AVD-total-coloring of complete equipartite graphs

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Definition

A **total-coloring** of G is an assignment of colors ϕ to the elements of G, such that each adjacent or incident elements of G have distinct colors.

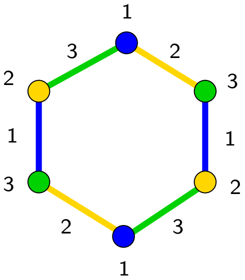


Figure: A total-coloring of cycle C_6 with 3 colors.

Introduction

- G a simple graph and ϕ a total-coloring of G.
- $C(u)$: set of colors that *occur* in a vertex u.
- Two vertices $u, v \in V(G)$ are **distinguishable** when $C(u) \neq C(v)$.

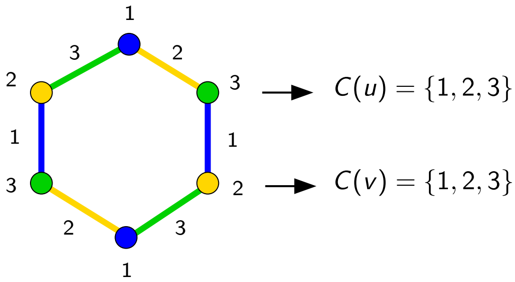
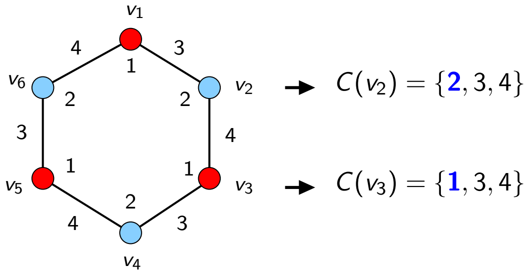


Figure: Total-coloring without distinguishable vertices.

AVD-total-coloring

Definition

- G a simple graph and ϕ a total-coloring of G.
- ϕ is an **AVD-total-coloring** if $\forall u, v \in V(G)$, uv adjacent, we have $C(u) \neq C(v)$.



*AVD = Adjacent Vertex Distinguishing.

AVD-total-chromatic number

The **AVD-total-chromatic number**, $\chi''_a(G)$, is the smallest number of colors for which G admits an AVD-total-coloring.

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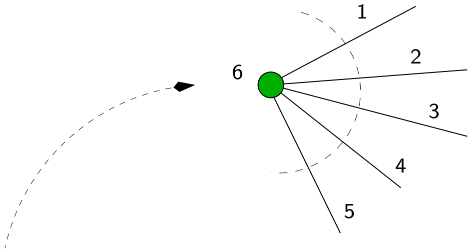

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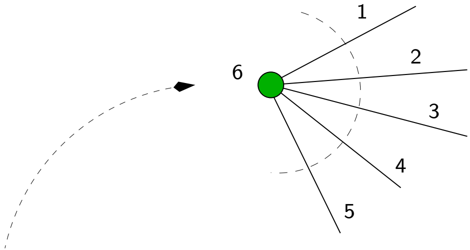


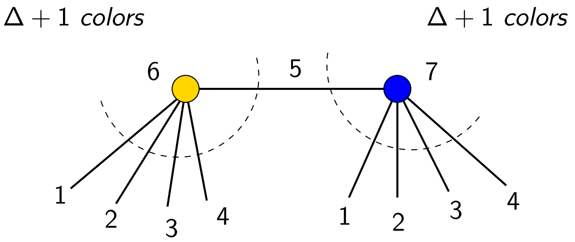
$$\chi''_a(G) \geq \Delta(G) + 1$$

AVD-total-coloring - Lower bound

Zhang et al. [1]

If G is a graph with at least two adjacent vertices of maximum degree, then

$$\chi''_a(G) \geq \Delta(G) + 2.$$

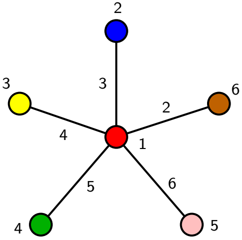


AVD-total-coloring conjecture

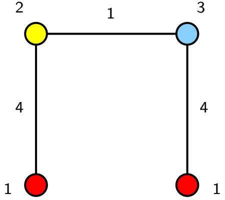
Zhang et al. [1]

If G is a simple graph, then

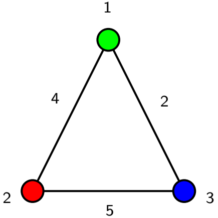
$$\chi''_a(G) \leq \Delta(G) + 3.$$



$\Delta + 1$



$\Delta + 2$



$\Delta + 3$

Related work

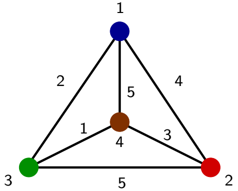
The AVD-total-coloring conjecture was verified for some classes of graphs, including:

- Complete graphs, complete bipartite graphs, and trees (Zhang et al. [1]).
- Graphs with $\Delta(G) = 3$ (J. Hulgan [8]).
- Hypercubes (M. Chen and X. Guo [6]).
- Outerplanar graphs (Y. Wang and W. Yang [5]).
- Indifference graphs (V. Pedrotti and C. P. de Mello [9]).
- Halin graphs (X. Chen and Z. Zhang [7]).

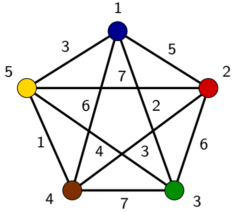
Complete Graphs

Zhang et al. [1]

$$\chi''_a(K_n) = \begin{cases} \Delta(K_n) + 2 & \text{if } n \text{ is even,} \\ \Delta(K_n) + 3 & \text{if } n \text{ is odd.} \end{cases}$$



$\chi''_a(K_4) = \Delta(K_4) + 2 = 5$



$\chi''_a(K_5) = \Delta(K_5) + 3 = 7$

Complete equipartite graphs

Definition

A *complete equipartite graph*, $K_{r(n)}$, is a graph whose vertex set can be partitioned into r independent sets *(parts)* of cardinality n, such that any two vertices belonging to different parts are joined by an edge.

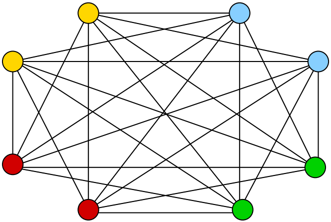


Figure: Complete equipartite graph $K_{4(2)}$.

Main Result

Theorem

Let $G = K_{r(n)}$, $r \geq 2$ and $n \geq 2$. Then,

- $\chi''_a(G) = \Delta(G) + 2$ if G has even order, or
- $\chi''_a(G) \leq \Delta(G) + 3$ if G has odd order.

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- Proof in 4 cases:
  - r even and n even.
  - r odd and n even.
  - r even and n odd.
  - r odd and n odd.

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Case 1 - n even and r even.

Sketch

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- $\chi''_a(G) \geq \Delta(G) + 2$.
- Build an AVD-total-coloring of $K_{r(n)}$ with $\Delta(G) + 2$ colors.

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- $\chi''_a(G) \geq \Delta(G) + 2$.
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  - **How?** - Canonical-decomposition of $K_{r(n)}$.

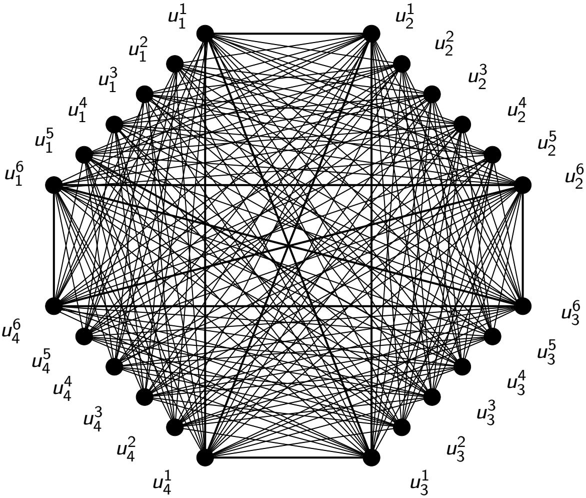
Case 1 - n even and r even.

Sketch

- $\chi''_a(G) \geq \Delta(G) + 2$.
- Build an AVD-total-coloring of $K_{r(n)}$ with $\Delta(G) + 2$ colors.
  - **How?** - Canonical-decomposition of $K_{r(n)}$.
- Illustration of the proof using the $K_{4(6)}$.

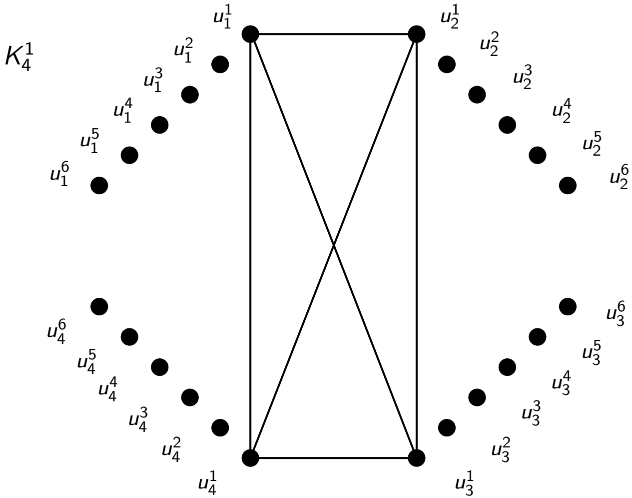
Canonical-decomposition of $K_{r(n)}$

Example: $K_{4(6)}$, $r = 4$ and $n = 6$. Label the vertices: u_i^k.



Canonical-decomposition of $K_{r(n)}$

$K_{r(n)}$ has n disjoint copies of K_r.



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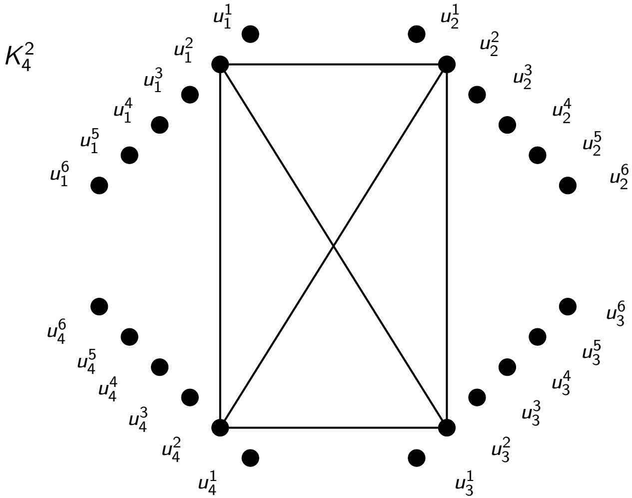

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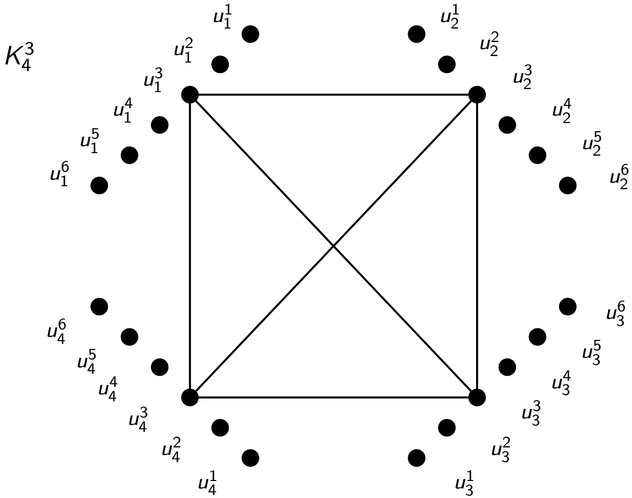

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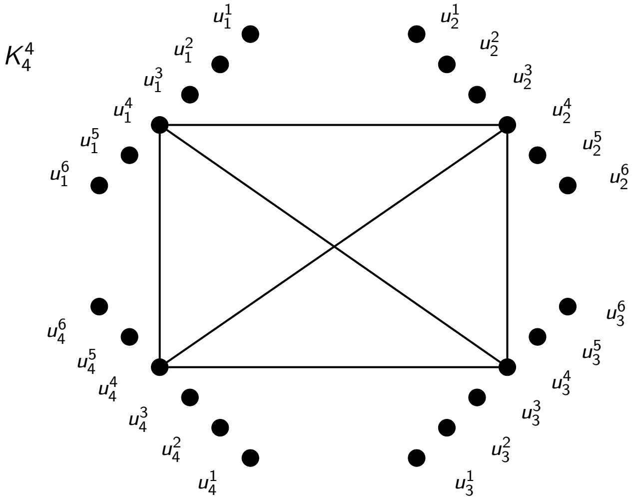

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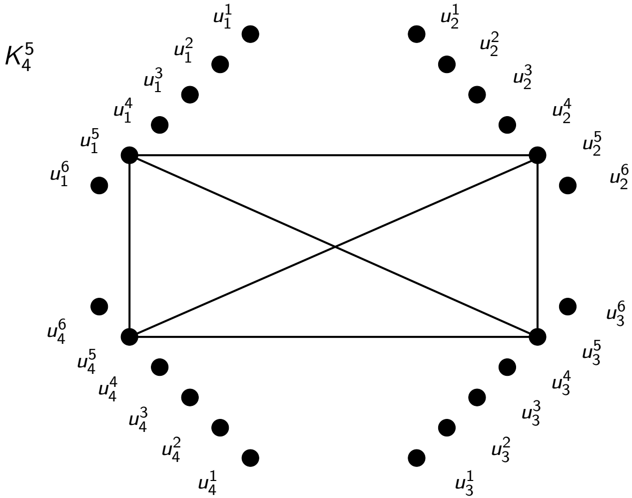

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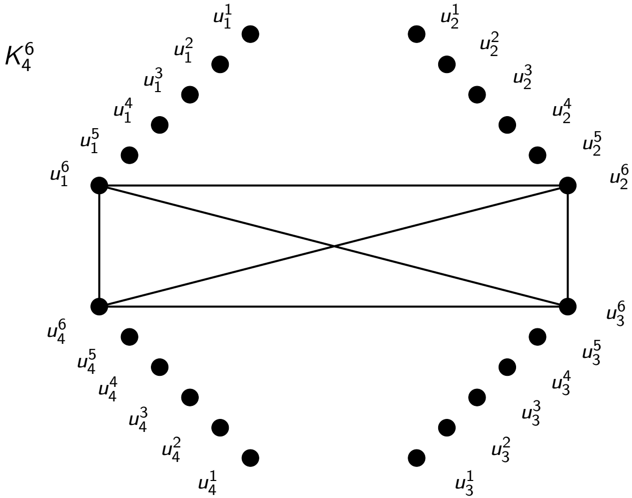
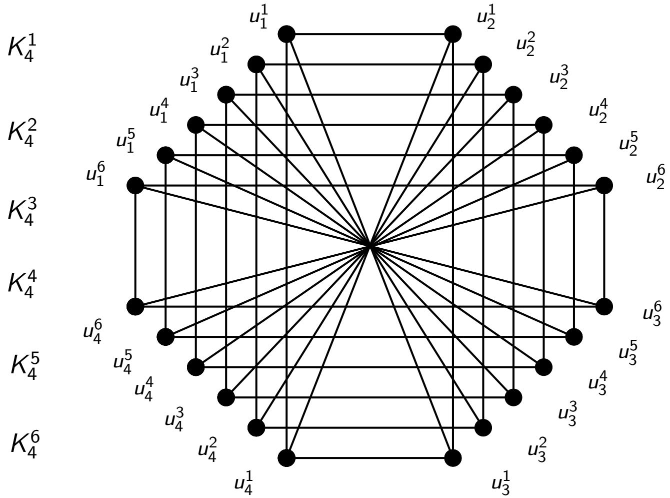


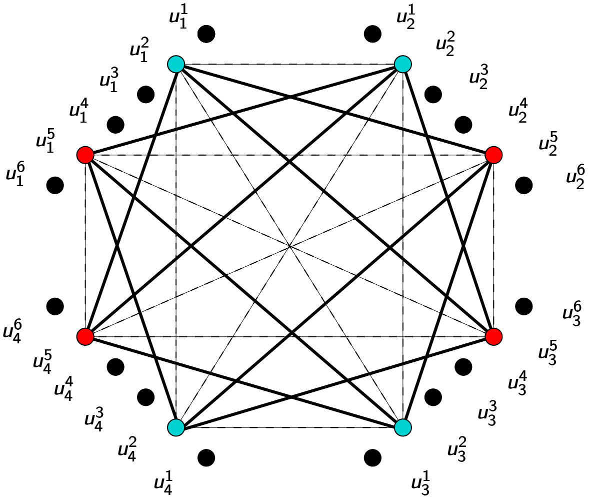
Canonical-decomposition of $K_{r(n)}$

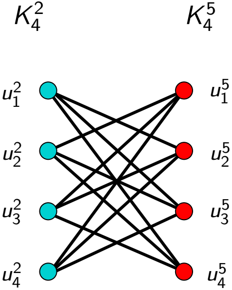
The 6 disjoint subgraphs of $K_{4(6)}$ that are isomorphic to K_4.



Canonical-decomposition of $K_{r(n)}$

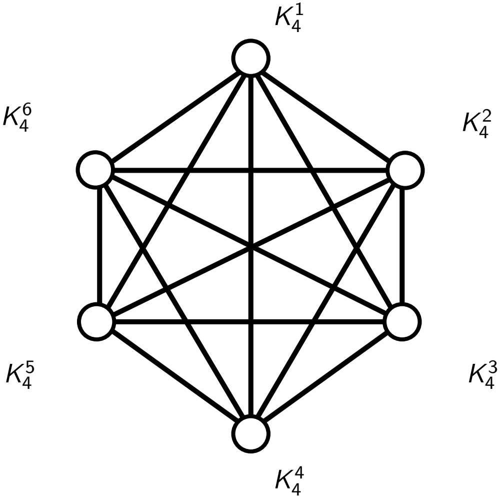
Edges joining vertices from K_r^i to vertices of K_r^j ($i \neq j$) induce an $(r-1)$-regular bipartite graph.





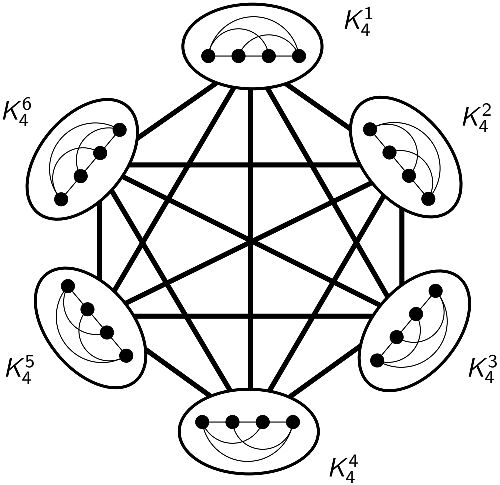
Representative graph of $K_{r(n)}$

Representative graph of $K_{r(n)}$ is isomorphic to K_n

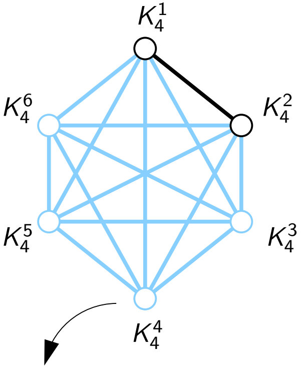


Representative graph of $K_{r(n)}$

Representative graph of $K_{4(6)}$

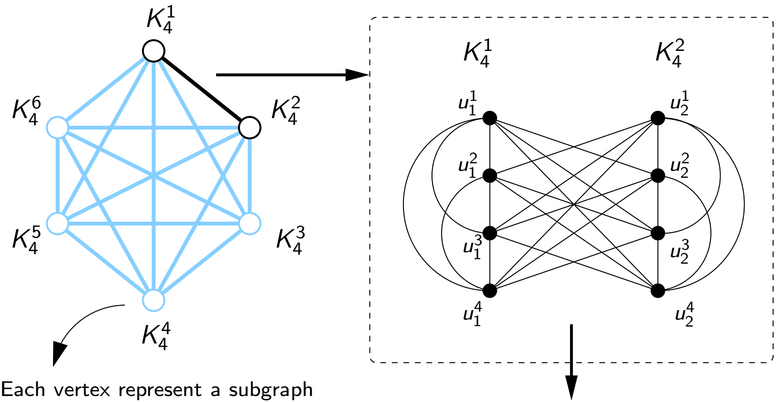


Representative graph of $K_{4(6)}$



Each vertex represent a subgraph K_4^i isomorphic to K_4

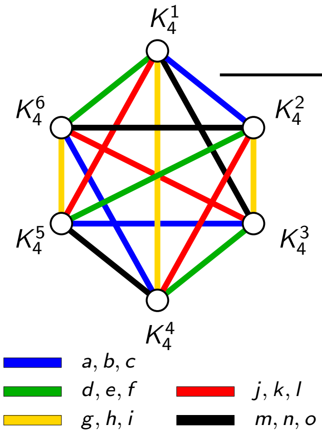
Representative graph of $K_{4(6)}$

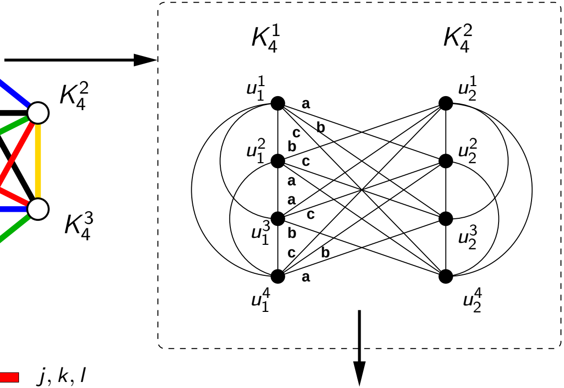


Each vertex represent a subgraph K_4^i isomorphic to K_4

Each edge represent an $(r-1)$-regular bipartite graph

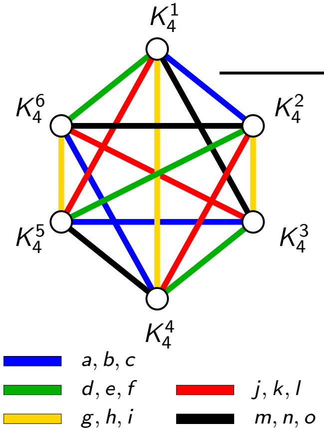
Representative graph of $K_{r(n)}$ has $n-1$ perfect matchings

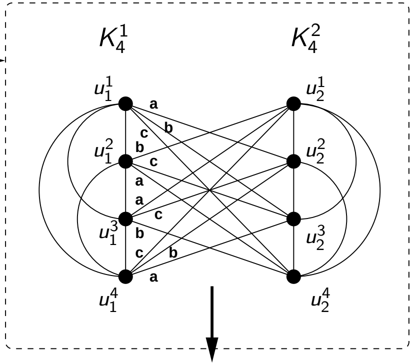




Each bipartite graph can be edge-colored with $(r-1)$ colors

Edge-coloring of bipartite graphs



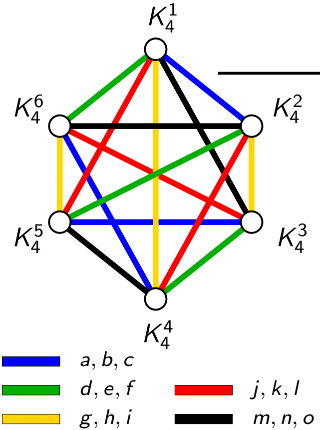


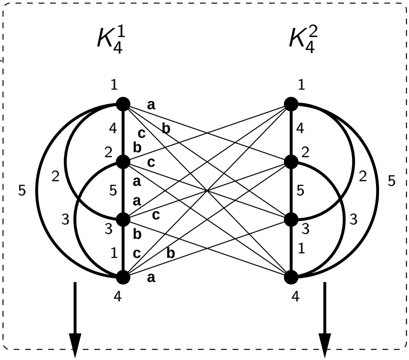
Each bipartite graph can be edge-colored with $(r-1)$ colors

Total of colors for bipartite graphs $= (r-1) * (n-1)$

Illustration of the proof

AVD-total-coloring of complete graphs K_r^i



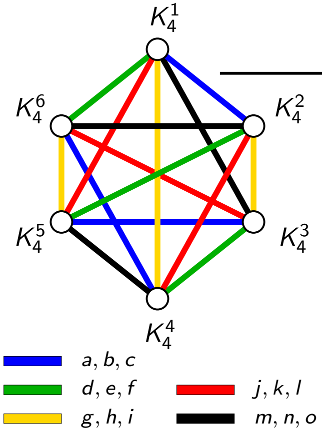


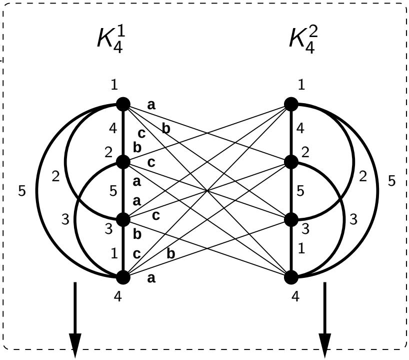
Since $K_r^i \cong K_r$, each subgraph K_r^i has an AVD-total-coloring with $r + 1$ colors

Each subgraph K_4^i has an AVD-total-coloring with 5 colors

Illustration of the proof

AVD-total-coloring of $K_{r(n)}$





Since $K_r^i \cong K_r$, each subgraph K_r^i has an AVD-total-coloring with $r+1$ colors

Total of colors $= (n-1)*(r-1)+(r+1) = n*(r-1)+2 = \mathbf{\Delta(K_{r(n)})+2}$

Concluding remarks

Theorem

Let $G = K_{r(n)}$, $r \geq 2$ and $n \geq 2$. Then,

- $\chi''_a(G) = \Delta(G) + 2$ if G has even order, or
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Conjecture

$\chi''_a(G) = \Delta(G) + 2$ for $G = K_{r(n)}$ of odd order.

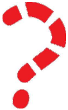
Acknowledgements

CNPq

FAPESP

Institute of Computing - UNICAMP - Brazil

Questions



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