Fundamentals of Image Processing (part IV)

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- The framework to design such transformations is named Image Foresting Transform [1].

- Applications involve segmentation [2, 3, 4, 5, 6, 7, 8], clustering [9, 10], classification [11, 12, 13, 14], distance transforms [15, 16], morphological reconstructions [17], multiscale skeletons [15, 16], shape saliences [18], etc.

Agenda

- Image Foresting Transform (IFT).

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- General IFT algorithm.

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- General IFT algorithm.
- Examples and main properties.

Image Foresting Transform

For a given image graph $(\mathcal{N}, \mathcal{A}, \mathrm{I})$, $\mathcal{N} \subseteq D_I$,

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- The set Π contains trivial paths $\pi_p = \langle p \rangle$ and paths $\pi_p \cdot \langle p, q \rangle$ that represent the extension of π_p by an arc $(p, q) \in \mathcal{A}$.
- The IFT algorithm minimizes a path-cost map V,

$$V(p) \quad = \quad \min_{\pi_p \in \Pi} \{f(\pi_p)\},$$

for all $p \in \mathcal{N}$, irrespective to its root node.

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- The map P will be optimal (i.e., an optimum-path forest) whenever f satisfies the conditions stated in [19].
- However, there are applications for the case P is just a rooted spanning forest [3, 4, 16].
- The image transformations derive from attributes of the forest: paths, costs, root labels, etc.

The IFT algorithm

- It starts from all nodes $p \in \mathcal{N}$ as trivial paths with values $V(p) \leftarrow f(\pi_p)$ in a priority queue Q. The roots will derive from the minima of this initial path-cost map.

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- By removing the nodes p in a non-decreasing order of path values from Q, it verifies for each adjacent $q \in \mathcal{A}(p)$

$$\begin{aligned} \textit{if} \quad f(\pi_p \cdot \langle p, q \rangle) < V(q), & \text{ then} \\ \pi_q \leftarrow \pi_p \cdot \langle p, q \rangle & \text{ and} \\ V(q) \leftarrow f(\pi_p \cdot \langle p, q \rangle). & \end{aligned}$$

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- It stops when Q is empty and the optimum paths π^*_p for all $p \in \mathcal{N}$ can be retrieved from P.

Input: Image graph $(\mathcal{N}, \mathcal{A}, \mathrm{I})$ and connectivity function f.
Output: Cost map V and predecessor map P.

1 For each $q \in \mathcal{N}$, set $V(q) \leftarrow f(\langle q \rangle)$ and $P(q) \leftarrow nil$, and insert q in Q.
2 While $Q \neq \emptyset$ do
3 Remove from Q the node $p = arg \min_{q \in Q}\{V(q)\}$.
4 For each $q \in \mathcal{A}(p)$, $q \in Q$, do
5 If $V(q) > f(\pi_p \cdot \langle p, q \rangle)$, then
6 update $V(q) \leftarrow f(\pi_p \cdot \langle p, q \rangle)$ and $P(q) \leftarrow p$.

Consider, for example, a max-arc-weight function f_{max} that forces optimum paths to start in a seed set $\mathcal{S} = \{a, b\} \subset \mathcal{N} = D_I$.

$$
\begin{aligned}
f_{\text{max}}(\langle p \rangle) &= \begin{cases} 0 & \text{if } p \in \mathcal{S}, \\ +\infty & \text{otherwise.} \end{cases} \\
f_{\text{max}}(\pi_p \cdot \langle p, q \rangle) &= \max\{f_{\text{max}}(\pi_p), \|\mathbf{I}(q), \mathbf{I}(p)\|_2\}.
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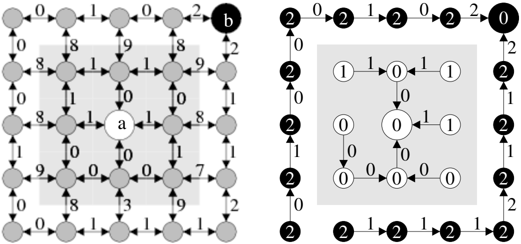
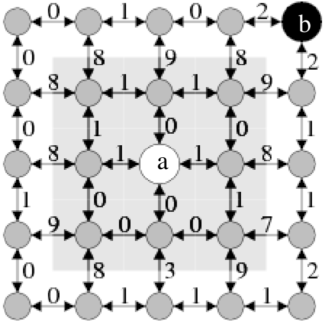
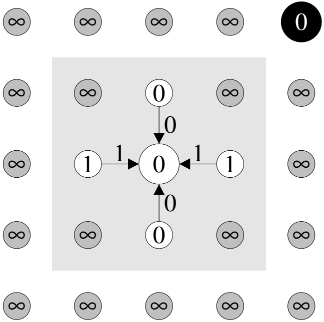
For the graph on the left, the output is the forest on the right.



From iteration 1 to 5, iteration 12, 20, and 25.





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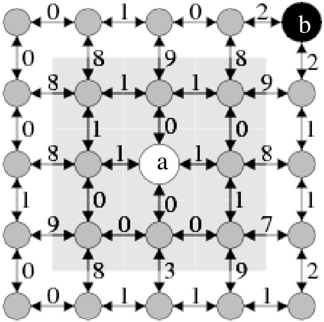
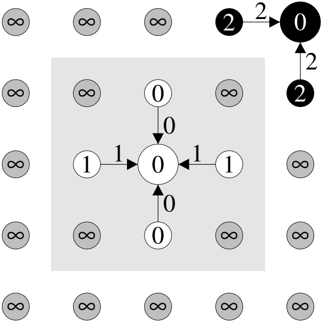
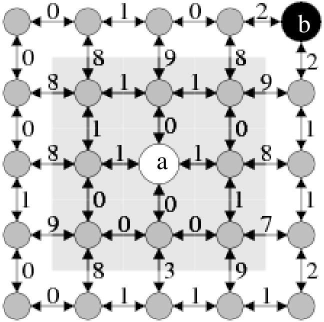
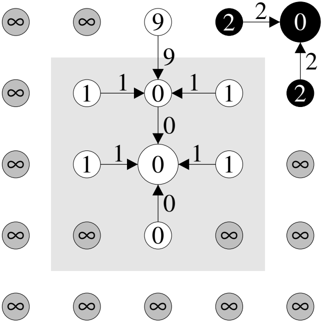




Example of optimum path propagation

From iteration 1 to 5, iteration 12, 20, and 25.





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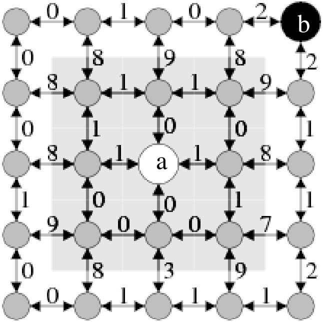

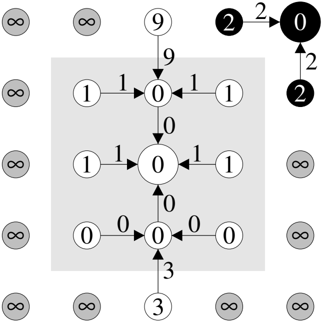

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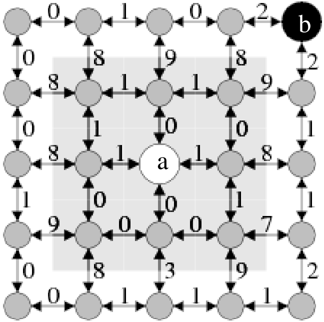

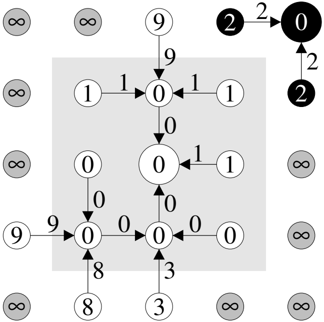

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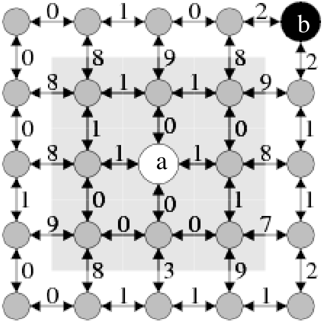

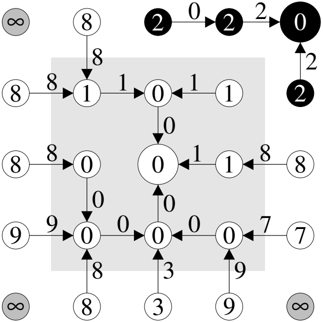

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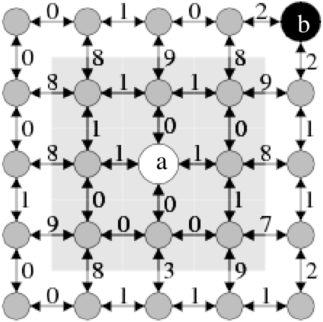

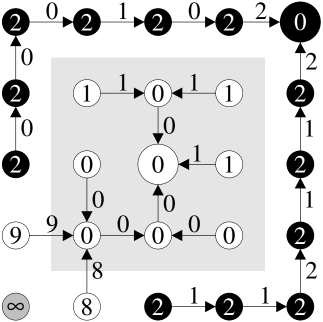

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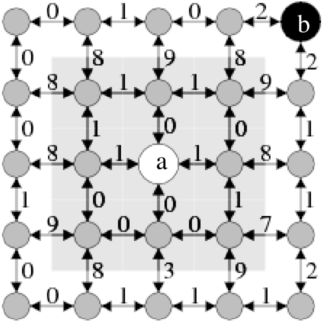
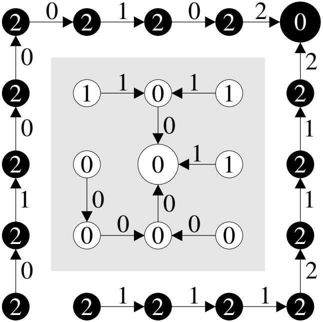
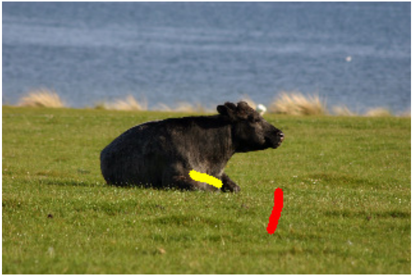




Example of seed-based segmentation

Using the same function $f_{\max}$, the roots may be forced to start from internal and external markers (location) for object delineation.



The object is defined by the optimum-path forest rooted at its internal markers.

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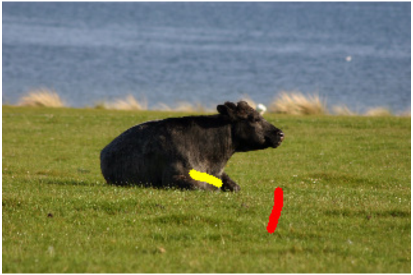
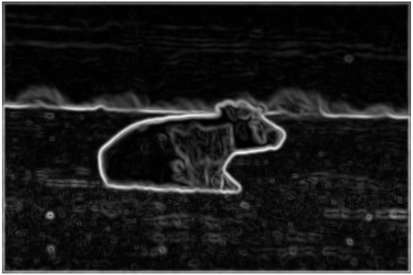


- Image with internal and external markers.

Example of seed-based segmentation

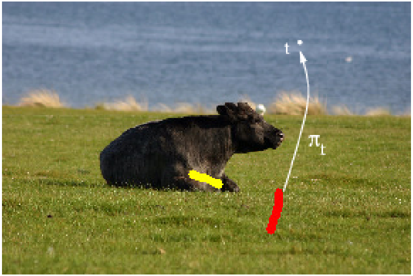


- Image with internal and external markers.
- Arc-weight image.

Example of seed-based segmentation

- Image with internal and external markers.
- Arc-weight image.
- Optimum-paths to foreground pixels.

Example of seed-based segmentation



- Image with internal and external markers.
- Arc-weight image.
- Optimum-paths to foreground pixels.
- Optimum-paths to background pixels.

Example of seed-based segmentation

- Image with internal and external markers.
- Arc-weight image.
- Optimum-paths to foreground pixels.
- Optimum-paths to background pixels.
- Segmentation result.

Exercise

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- Change the general IFT algorithm to receive $(\mathcal{S}, \lambda)$ and return in a label map L with the label $L(p) \in \{0, 1\}$ of background and object pixels.
- Change it now to output a root map R that assigns to every $p \in \mathcal{N}$ the root node $R(p)$ in the optium path π^*_p.

Main properties

- When p is removed from Q,
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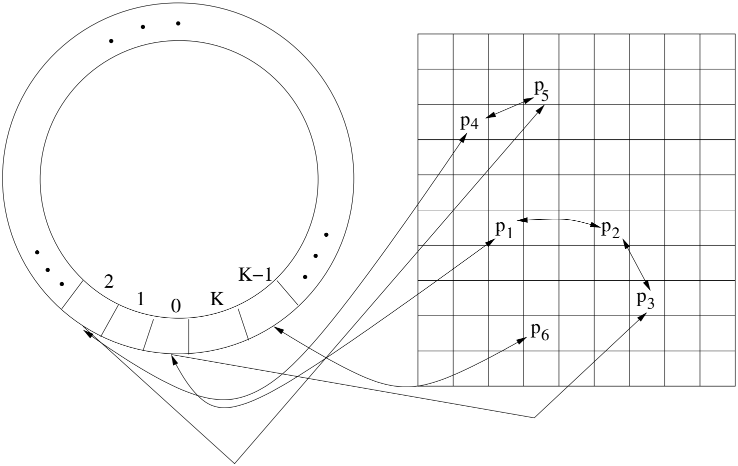
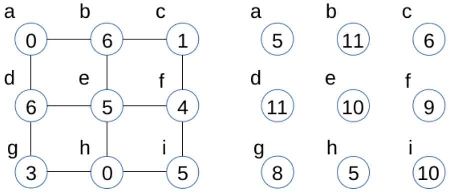
- The algorithm takes $O(|\mathcal{A}| + |\mathcal{N}|^2)$.
- If $|\mathcal{A}| \ll |\mathcal{N}|^2$ and Q is a binary heap, it takes $O(|\mathcal{N}| \log |\mathcal{N}|)$.
- If $|\mathcal{A}| \ll |\mathcal{N}|^2$ and $f(\pi_p \cdot \langle p, q \rangle) - f(\pi_p) \in [0, K]$, $K \ll |\mathcal{N}|$, it can take $O(|\mathcal{N}|)$ using bucket sort.



Nodes p are inserted in and removed from bucket $V(p)$ mod $K+1$ in $O(1)$.

Exercise

Let $\hat{I} = (D_I, I)$ be the image on the left, where the numbers indicate $I(t)$, $\mathcal{N} = D_I$ and $\mathcal{A}$ is defined by the four neighbors.



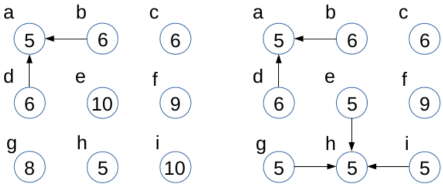
On the right, the trivial forest of the connectivity function

$$
\begin{aligned}
f(\langle t \rangle) &= I(t) + 5, \\
f(\pi_s \cdot \langle s, t \rangle) &= \max\{f(\pi_s), I(t)\}.
\end{aligned}
$$

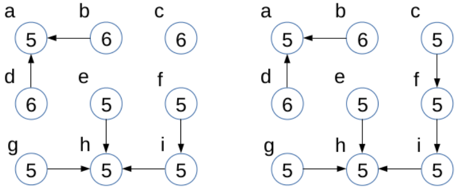
Can you tell which nodes will be in the root set $\mathcal{R}$? Change the algorithm to propagate a distinct label per optimum-path tree.

Subsequent iterations

After the first iteration (left) and second (right).



After the third (left) and from 4–9 (right).



Let's see Watershed.ipynb in notebooks.tar.gz

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