MC542

Organização de Computadores
Teoria e Prática

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MC542

Circuitos Lógicos

Representação de Números
e Circccuitos Aritméticos

"Fundamentals of Digital Logic with VHDL Design" - (Capítulo 5)

Título do Capítulo Abordado
Sumário

- Representação Posicional
- Adição de N´´umeros sem Sinal
  Números com Sinal (sinalizados)
- Somadores Rápidos
- Multiplicação
- Representações de Números Reais

Representação Posicional

- Representação de Inteiros sem Sinal

$$D = d_{n-1}\ d_{n-2}\ \dots\ d_1\ d_0$$

$$V(D) = d_{n-1} \times 10^{n-1} + d_{n-2} \times 10^{n-2} + \dots + d_1 \times 10^1 + d_0 \times 10^0$$

- Sistema Binário:

$$B = b_{n-1}\ b_{n-2}\ \dots\ b_1\ b_0$$

$$V(B) = b_{n-1} \times 2^{n-1} + b_{n-2} \times b^{n-2} + \dots + b_1 \times 2^1 + b_0 \times 2^0$$

$$= \sum_{i=0}^{n-1} b_i \times 2^i$$

Representação Posicional
Conversão entre Decimal e Binário

$$V(B) = b_{n-1}x2^{n-1} + b_{n-2}xb^{n-2} + \ldots + b_1x2^1 + b_0x2^0$$

$$V(B) = b_{n-1}x2^{n-1} + b_{n-2}xb^{n-2} + \ldots + b_1x2^1 + b_0$$

$$\frac{V(B)}{2} = b_{n-1}x2^{n-2} + b_{n-2}xb^{n-3} + \ldots + b_1 + \frac{b_0}{2}$$

Conversão de Decimal para Binário: Divisão sucessiva por 2

Exemplo

Convert $(857)_{10}$

			Remainder	
$857 \div 2$	$=$	428	1	LSB
$428 \div 2$	$=$	214	0	
$214 \div 2$	$=$	107	0	
$107 \div 2$	$=$	53	1	
$53 \div 2$	$=$	26	1	
$26 \div 2$	$=$	13	0	
$13 \div 2$	$=$	6	1	
$6 \div 2$	$=$	3	0	
$3 \div 2$	$=$	1	1	
$1 \div 2$	$=$	0	1	MSB

Result is $(1101011001)_2$

Octal e Hexadecimal

Decimal	Binary	Octal	Hexadecimal
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	01011	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Adição de Números sem Sinal

$$\begin{array}{ccccc} & 0 & 1 & 0 & 1 \\ + & 0 & 0 & 1 & 1 \\ \hline & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{ccc} & 0 & x \\ + & 0 & y \\ \hline & c & s \end{array}$$

		Carry	Soma
x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Adição de Números sem Sinal

$X = x_4 x_3 x_2 x_1 x_0$ 0 1 1 1 1 $(15)_{10}$

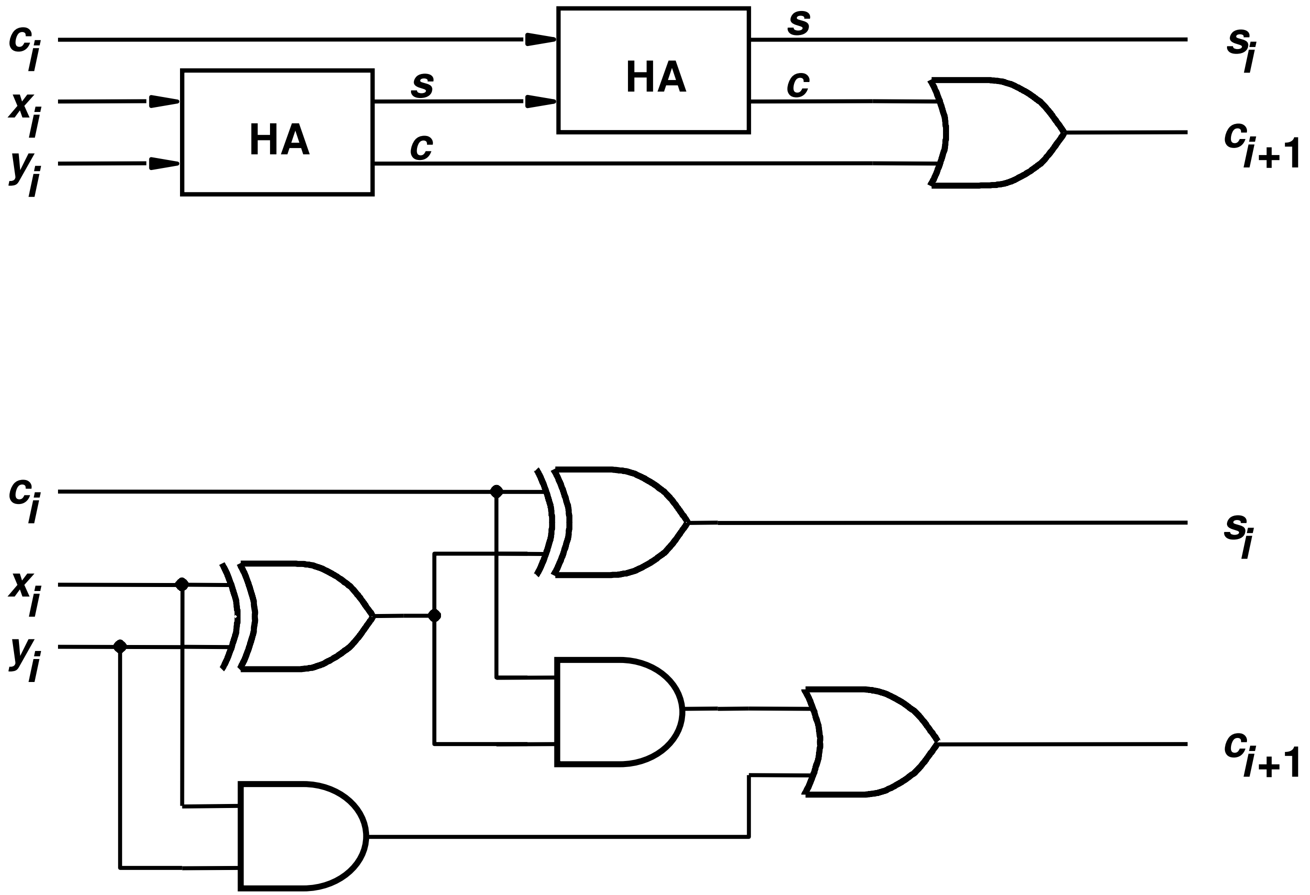
$+ \; Y = y_4 y_3 y_2 y_1 y_0$ 0 1 0 1 0 $(10)_{10}$

1 1 1 0 ← Generated carries

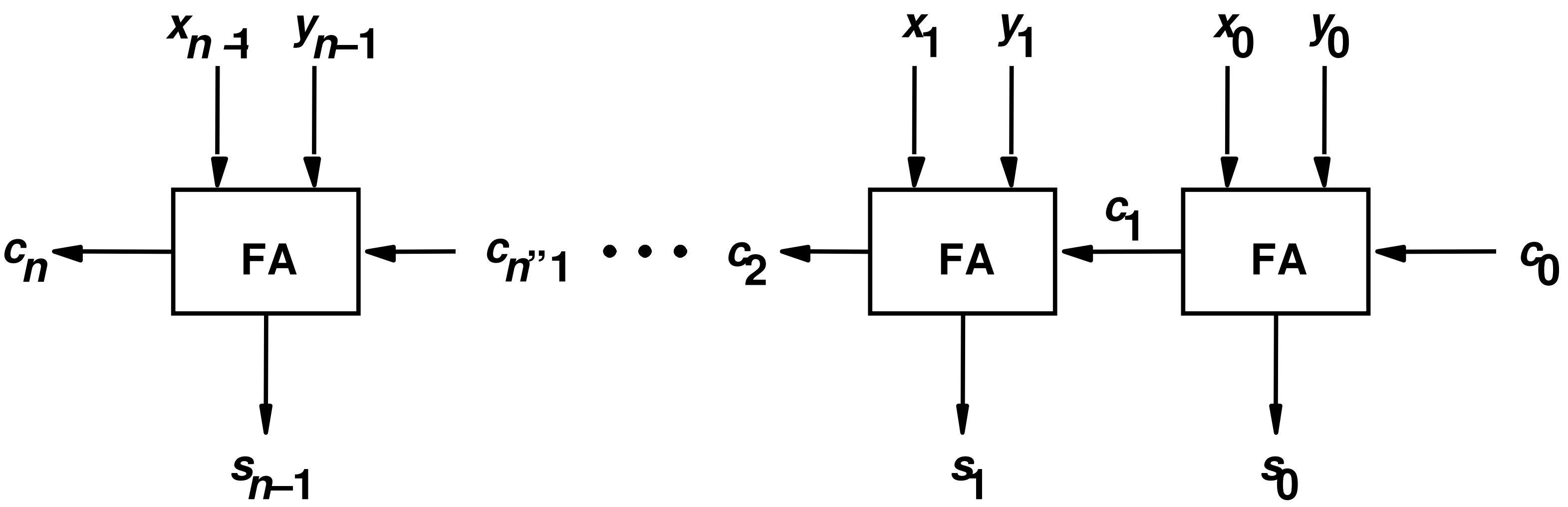
$S = s_4 s_3 s_2 s_1 s_0$ 1 1 0 0 1 $(25)_{10}$

Adição de Números sem Sinal

Somador Completo a partir de ½ Somador



Somador Ripple-Carry

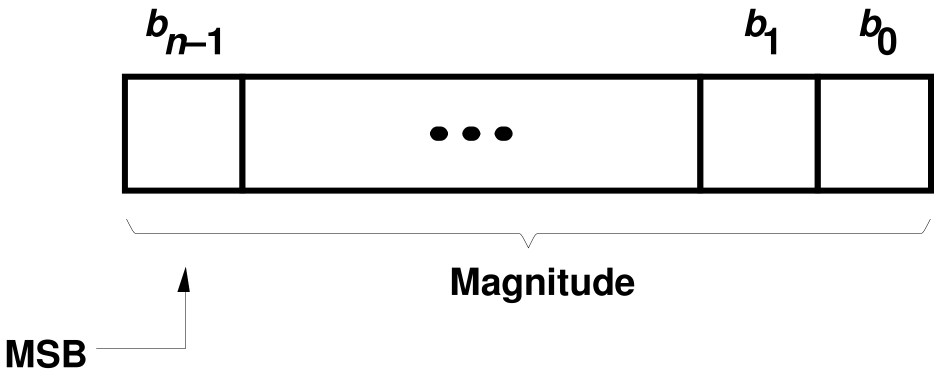


MSB position

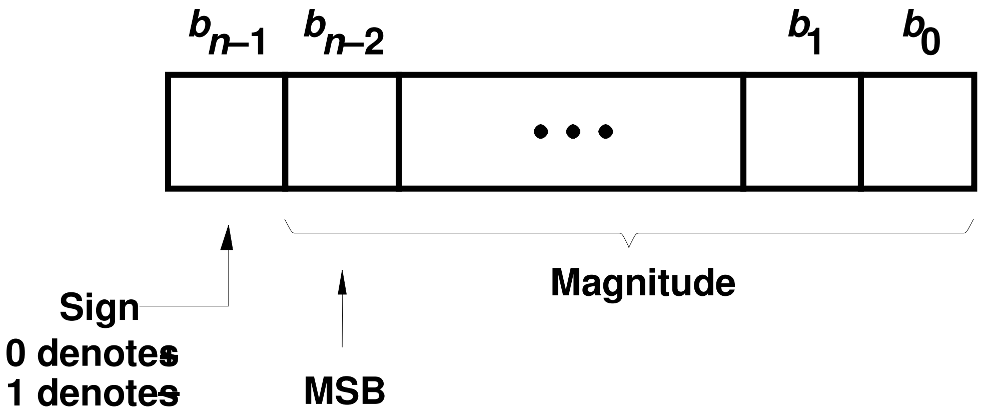
LSB position

Representação de Números Negativos

Sinal e Magnitude



Número sem Sinal



Número com Sinal

Representação de Números Negativos

Complemento de 1

Em complemento de “Um” o número negativo K, com n-bits, é obtido subtraíndo seu positivo P de $2^n - 1$

$$K = (2^n - 1) - P$$

Exemplo: se n = 4
então:

$K = (2^4 - 1) - P$
$K = (16 - 1) - P$
$K = (1111)_2 - P$

K = -7 -> P = 7
$7 = (0111)_2$
$-7 = (1111)_2 - (0111)_2$
$-7 = (1000)_2$

Representação de Números Negativos

Complemento de 2

Em complemento de “Dois” o número negativo K, com n-bits, é obtido subtraíndo seu positivo P de 2^n

$$K = 2^n - P \longrightarrow K = (2^n - 1) + 1 - P$$

$$K = (2^n - 1) - P + 1$$

Exemplo: se n = 4
então:

$$K = 2^4 - P$$
$$K = 16 - P$$
$$K = (10000)_2 - P$$

$K = -7$ -> $P = 7$

$7 = (0111)_2$

$-7 = (10000)_2 - (0111)_2$

$-7 = (1001)_2$

Representação de Números Negativos

$b_3 b_2 b_1 b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	−0	−7	−8
1001	−1	−6	−7
1010	−2	−5	−6
1011	−3	−4	−5
1100	−4	−3	−4
1101	−5	−2	−3
1110	−6	−1	−2
1111	−7	−0	−1

Adição e Subtração
Complemento de 1

$$\begin{array}{r} (+5) \\ +\,(+2) \\ \hline (+7) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \end{array} \qquad\qquad \begin{array}{r} (-5) \\ +\,(+2) \\ \hline (-3) \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\,0\,0\,1\,0 \\ \hline 1\,1\,0\,0 \end{array}$$

$$\begin{array}{r} (+5) \\ +\,(-2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,0\,1\,0 \\ \llcorner\!\!\!\longrightarrow \boxed{1} \\ \hline \boxed{0\,0\,1\,1} \end{array} \qquad\qquad \begin{array}{r} (-5) \\ +\,(-2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,1\,1\,1 \\ \llcorner\!\!\!\longrightarrow \boxed{1} \\ \hline \boxed{1\,0\,0\,0} \end{array}$$

Adição e Subtração
Complemento de 2

$$\begin{array}{r} (+5)\\ +\ (+2)\\ \hline (+7) \end{array} \qquad \begin{array}{r} 0\ 1\ 0\ 1\\ +\ 0\ 0\ 1\ 0\\ \hline 0\ 1\ 1\ 1 \end{array} \qquad\qquad \begin{array}{r} (-5)\\ +\ (+2)\\ \hline (-3) \end{array} \qquad \begin{array}{r} 1\ 0\ 1\ 1\\ +\ 0\ 0\ 1\ 0\\ \hline 1\ 1\ 0\ 1 \end{array}$$

$$\begin{array}{r} (+5)\\ +\ (-2)\\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\ 1\ 0\ 1\\ +\ 1\ 1\ 1\ 0\\ \hline \underset{\overset{\uparrow}{\text{ignore}}}{1}\ 0\ 0\ 1\ 1 \end{array} \qquad\qquad \begin{array}{r} (-5)\\ +\ (-2)\\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\ 0\ 1\ 1\\ +\ 1\ 1\ 1\ 0\\ \hline \underset{\overset{\uparrow}{\text{ignore}}}{1}\ 1\ 0\ 0\ 1 \end{array}$$

Subtração em Complemento de 2

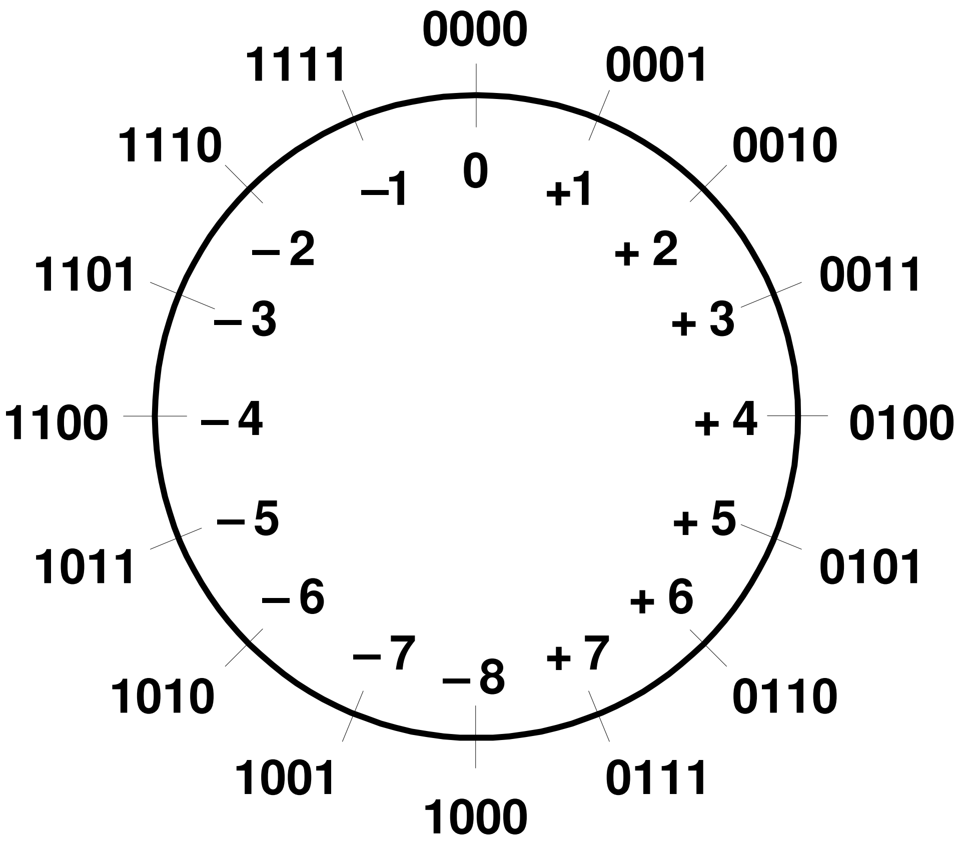
$$\begin{array}{r} (+5) \\ -\,(+2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ -\,0\,0\,1\,0 \\ \hline \\ \end{array} \quad \Rightarrow \quad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,1\,0 \\ \hline \underset{\substack{\uparrow \\ \text{ignore}}}{1}\,0\,0\,1\,1 \end{array}$$

$$\begin{array}{r} (-5) \\ -\,(+2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ -\,0\,0\,1\,0 \\ \hline \\ \end{array} \quad \Rightarrow \quad \begin{array}{r} 1\,0\,1\,1 \\ +\,1\,1\,1\,0 \\ \hline \underset{\substack{\uparrow \\ \text{ignore}}}{1}\,1\,0\,0\,1 \end{array}$$

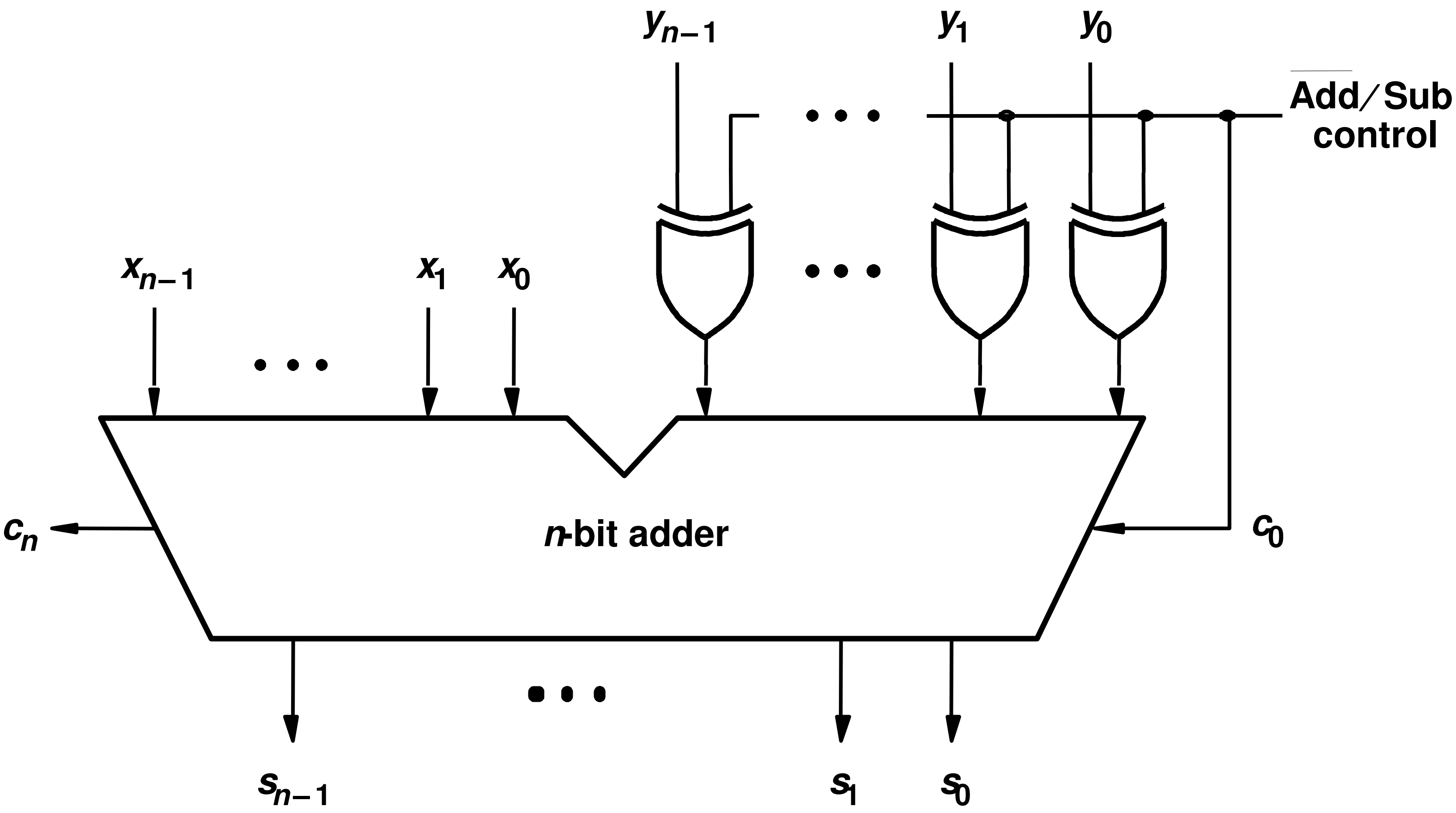
$$\begin{array}{r} (+5) \\ -\,(-2) \\ \hline (+7) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ -\,1\,1\,1\,0 \\ \hline \\ \end{array} \quad \Rightarrow \quad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \end{array}$$

$$\begin{array}{r} (-5) \\ -\,(-2) \\ \hline (-3) \end{array} \qquad \begin{array}{r} 1\,0\,1\,1 \\ -\,1\,1\,1\,0 \\ \hline \\ \end{array} \quad \Rightarrow \quad \begin{array}{r} 1\,0\,1\,1 \\ +\,0\,0\,1\,0 \\ \hline 1\,1\,0\,1 \end{array}$$

Complemento de 2



Unidade Somador-Subtrator



Overflow

Quando há overflow?
Como detectar se houve overrflow?

$$\begin{array}{r} (+7) \\ +\ (+2) \\ \hline (+9) \end{array} \qquad \begin{array}{r} 0\,1\,1\,1 \\ +\ 0\,0\,1\,0 \\ \hline 1\,0\,0\,1 \end{array}$$

$c_4 = 0$
$c_3 = 1$

$$\begin{array}{r} (-7) \\ +\ (+2) \\ \hline (-5) \end{array} \qquad \begin{array}{r} 1\,0\,0\,1 \\ +\ 0\,0\,1\,0 \\ \hline 1\,0\,1\,1 \end{array}$$

$c_4 = 0$
$c_3 = 0$

$$\begin{array}{r} (+7) \\ +\ (-2) \\ \hline (+5) \end{array} \qquad \begin{array}{r} 0\,1\,1\,1 \\ +\ 1\,1\,1\,0 \\ \hline 1\,0\,1\,0\,1 \end{array}$$

$c_4 = 1$
$c_3 = 1$

$$\begin{array}{r} (-7) \\ +\ (-2) \\ \hline (-9) \end{array} \qquad \begin{array}{r} 1\,0\,0\,1 \\ +\ 1\,1\,1\,0 \\ \hline 1\,0\,1\,1\,1 \end{array}$$

$c_4 = 1$
$c_3 = 0$

Somadores Rápidos
Carry-Lookahead

- Desempenho do somador Ripple Carry de n-bits
  - Se cada somador completo tem um atraso de Δt
  - Então, o atraso de todo o somador será de n Δt

Somadores Rápidos

Carry-Lookahead

$$C_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

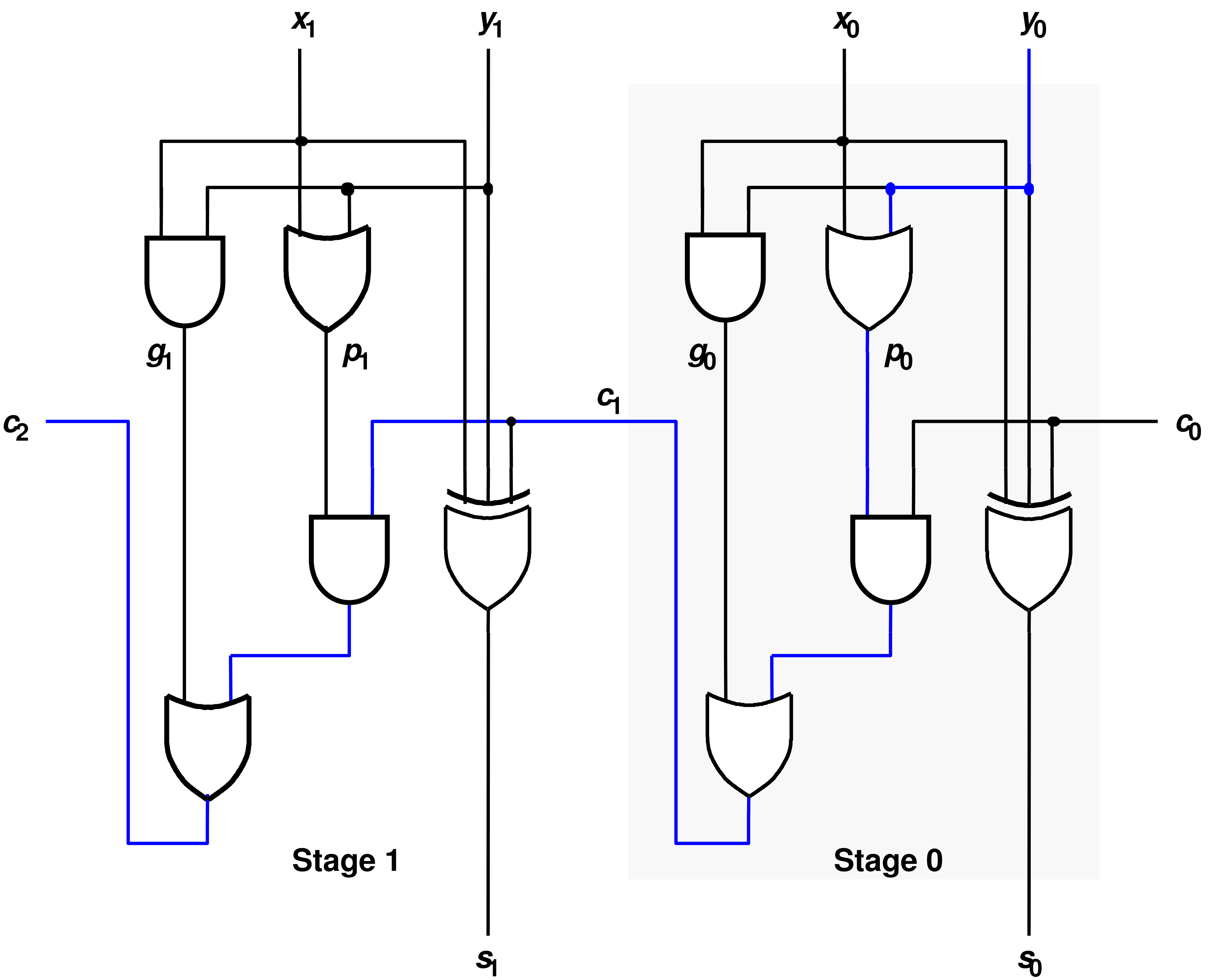
$$C_{i+1} = x_i y_i + (x_i + y_i) c_i \quad \Longrightarrow \quad C_{i+1} = g_i + p_i c_i$$

$$C_{i+1} = g_i + p_i (g_{i-1} + p_{i-1} c_{i-1})$$

$$C_{i+1} = g_i + p_i g_{i-1} + p_i p_{i-1} c_{i-1}$$

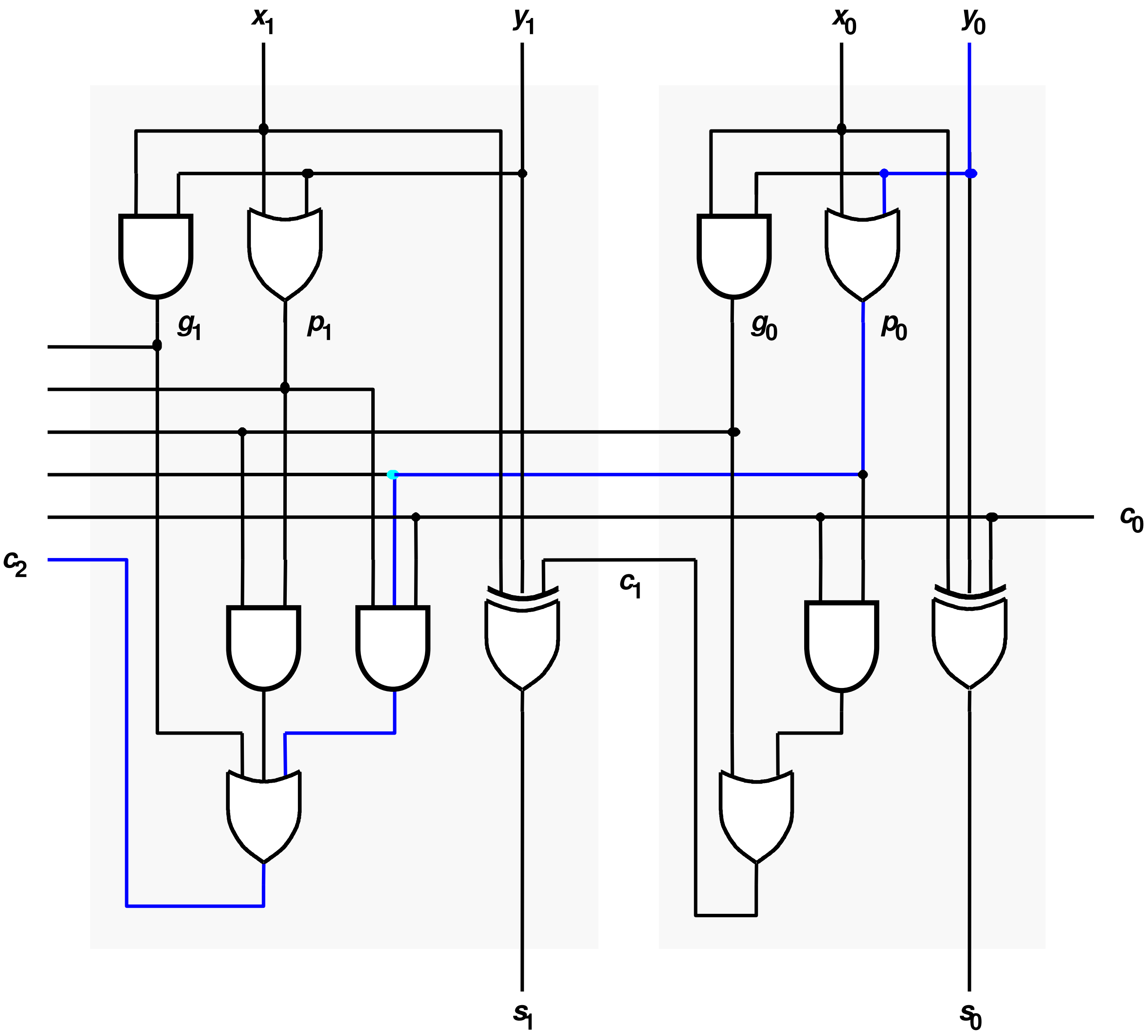
Somadores Rápidos

Ripple Carry em termos de g e p

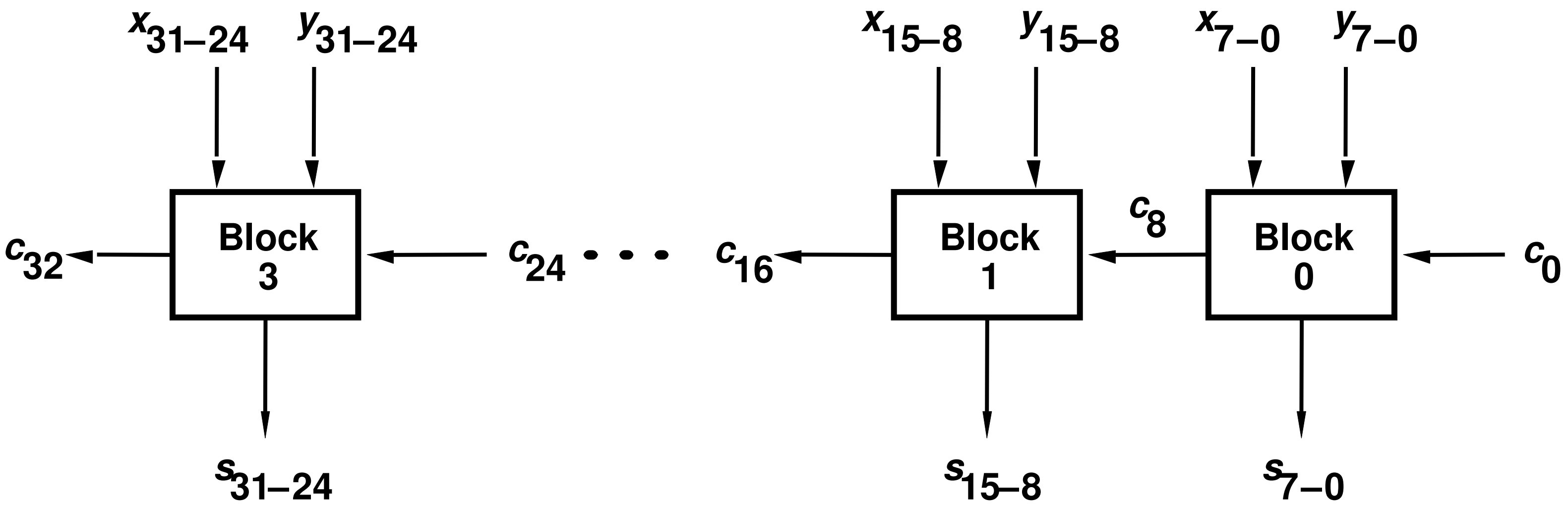


Somadores Rápidos

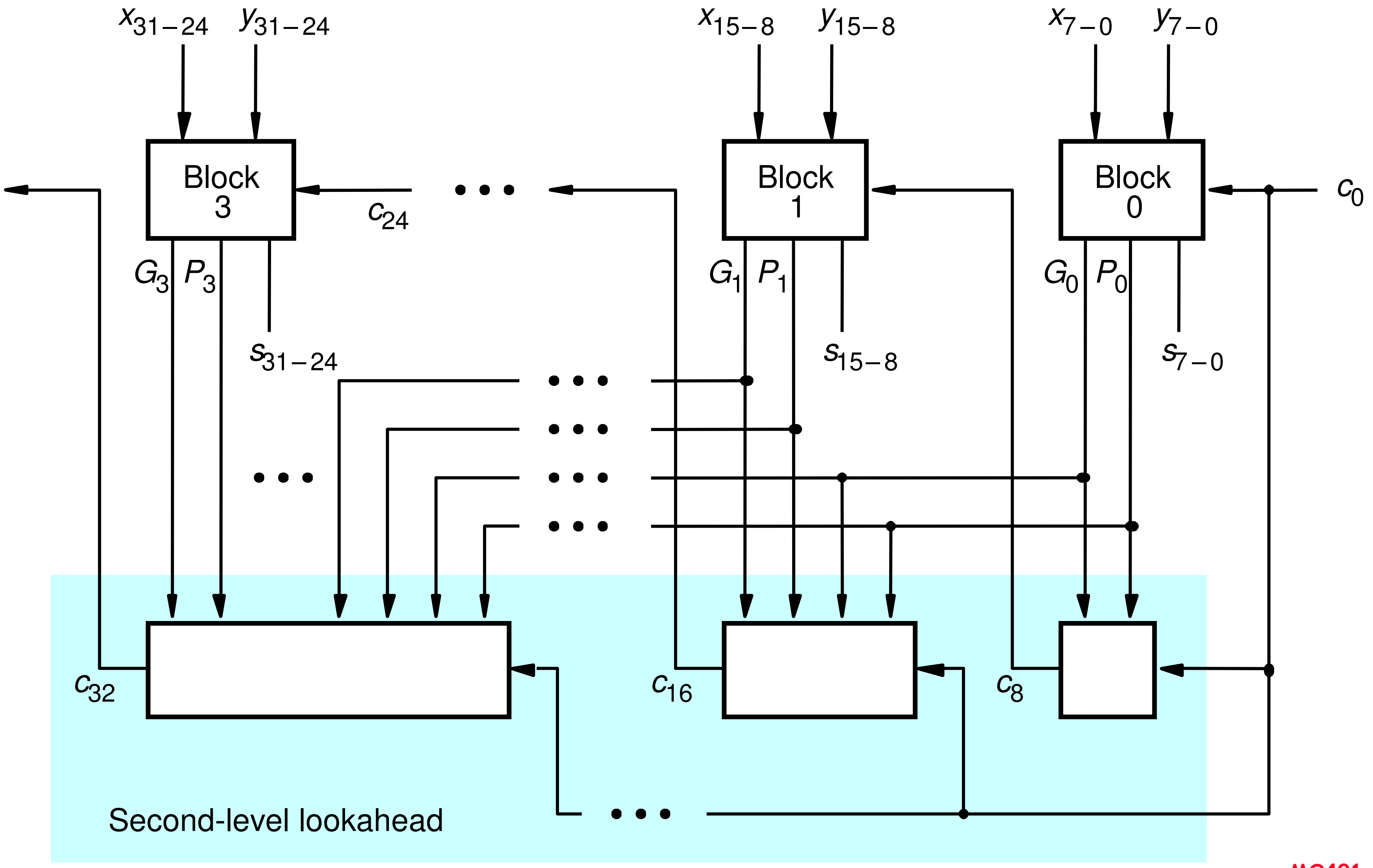
Carry-Lookahead



Somadores Rápidos
Carry-Lookahead



Carry-Lookahead Hierarquico

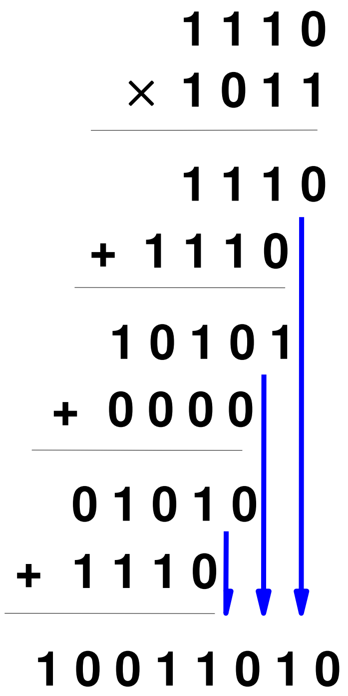


Multiplicação

$$\begin{array}{llr}
\text{Multiplicand M} & (14) & 1\,1\,1\,0 \\
\text{Multiplier Q} & (11) & \times\ 1\,0\,1\,1 \\
\cline{3-3}
 & & 1\,1\,1\,0 \\
 & & 1\,1\,1\,0 \\
 & & 0\,0\,0\,0 \\
 & & 1\,1\,1\,0 \\
\cline{3-3}
\text{Product P} & (154) & 1\,0\,0\,1\,1\,0\,1\,0
\end{array}$$

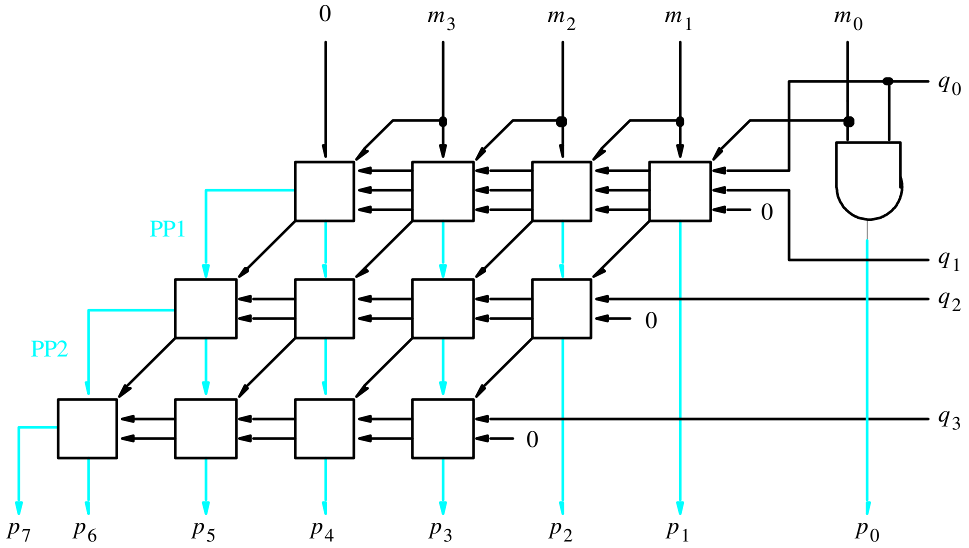
Multiplicação: lapis e papel

Multiplicação

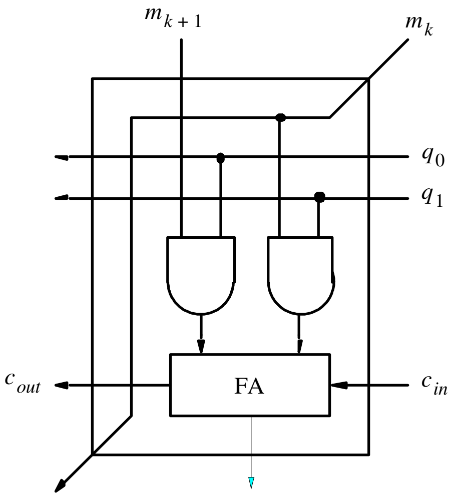


Multiplicand M	**(11)**	**1 1 1 0**
Multiplier Q	**(14)**	**× 1 0 1 1**
Partial product 0		**1 1 1 0**
		+ 1 1 1 0
Partial product 1		**1 0 1 0 1**
		+ 0 0 0 0
Partial product 2		**0 1 0 1 0**
		+ 1 1 1 0
Product P	**(154)**	**1 0 0 1 1 0 1 0**

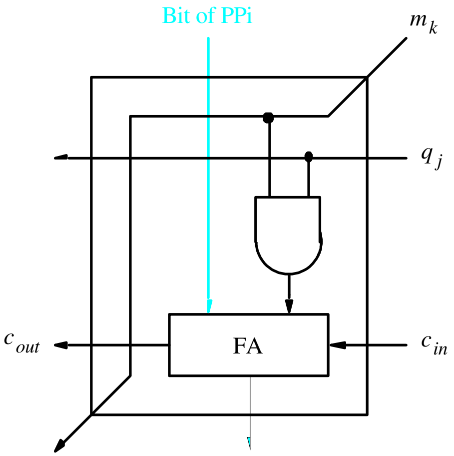
Multiplicação



(a) Structure of the circuit

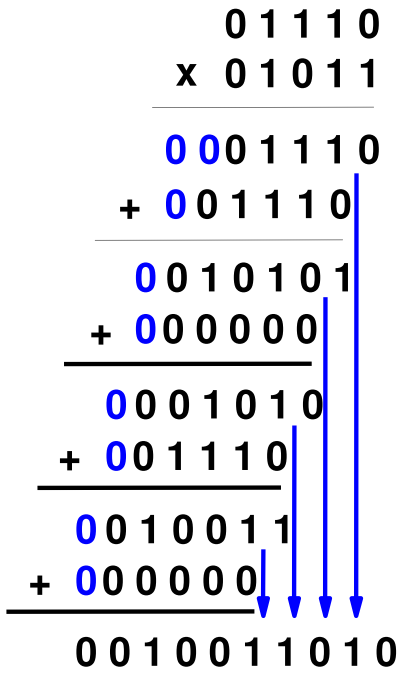


(b) A block in the top row



(c) A block in the bottom two rows

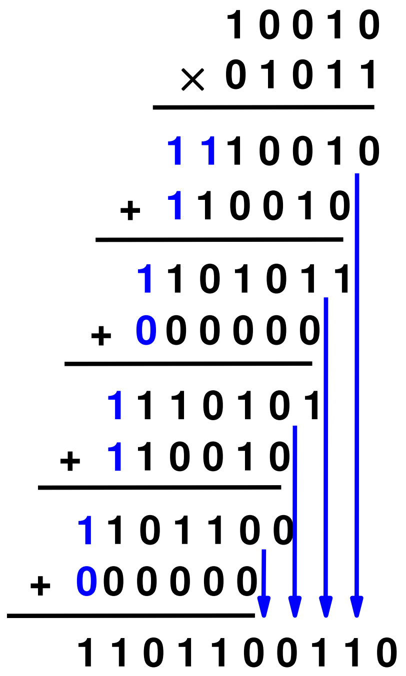
Multiplicação
Números com Sinal



Multiplicand M	(+14)	0 1 1 1 0
Multiplier Q	(+11)	x 0 1 0 1 1
Partial product 0		0 0 0 1 1 1 0
		+ 0 0 1 1 1 0
Partial product 1		0 0 1 0 1 0 1
		+ 0 0 0 0 0 0
Partial product 2		0 0 0 1 0 1 0
		+ 0 0 1 1 1 0
Partial product 3		0 0 1 0 0 1 1
		+ 0 0 0 0 0 0
Product P	(+154)	0 0 1 0 0 1 1 0 1 0

(a) Positive multiplicand

Multiplicação
Números com Sinal



Multiplicand M	(–14)	1 0 0 1 0
Multiplier Q	(+11)	× 0 1 0 1 1
Partial product 0		1 1 1 0 0 1 0
		+ 1 1 0 0 1 0
Partial product 1		1 1 0 1 0 1 1
		+ 0 0 0 0 0 0
Partial product 2		1 1 1 0 1 0 1
		+ 1 1 0 0 1 0
Partial product 3		1 1 0 1 1 0 0
		+ 0 0 0 0 0 0
Product P	(–154)	1 1 0 1 1 0 0 1 1 0

(b) Negative multiplicand

Representações de Números Reais

- Notação Científica:

$$\pm \text{ Mantissa} \times 10^{E}$$

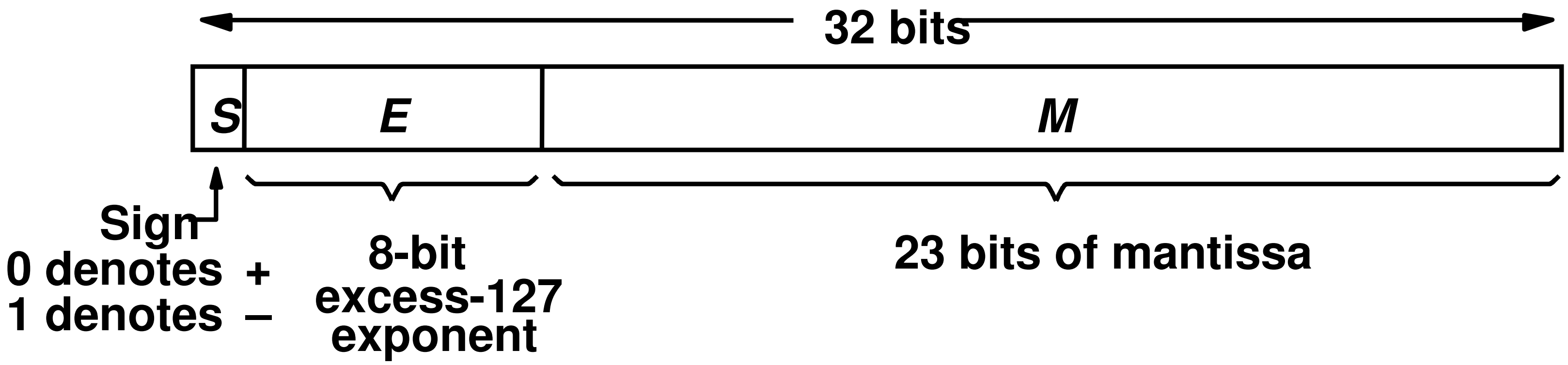
Mantissa = x.yyyyyy

- Se Mantissa não possui zeros a esquerda do ponto decimal -> normalizado

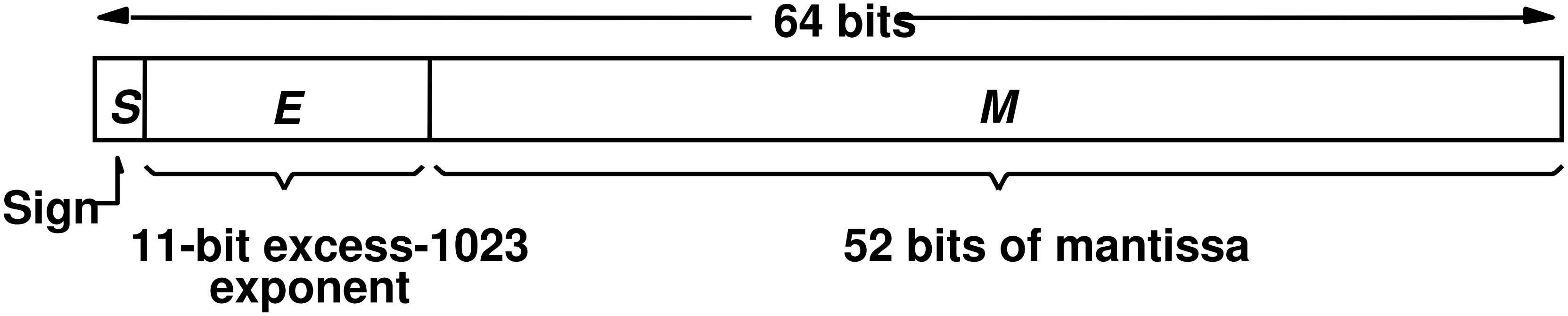
- Padrão IEEE-754
  - Normalizado
  - Bit Escondido

$$(-1)^{S} \times (1 + \text{Fração}) \times 2^{E}$$

IEEE-754 de Precisão Simples



IEEE-754 de Precisão Dupla



IEEE-754
Valores Representados

Exponent	Fraction	Represents
$e = 0 = E_{mim} - 1$	$f = 0$	± 0
$e = 0 = E_{mim} - 1$	$f \neq 0$	$0.f \times 2^{E_{mim}}$
$E_{mim} \leq e \leq E_{max}$		$1.f \times 2^{e}$
$e = E_{max} + 1$	$f = 0$	$\pm \infty$
$e = E_{max} + 1$	$f \neq 0$	NaN

IEE-754: Exemplo

$-0{,}75_{10} = -0{,}11_2$

Normalizando → $1{,}1 \times 2^{-1}$

31	30 23	22 0
1	0 1 1 1 1 1 1 0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

IEE-754: Exemplo

Qual o decimal correspondente ?

31	30 23	22 0
1	1 0 0 0 0 0 0 1	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

$$N= -\ (\ 1+0.25)\ x\ 2^{(129-127)} = -1{,}25\ x\ 4 = -5{,}0$$

BCD
Binary-Coded-Decimal

Decimal digit	BCD code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Adição Usando BCD

$$\begin{array}{rcrcr} X & \quad & 0\,1\,1\,1 & \quad & 7 \\ +\;Y & & +\;0\,1\,0\,1 & & +\;5 \\ \hline Z & & 1\,1\,0\,0 & & 12 \\ & & +\;0\,1\,1\,0 & & \\ \cline{3-3} \end{array}$$

$$\text{carry} \longrightarrow 1\,\underbrace{0\,0\,1\,0}_{S\,=\,2}$$

$$\begin{array}{rcrcr} X & \quad & 1\,0\,0\,0 & \quad & 8 \\ +\;Y & & +\;1\,0\,0\,1 & & +\;9 \\ \hline Z & & 1\,0\,0\,0\,1 & & 17 \\ & & +\;0\,1\,1\,0 & & \\ \cline{3-3} \end{array}$$

$$\text{carry} \longrightarrow 1\,\underbrace{0\,1\,1\,1}_{S\,=\,7}$$

Adição Usando BCD