin{cases} -1 & -T < t < 0 \\ +1 & 0 < t < T \\ 0 & \text{Otherwise} \end{cases}$$

since

$$X_{n+1}^T - X_n^T = \int_{-T}^{T} \psi(t - nT)\,dN(t)$$

and $\mathrm{E}[X_{n+1}^T - X_n^T] = 0$. The ability to view FPPs in the wavelet coefficients domain is invaluable since the versatility of wavelet analysis has been widely evidenced [1, 2, 8].

4 Traffic Descriptions for FPPs

We obtain mathematical relations between relevant statistics and the model parameters of each FPP so that we understand how different choices of the values of the model parameters affect the statistical behavior of a particular FPP in a quantitative manner. For each FPP, we first derive the equations that relate the TFP (λ, H, T_0) and the parameters of the FPP. If the FPP under study has more than three parameters, other statistic(s) is introduced and the corresponding equation(s) is derived for complete characterization.

4.1 FRP and Sup-FRP

The Sup-FRP model is constructed as the superposition of M i.i.d. FRPs where each FRP is completely characterized by the following power-law probability density function (pdf) for inter-arrival times:

$$p(t) = \begin{cases} \gamma A^{-1} e^{-\gamma t/A} & 0 \le t \le A \\ \gamma e^{-\gamma} A^{\gamma} t^{-(\gamma+1)} & t > A \end{cases} \quad (7)$$

with $1 < \gamma < 2$. Note that the Sup-FRP has three parameters (γ, A, M), which are mapped into the TFP as follows [20]:

$$\begin{aligned} H &= (3-\gamma)/2 \\ \lambda &= M\gamma\left[1+(\gamma-1)^{-1}e^{-\gamma}\right]^{-1} A^{-1}, \\ T_0^{\alpha} &= 2^{-1}\gamma^{-2}e^{-\gamma}(\gamma-1)^{-1}(2-\gamma)(3-\gamma)\cdot \\ & \quad [1+(\gamma-1)e^{\gamma}]^2 A^{\alpha} \end{aligned} \quad (8)$$

where $\gamma = 2 - \alpha$. We note that the FRP is the special case of the Sup-FRP with $M = 1$.

4.2 On-Off FMPPs

4.2.1 PowON-PowOFF

The rate process $I(t)$ of the PowON-PowOFF FPP is the superposition of M i.i.d. On-Off processes where both ON and OFF periods are i.i.d. are drawn from the same pdf, and are independent of each other. When an On-Off process is ON, it takes the value of R, resulting in MR as the peak value of $I(t)$. With Eq. (7) as the pdf for both ON and OFF periods, this model has the four parameters (γ, A, M, R), and are mapped into the TFP as [20]:

$$\begin{aligned} H &= (3-\gamma)/2, \\ \lambda &= RM/2, \\ T_0^{\alpha} &= \gamma^{-1}(2-\gamma)(3-\gamma)[(\gamma-1)e^{\gamma}+1]R^{-1}A^{\alpha-1} \end{aligned} \quad (9)$$

with $\gamma = 2 - \alpha$. To completely characterize the model, an additional relation is needed. Suppose a traffic source is characterized by this model. Then, this source is inactive (i.e., $I(t) = 0$) with the probability of 2^{-M}. In other words, this source is active (i.e., ON) $100 * (1 - 2^{-M})$ % of the time on the average. This quantity, namely *source activity ratio*, is useful for a traffic source description. In this way, the FBNDP model can be completely characterized by the TFPs and *source activity ratio*.

4.2.2 PowON-ExpOFF and ExpON-PowOFF

These two models are identical to the PowON-PowOFF model described previously except that either ON or OFF are drawn from the exponential distribution, i.e., the pdf given by $p_e(t) = \nu^{-1}e^{-t/\nu}$. Then, the average dwell times, $\langle ON \rangle$ and $\langle OFF \rangle$, are given by

Model	$\langle ON \rangle$	$\langle OFF \rangle$
PowON-ExpOFF	$f(\gamma)A$	ν
ExpON-PowOFF	ν	$f(\gamma)A$

with $f(\gamma) \equiv \gamma^{-1}[1 + (\gamma-1)^{-1}e^{-\gamma}]$. Then, the TFPs for the PowON-ExpOFF and ExpON-PowOFF are given by

$$\begin{aligned} H &= (3-\gamma)/2 \\ \lambda &= \frac{\langle ON \rangle}{\langle ON \rangle + \langle OFF \rangle} RM \\ T_0^{\alpha} &= 2^{-1}\nu^{-2}R^{-1}e^{\gamma}(3-\gamma)(2-\gamma)(\gamma-1)\cdot \\ & \quad (\langle ON \rangle + \langle OFF \rangle)^2 A^{-\gamma} \langle ON \rangle, \end{aligned} \quad (10)$$

where $\gamma = 2 - \alpha$, R is the height (rate) during the ON period for a single On-Off process, and M is the number of On-Off processes superposed. The proof can be found in [21]. Since both models have five parameters each (α, A, M, R, ν), we need two additional relations to completely characterize them. As was in the case for the PowON-PowOFF model, we employ the traffic activity ratio. Let p denote the probability that a single On-Off process is ON, which is given by

$$p \equiv \frac{\langle ON \rangle}{\langle ON \rangle + \langle OFF \rangle} \quad (11)$$

Then, the traffic activity ratio, τ, is given by

$$\tau = 1 - (1-p)^M \quad (12)$$

Finally, we employ the peak-to-mean ratio η which is given by

$$\eta \equiv MR/\lambda = 1/p \quad (13)$$

4.3 FSNDP (Shot-Noise FMPP)

The characterization of a FSNDP begins with obtaining a general expression for the power spectral density (PSD) of its rate process $I(t)$, denoted as $P_I(\omega)$. Let $P_{dN_1}(\omega)$ denote the PSD of the input point process $dN_1(t)$, and let $H(\omega)$ denote the Fourier transform of the linear filter $h(t)$. Then, we show that $P_I(\omega)$ is given by

$$P_I(\omega) = \lambda_1 \mathrm{E}\left[|H(\omega)|^2\right] + [P_{dN_1}(\omega) - \lambda_1]\Big|\mathrm{E}[H(\omega)]\Big|^2 \quad (14)$$

where λ_1 is the average arrival rate of the $dN_1(t)$ [21].

We emphasize that Eq. (14) reveals how the input point process $dN_1(t)$ and the filter $h(t)$ contribute to the fractal nature of the resulting FSNDP in a quantitative manner: the rate process $I(t)$ is fractal as long as at least one of the three quantities ($P_{dN_1}(\omega)$, $\Big|\mathrm{E}[H(\omega)]\Big|^2$, $\mathrm{E}[|H(\omega)|^2]$) takes a power-law decaying form with its power-law exponent between -1 and 0 (i.e., $1/f$-noise behavior).

We have obtained the expressions for $P_I(\omega)$ for two types of the FSNDPs by evaluating $P_{dN_1}(\omega)$, $\Big|\mathrm{E}[H(\omega)]\Big|^2$, and $\mathrm{E}[|H(\omega)|^2]$ for each process. The first FSNDP, denoted as H-FSNDP, has the homogeneous Poisson Point Process (HPP) as $dN_1(t)$, while the second FSNDP, denoted as F-FSNDP, has the Sup-FRP as $dN_1(t)$. Both FSNDPs employ the following random rectangular filter $h(t)$:

$$h_i(t) = \begin{cases} K_i & 0 < t < B_i \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

where $\{K_i\}$ and $\{B_i\}$ are i.i.d. random variables, they are uncorrelated, and the first and second moments of K_i are finite.

4.3.1 H-FSNDP

For the H-FSNDP, the pdf for $\{B_i\}$ takes the smoothed Pareto form as follows:

$$p_B(t) = \begin{cases} \gamma_B A_B^{-1} e^{-\gamma_B t/A_B} & 0 \le t \le A_B \\ \gamma_B e^{-\gamma_B} A_B^{\gamma_B} t^{-(\gamma_B+1)} & t > A_B \end{cases} \quad (16)$$

After a rigorous analysis in the frequency domain, the TFPs of the H-FSNDP are given by [21]

$$\begin{aligned} H &= (3-\gamma_B)/2 \\ \lambda_2 &= \lambda_1 \mathrm{E}[K]\mathrm{E}[B] \\ T_0^{\alpha_B} &= 2^{-1} e_B^{\gamma}(3-\gamma_B)(2-\gamma_B)(\gamma_B-1)\frac{\mathrm{E}[K]\mathrm{E}[B]}{\mathrm{E}[K^2]} A^{-\gamma_B} \end{aligned} \quad (17)$$

where $\alpha_B = 2 - \gamma_B$.

4.3.2 F-FSNDP

For the F-FSNDP, we employ the Sup-FRP as the input point process $dN_1(t)$.

We first analyze the F-FSNDP with the exponential filter (F-FSNDP-EF). Again, after a rigorous analysis, we obtain the following expressions for its TFPs:

$$\begin{aligned} H &= (3-\gamma_1)/2 \\ \lambda_2 &= \lambda_1 \mathrm{E}[K]\mathrm{E}[B] \\ T_0 &= \left(\frac{\mathrm{E}[K] - 2\mathrm{E}[K^2]E[B]}{\mathrm{E}^2[K]\mathrm{E}[B]}\right)^{1/\alpha_1} T_{0,1} \end{aligned} \quad (18)$$

where $\alpha_1 = 2 - \gamma_1$ and $T_{0,1}$ is the fractal onset time scale (from IDC) of the input point process Sup-FRP.

Next, we consider the F-FSNDP with the fractal filter (F-FSNDP-FF). The analysis of this model is tricky since it involves two fractal behaviors (one present in the input point process and the other in the filter) with potentially different fractal exponent and FOTS. Instead of presenting the analysis results (which are available in [21], we discuss its IDC only.

The IDC of the F-FSNDP-FF is given in the form of

$$F(T) \approx 1 + (T/T_p)^{\alpha_p} + (T/T_q)^{\alpha_q} + (T/T_r)^{\alpha_r} \quad (19)$$

where T_p, T_q, and T_r can be considered as FOTSs for each fractal behavior. It is easy to see from Eq. (19) that this model can exhibit up to three different scaling behaviors occurring over different but possibly overlapping ranges of time with the corresponding scaling exponents $\alpha_p = 2-\gamma_B$, $\alpha_q = 2-\gamma_1$, $\alpha_r = 3-\gamma_1-\gamma_B$, respectively. Considering the multi-fractal patterns observed in various Internet traffic traces, this model may provide insights into explaining such a complex behavior. This issue is under an active research.

5 Concluding Remarks

We presented the unified framework of fractal point processes and a detailed analysis of fractal traffic models derived from the framework. Under this framework, the Three Fundamental Parameters were defined which include the Fractal Onset Time Scale that marks the beginning of the fractal behavior in the time domain. For each model, the mathematical relations between the TFP and the model parameters are obtained.

The applications of these parsimoniously parameterized models for Internet traffic engineering are immense. To begin with, all of these models can be employed for packet-level fractal traffic generators with the full control of the main first- and second-order statistics; see [23, 17] for some of examples. In addition, these models can be used for characterizing session- or application-level fractal behavior; see [19] for the use of the Sup-FRP model for characterizing the web requests generated by a group of users. Moreover, the FSNDP family serves as the intuitive modeling tool for Internet flow characterization [4, 19].

Another important usefulness of the FSNDP is for understanding the Causes for the Fractal Nature of Internet Traffic. Throughput this paper, we have emphasized that much effort has been devoted to explaining the causes of fractal behavior in both LAN and WAN traffic. While several plausible explanations are suggested [5, 18, 27], they are qualitative in nature rather than being quantitative. We argue that *quantitative* understanding of the impact of session-level fractal characteristics (such as session arrivals and/or heavy-tailed file size distributions) is essential for making the knowledge of the fractal nature of Internet traffic more tangible and practical for tackling various Internet traffic engineering issues.

Indeed, we have achieved this goal based on the structure of the FSNDP model with the fractal rectangular filter analyzed in Section 4.3.1. For the H-FSNDP and F-FSNDP, we have obtained expressions for the TFPs as a function of the model parameters. The structure of the H-FSNDP represents scenarios where session or flow arrivals follow a Poisson model and session or flow duration and/or volume are heavy-tailed. On the other hand, the F-FSNDP represents the case with fractal session arrivals as reported in [3, 19], regardless of whether the session/flow duration/volume is fractal or not. When it is not fractal, the resulting FSNDP exhibits a single scaling pattern (mono-fractal). On the other hand, the case of heavy-tailed session duration and/or volume may yield multiple scaling patterns that may occur over different but overlapping ranges of time scales. We base this finding on Eq. (19). For example, with $0 < \alpha_{\{p,q,r\}} 1$ and $T_p \ll T_q \ll T_r$, this traffic model captures three scaling patterns: the first one over $T_p \ll T \ll T_q$ with the scaling exponent α_p, the second one over $T_q \ll T \ll T_r$ with α_q, and the third one over $T_r \ll T$ with α_r. This demonstrates that the structure of the FSNDP provides the *quantitative* link between the session- or flow-level and the packet-level dynamics, and therefore shed further light into understanding the causes of the Internet traffic self-similarity. We are actively analyzing various Internet traces to determine if any of them exhibits this multi-fractal pattern. Results will be reported elsewhere. These selected applications demonstrate that the traffic models presented in this paper do a lot more than serving as data-fitting exercise tools.

We are currently working on extending this work as follows. One is to conduct cases studies with representative, recently collected, and widely varying Internet traces in order to establish strong evidence that the FPP models are highly effective as traffic modeling and characterization tools. The second effort is being devoted to make these traffic models available in popular networking simulation platforms. All of these models have been implemented in OPNET and will be released as part of 7.0 in the form of *Raw Packet Generators* (RPG). A unique feature of RPGs is that they are IP-routable with the ability to select destinations. In this way, they can be readily used for generating background and/or aggregate traffic. We are also working on porting these models in *ns* as well. By sharing these tools with other researchers for studying a wide range of Internet traffic engineering issues, we hope to see these tools make substantial contributions to making our Internet faster, more reliable, and more flexible.

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