

# How to Update Documents *Verifiably* in Searchable Symmetric Encryption

Kaoru Kurosawa and Yasuhiro Ohtaki

Ibaraki University, Japan

# Outline

- (1) Introduction
- (2) Our (previous) **verifiable** SSE scheme
- (3) Extend it to a **dynamic** SSE scheme  
(but not yet fully verifiable)
- (4) **Verifiable dynamic** SSE scheme
- (5) Summary

# A Searchable Symmetric Encryption (SSE) scheme

- Consists of a **store** phase and a **search** phase
- Suppose that we have 5 documents and 3 keywords.

keyword	Documents
Austin	$D_1, D_3, D_5$
Boston	$D_2, D_4$
Washington	$D_1, D_2, D_4$

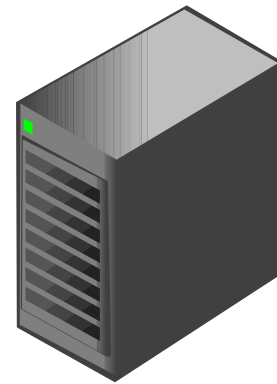
# In the **store** phase,

- A client stores encrypted files (or documents) and an index  $I$  (in an encrypted form) on a server.



Client

$E(D_1), \dots, E(D_5), I$



Server

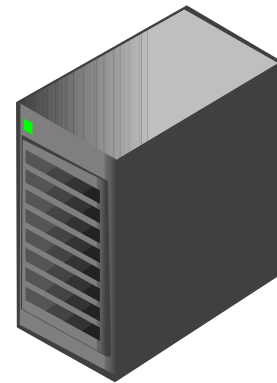
# In the **search** phase,

- the client sends an encrypted **keyword** to the server.



Client

$E(\text{Austin})$



Server

# The server somehow returns

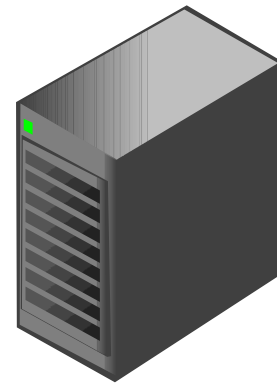
- the encrypted files  $E(D_1)$ ,  $E(D_3)$ ,  $E(D_5)$  which contain the keyword.



Client



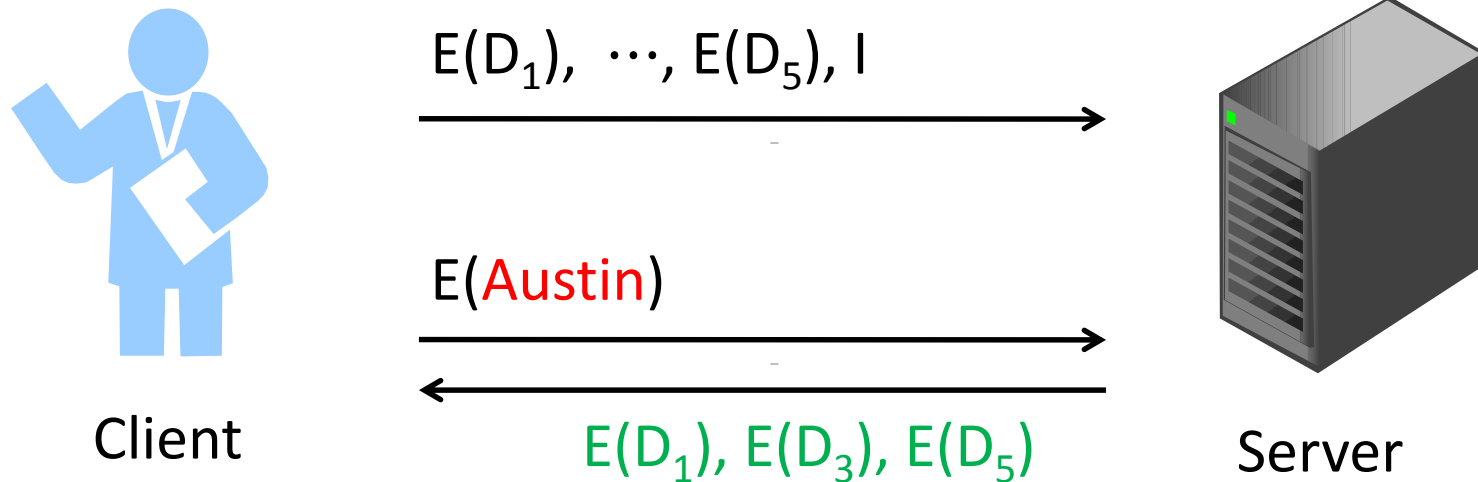
$E(D_1)$ ,  $E(D_3)$ ,  $E(D_5)$



Server

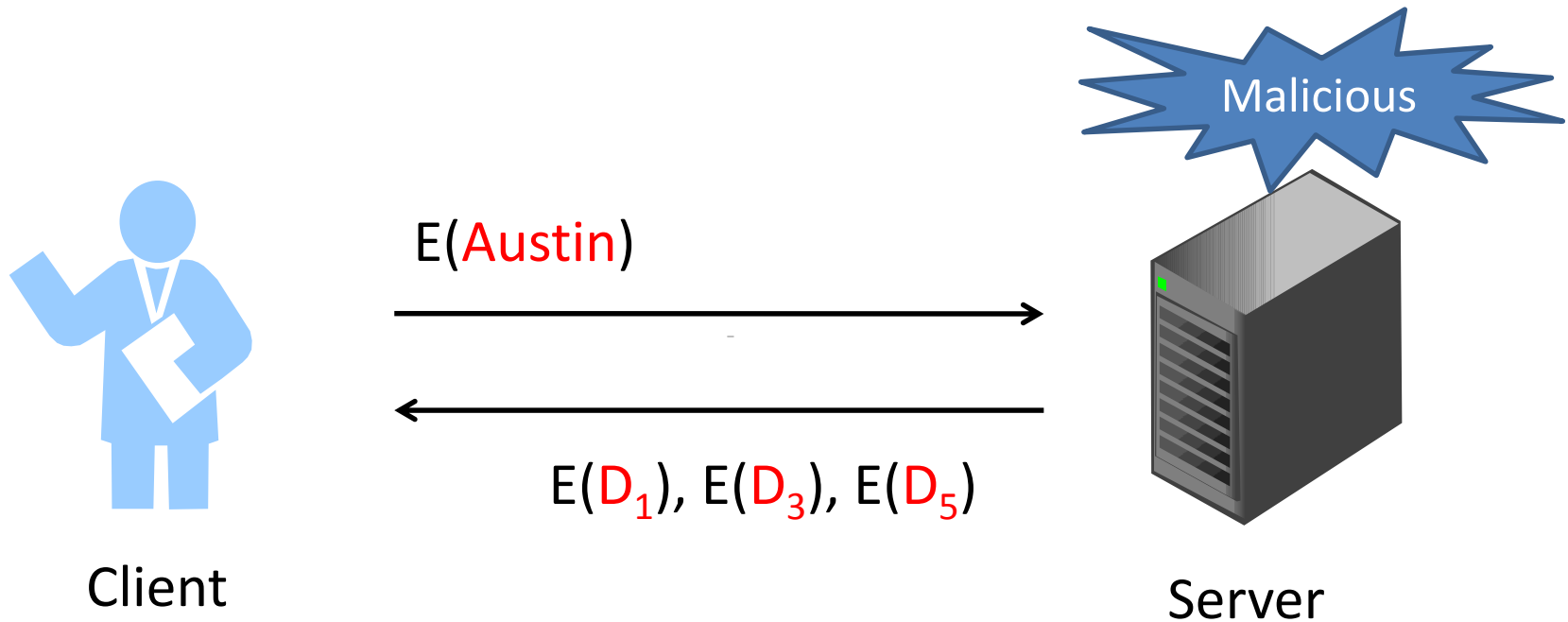
# So the client can

- retrieve **the encrypted files** which contain a specific **keyword**, keeping the **keyword** and document secret to the server.



# By Passive Attack

- A malicious server breaks the **privacy**.
- She tries to **find** the keyword and the documents.



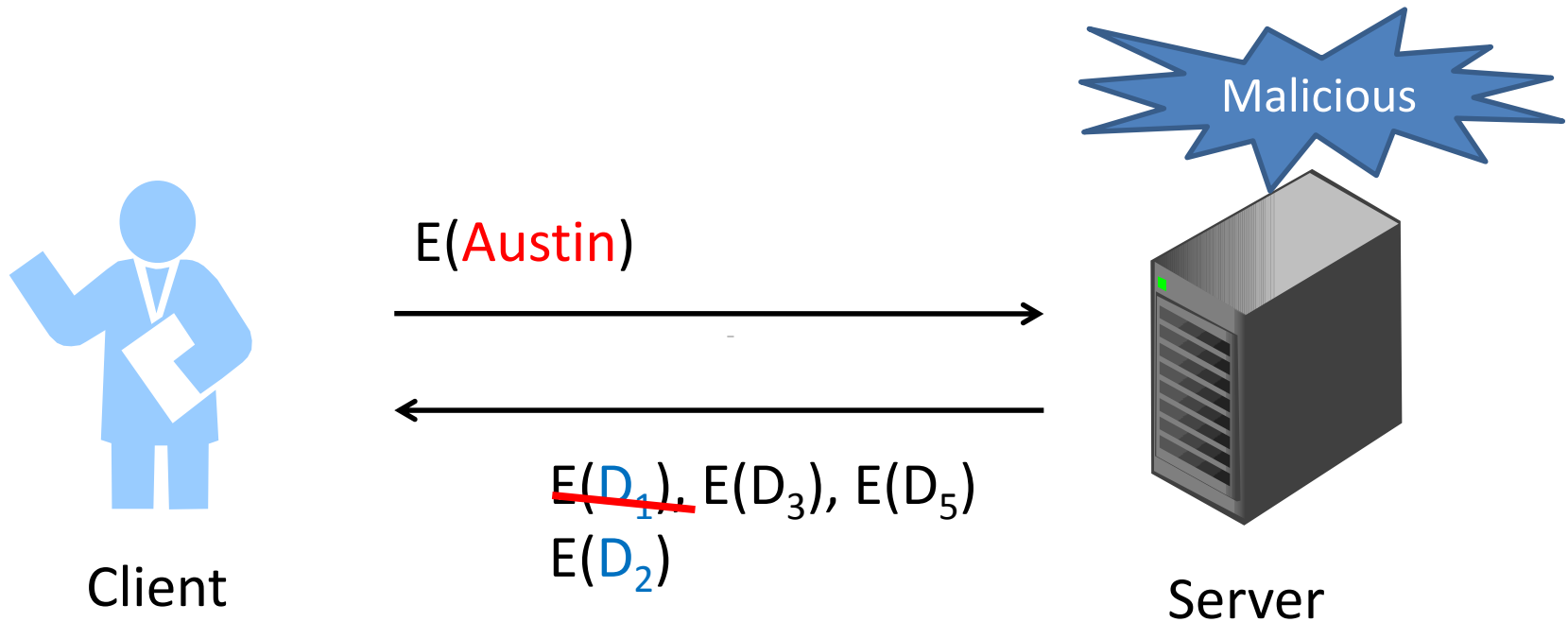


# The security against **passive attacks**

- has been studied by several researchers.
- Curtmola, Garay, Kamara and Ostrovsky showed a rigorous definition of security against **passive attacks**.
- They also gave a scheme which satisfies their definition.

# By Active Attack

- A malicious server breaks the **reliability**.
- She tries to **forge/delete** some files or **replace**  $E(D_1)$  with another (valid)  $E(D_2)$ .



# The security against **active attacks**

- has been studied by Kurosawa et al. (2011)
- They showed a definition of security against **active attacks**.
- They proposed a **verifiable** SSE scheme
  - the client can detect any cheating behavior of malicious server.
  - **UC-secure**

# Dynamic SSE scheme

- Kamara et al. showed a **dynamic** SSE scheme.
  - the client can **add** and **delete** documents.
  - secure against adaptive chosen keyword attacks.
- Subsequently, they showed a **parallel** and **dynamic** SSE scheme.
- However, these dynamic schemes are not verifiable.

# Our contribution

## Comparison with Previous Works

	Verifiability	Dynamism
Curtmola et al.	X	X
Kurosawa et al.	O	X
Kamara et al.	X	O
<b>This paper</b>	<b>O</b>	<b>O</b>

# Outline

- (1) Introduction
- (2) Our (previous) verifiable SSE**
- (3) Extend it to a dynamic SSE scheme  
(but not yet fully verifiable)
- (4) Verifiable dynamic SSE scheme
- (5) Summary

# Contents of the encrypted index



keyword	Documents
Austin	$D_1, D_3, D_5$
Boston	$D_2, D_4$
Washington	$D_1, D_2, D_4$

label	document ID	Authenticator
$t(\text{Austin})$	$(1, 3, 5)$	$\text{tag}_{\text{Austin}}$ $=\text{MAC}(t(\text{Austin}), (C_1, C_3, C_5))$
$t(\text{Boston})$	$(2, 4)$	$\text{tag}_{\text{Boston}}$ $=\text{MAC}(t(\text{Boston}), (C_2, C_4))$
$t(\text{Washington})$	$(1, 2, 4)$	$\text{tag}_{\text{Washington}}$ $=\text{MAC}(t(\text{Washington}), (C_1, C_2, C_4))$

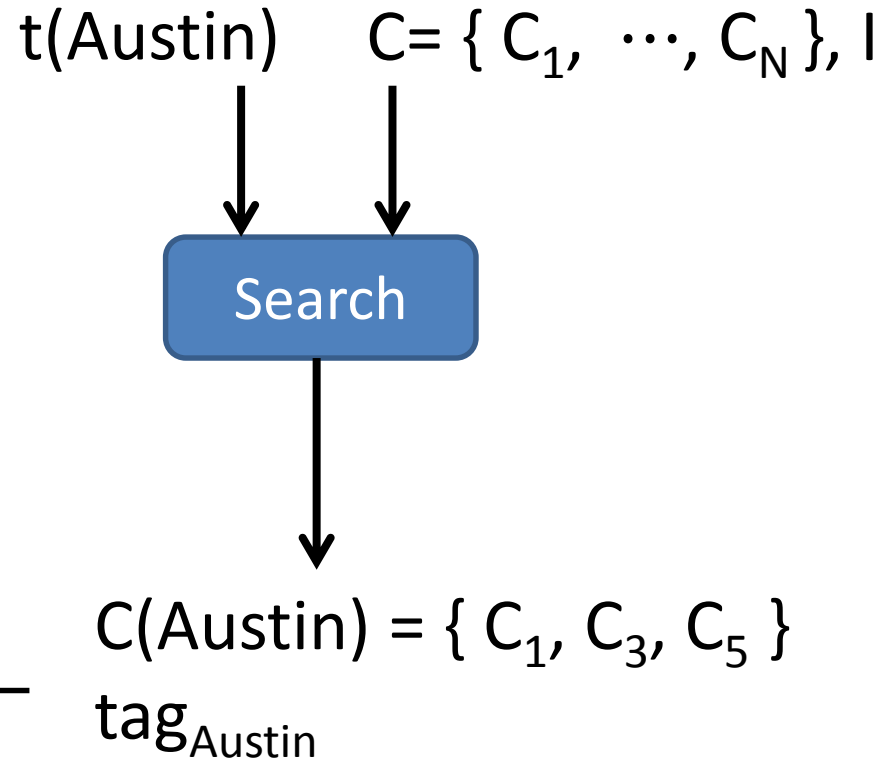
$t()$  : some trapdoor function

$\text{MAC}()$  : some Message Authentication Code

# In the search phase,

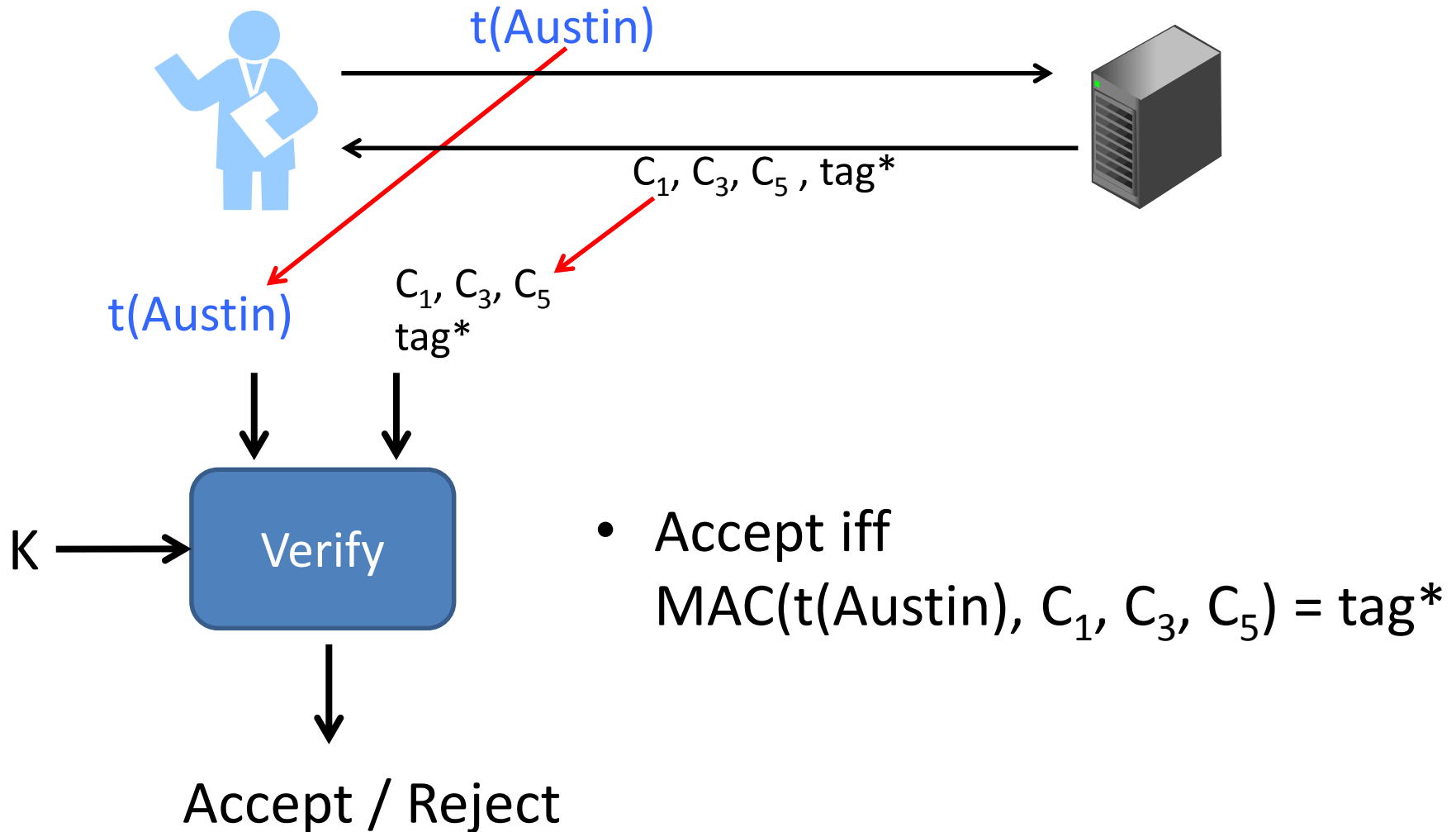
- The server receives  
 $t(\text{keyword})$   
and outputs  
 $C(\text{keyword})$   
 $\text{tag}_{\text{keyword}}$

and send them to the client.





# Verify algorithm



# Outline

- (1) Introduction
- (2) Our (previous) verifiable SSE
- (3) Extend it to a dynamic SSE scheme  
(but not yet fully verifiable)**
- (4) Verifiable dynamic SSE scheme
- (5) Summary

# Extending the previous scheme

- When we consider extending our previous scheme to a dynamic scheme, we noticed that the scheme cannot modify documents efficiently.

# Suppose the client wants to modify $C_1$ to $C'_1$

- The client must store  $C'_1$  and **two** updated authenticators to the server.

label	document ID	Authenticator
t(Austin)	(1, 3, 5)	$\text{tag}_{\text{Austin}}$ $=\text{MAC}(t(\text{Austin}), (C_1, C_3, C_5))$
t(Boston)	(2, 4)	$\text{tag}_{\text{Boston}}$ $=\text{MAC}(t(\text{Boston}), (C_2, C_4))$
t(Washington)	(1, 2, 4)	$\text{tag}_{\text{Washington}}$ $=\text{MAC}(t(\text{Washington}), (C_1, C_2, C_4))$

- If  $C_1$  includes more keywords, then the client must update more authenticators.

# To update efficiently

- Authenticate  
 $(t(\text{Austin}), (1, 3, 5))$   
and  
each  $(i, C_i)$   
separately.

	Authenticator for Document
1	$\text{MAC}(1, C_1)$
2	$\text{MAC}(2, C_2)$
3	$\text{MAC}(3, C_3)$
4	$\text{MAC}(4, C_4)$
...	...

label	doc ID	Authenticator for index
$t(\text{Austin})$	$(1, 3, 5)$	$\text{tag}_{\text{Austin}}$ $=\text{MAC}( t(\text{Austin}), (1, 3, 5) )$
$t(\text{Boston})$	$(2, 4)$	$\text{tag}_{\text{Boston}}$ $=\text{MAC}( t(\text{Boston}), (2, 4) )$
$t(\text{Washington})$	$(1, 2, 4)$	$\text{tag}_{\text{Washington}}$ $=\text{MAC}( t(\text{Washington}), (1, 2, 4) )$

# To update $C_1$ to $C'_1$

- the client has only to store just **one** authenticator on  $(1, C'_1)$  no matter how many keywords are included in  $C_1$ .

	Authenticator for Document
1	$MAC(1, C'_1)$
2	$MAC(2, C_2)$
3	$MAC(3, C_3)$
4	$MAC(4, C_4)$
...	...

label	doc ID	Authenticator for index
t(Austin)	(1, 3, 5)	$tag_{Austin} = MAC( t(Austin), (1, 3, 5) )$
t(Boston)	(2, 4)	$tag_{Boston} = MAC( t(Boston), (2, 4) )$
t(Washington)	(1, 2, 4)	$tag_{Washington} = MAC( t(Washington), (1, 2, 4) )$

# Deletion and Addition

- To *delete* a document  $C_2$  the client runs *update* with a special symbol  $C'_2 = \textit{delete}$ .
- To *add* a new document  $D_6$  which includes *Austin*, the client updates the authenticator  $\text{tag}_{\textit{Austin}} = \text{MAC}(t(\textit{Austin}), (1, 3, 5))$  to  $\text{tag}'_{\textit{Austin}} = \text{MAC}(t(\textit{Austin}), (1, 3, 5, 6))$ .

# However,

- This scheme is vulnerable against **replay attack**.
- A malicious server may return **old**

$(1, C_1, \text{MAC}(1, C_1))$

instead of the **latest**

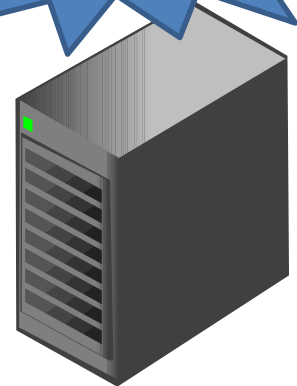
$(1, C'_1, \text{MAC}(1, C'_1))$ .



Client

$\text{tag}_{\text{Austin}}, (1, C'_1, x'_1), (3, C_3, x_3), (5, C_5, x_5)$

$(1, C_1, x_1)$



Server



# So the problem is

- How to timestamp on the *current/updated* ( $i, C_i$ )
- How to timestamp on the *current/updated* (label<sub>Austin</sub>, (1, 3, 5))

# We solved this problem by

- using an authentication scheme which possesses the timestamp functionality.
  - Merkle hash tree
  - Authenticated skip list
  - **RSA accumulator**

# These schemes

- Allows one to hash **a set of inputs**  
 $E = \{ x_1, x_2, \dots, x_n \}$   
into **one short accumulation value**  
 $\text{Acc}(E)$ .
- Infeasible to find a set  $E'$   
such that  $\text{Acc}(E) = \text{Acc}(E')$ ,  $E \neq E'$ .
- Able to construct a "witness"  $\square_j$   
that a given input  $x_j$  was incorporated into  $\text{Acc}(E)$ .

# RSA accumulator

- Let  $f$  be a function randomly chosen from a two-universal function family  
 $F = \{f_a : \{0,1\}^{3\lambda} \rightarrow \{0,1\}^\lambda\}$ .
- For any  $y \in \{0,1\}^\lambda$ ,  
we can compute a prime  $x \in \{0,1\}^{3\lambda}$   
such that  $f(x)=y$  by sampling  $O(\lambda^2)$  times  
with overwhelming probability.

# RSA accumulator

- Let  $p=2p'+1$  and  $q=2q'+1$  be two large primes such that  $p'$  and  $q'$  are also primes and  $|pq|>3\lambda$ .
- Let  $N=pq$ .
- Let  $QR_N=\{a \mid x^2 \pmod N \text{ for some } x \in Z_N^*\}$ .  
Then  $QR_N$  is a cyclic group of size  $(p-1)(q-1)/4$ .
- Let  $g$  be a generator of  $QR_N$ .

# RSA accumulator

- For a set  $E = \{y_1, y_2, \dots, y_n\}$  with  $y_i \in \{0,1\}^\lambda$ , RSA accumulator works as follows:
  - For each  $y_i$ , Alice chooses a prime  $x_i \in \{0,1\}^{3\lambda}$  randomly.  
Let  $prime(y_i)$  denote such a prime  $x_i$ .  
 $x_i = prime(y_i)$
  - Alice computes accumulated value of set  $E$  as
$$Acc(E) = g^{x_1 x_2 \dots x_n} \pmod N$$
and sends  $Acc(E)$  to Bob.

# RSA accumulator

- Later, Alice proves that  $y_j \in E$  to Bob as follows:
  - She computes
$$\pi_j = g^{x_1 x_2 \dots x_{j-1} x_{j+1} \dots x_n} \pmod N$$
$$x_j = \text{prime}(y_j)$$
and sends  $\pi_j$  and  $x_j$  to Bob
  - Bob verifies that
$$\text{Acc}(E) = \pi_j^{x_j} \pmod N$$

# To time stamp in our scheme

- We apply the RSA accumulator to the sets

$$E_c = \{ (i, C_i) \mid i=1, \dots, n \}$$

$$E_l = \{ (\text{label}_i, j, \overline{[\text{index}_i]_j}) \mid i=1, \dots, m, j=1, \dots, n \}$$

and compute their accumulated values

$$A_c = \text{Acc}(E_c)$$

$$A_l = \text{Acc}(E_l)$$

$[x]_j$  : j-th bit of x



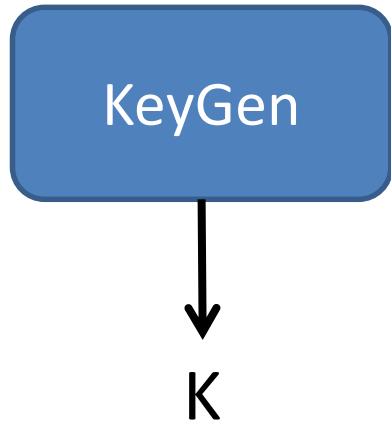
# Update and Verification

- The client updates  $A_C$  each time when he *modifies* or *deletes* a document.
- He also updates  $A_I$  each time when he *adds* a document.
- In the search phase, the client checks whether the server returned the valid latest ciphertexts, based on  $A_C$  and  $A_I$ .

# Outline

- (1) Introduction
- (2) Our (previous) verifiable SSE
- (3) Extend it to a dynamic SSE scheme  
(but not yet fully verifiable)
- (4) Verifiable dynamic SSE scheme**
- (5) Summary

# Key generation



The client first generates set of keys

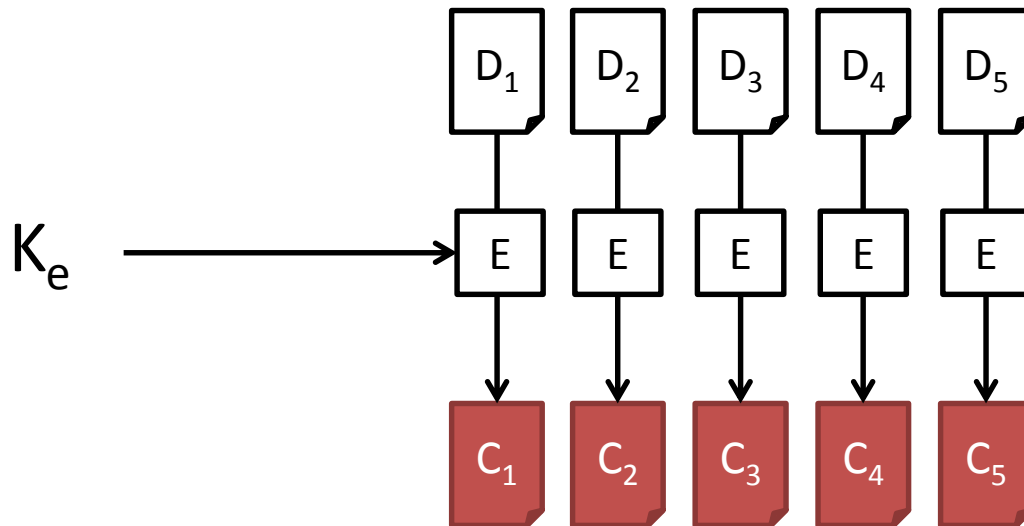
$K_e$  key of SKE

$K_0, K_1$  keys of PRF

and keeps them secret.

# Store phase: File encryption

The client first encrypts each document.

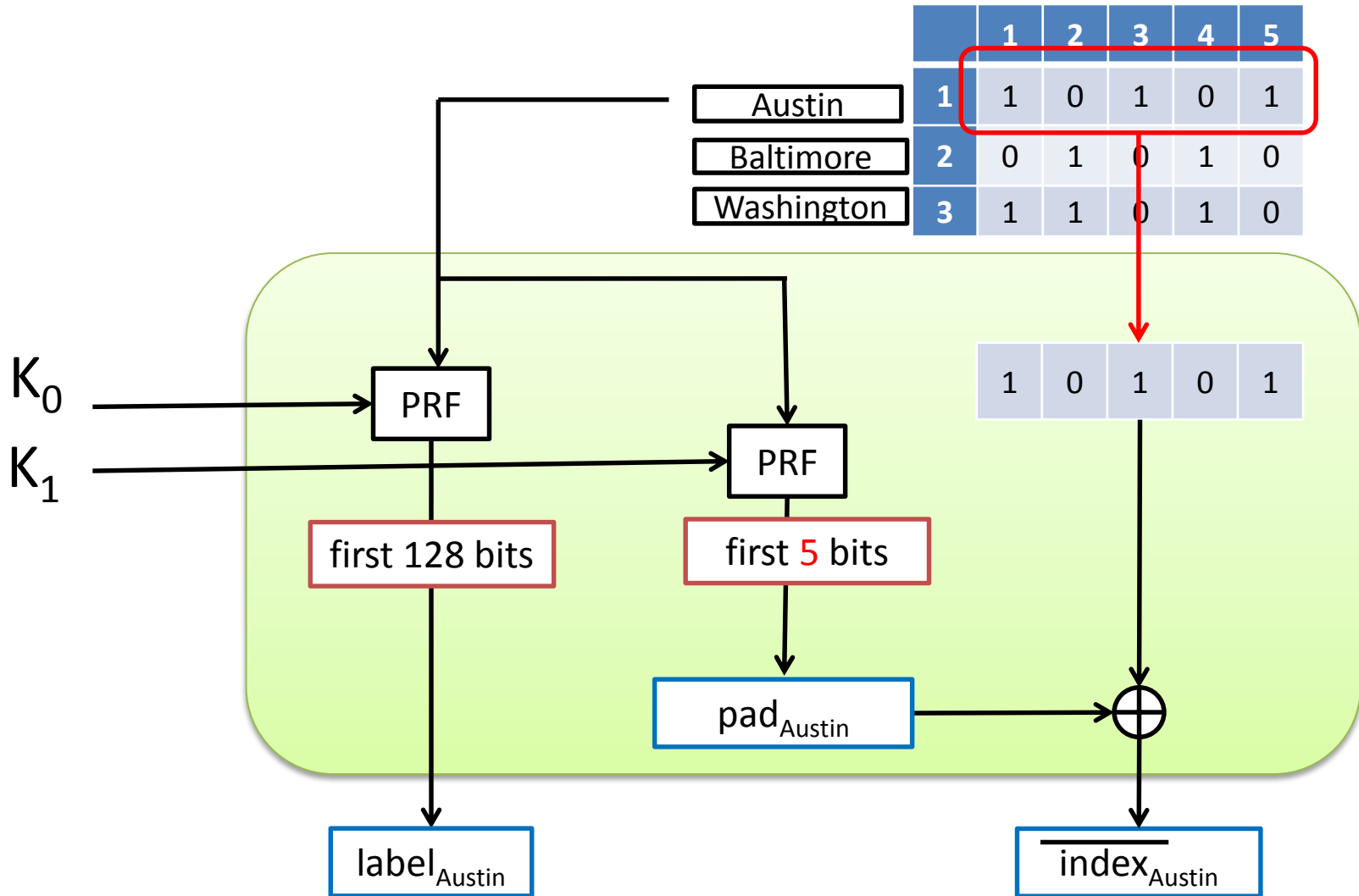


# Store phase: Index encryption

The plain index is an 5 x 3 binary matrix where  $e_{i,j} = 1$  if keyword<sub>i</sub> is contained in D<sub>j</sub>,  
0 otherwise

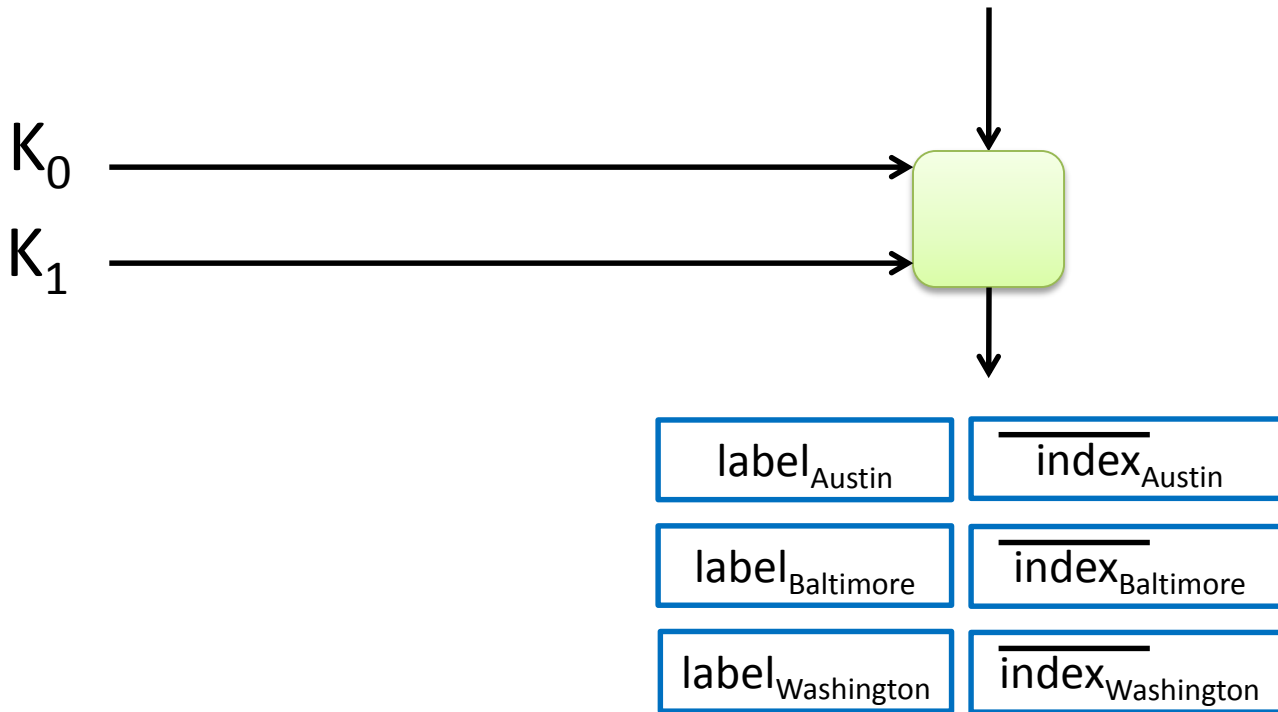
		D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>
		1	2	3	4	5
Austin	1	1	0	1	0	1
Baltimore	2	0	1	0	1	0
Washington	3	1	1	0	1	0

# Store phase: Index encryption



# Store phase: Index encryption

	1	2	3	4	5
Austin	1	0	1	0	1
Baltimore	0	1	0	1	0
Washington	1	1	0	1	0



# Store phase: Compute $A_C$

The client maps each document status into a prime number and computes  $A_C$

$$x_1 = \textit{prime}( H(1, H(C_1)) )$$

$$x_2 = \textit{prime}( H(2, H(C_2)) )$$

$$x_3 = \textit{prime}( H(3, H(C_3)) )$$

$$x_4 = \textit{prime}( H(4, H(C_4)) )$$

$$x_5 = \textit{prime}( H(5, H(C_5)) )$$

$H()$  : collision-resistant  
hash function

$$A_C = g^{x_1 x_2 x_3 x_4 x_5} \quad \text{mod } N$$



# Store phase: Compute $A_i$

The client also maps each element of the encrypted index to a prime number.

$$\begin{aligned} p_{1,1} &= \textit{prime}(\text{H}(\text{label}_{\text{Austin}}, 1, 0)) \\ p_{1,2} &= \textit{prime}(\text{H}(\text{label}_{\text{Austin}}, 2, 1)) \\ p_{1,3} &= \textit{prime}(\text{H}(\text{label}_{\text{Austin}}, 3, 1)) \\ p_{1,4} &= \textit{prime}(\text{H}(\text{label}_{\text{Austin}}, 4, 1)) \\ p_{1,5} &= \textit{prime}(\text{H}(\text{label}_{\text{Austin}}, 5, 0)) \end{aligned}$$

Document ID

index <sub>Austin</sub>					
	1	2	3	4	5
1	0	1	1	1	0

# Store phase: Compute $A_i$

Do same for all encrypted index element.

	1	2	3	4	5
1	0	1	1	1	0
2	1	0	0	1	1
3	1	0	1	0	0

$$p_{1,1} = \text{prime}(H(\text{label}_{\text{Austin}}, 1, 1))$$

$$p_{1,2} = \text{prime}(H(\text{label}_{\text{Austin}}, 1, 0)) \quad p_{2,1} = \text{prime}(H(\text{label}_{\text{Baltimore}}, 1, 0))$$

$$p_{1,3} = \text{prime}(H(\text{label}_{\text{Austin}}, 2, 1)) \quad p_{2,2} = \text{prime}(H(\text{label}_{\text{Baltimore}}, 2, 0))$$

$$p_{1,4} = \text{prime}(H(\text{label}_{\text{Austin}}, 2, 0)) \quad p_{2,3} = \text{prime}(H(\text{label}_{\text{Baltimore}}, 3, 1)) \quad p_{3,1} = \text{prime}(H(\text{label}_{\text{Washington}}, 1, 1))$$

$$p_{1,5} = \text{prime}(H(\text{label}_{\text{Austin}}, 3, 1)) \quad p_{2,4} = \text{prime}(H(\text{label}_{\text{Baltimore}}, 3, 0)) \quad p_{3,2} = \text{prime}(H(\text{label}_{\text{Washington}}, 2, 0))$$

$$p_{2,5} = \text{prime}(H(\text{label}_{\text{Baltimore}}, 4, 1)) \quad p_{3,3} = \text{prime}(H(\text{label}_{\text{Washington}}, 3, 1))$$

$$p_{3,4} = \text{prime}(H(\text{label}_{\text{Washington}}, 4, 0))$$

$$p_{3,5} = \text{prime}(H(\text{label}_{\text{Washington}}, 5, 0))$$

# Store phase: Compute $A_i$

The client computes  $A_i$

$$A_i = g^{p_{1,1}p_{1,2}\cdots p_{1,5} p_{2,1}\cdots p_{2,5} p_{3,1}\cdots p_{3,5}} \text{ mod } N$$

The client keeps  $n(=5)$ ,  $A_i$  and  $A_c$ .

# Store phase: Store data

$C =$

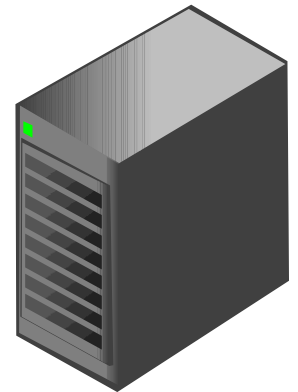
- $(1, C_1)$
- $(2, C_2)$
- $(3, C_3)$
- $(4, C_4)$
- $(5, C_5)$

$I =$

- $(\text{label}_{\text{Austin}}, \overline{\text{index}}_{\text{Austin}})$
- $(\text{label}_{\text{Baltimore}}, \overline{\text{index}}_{\text{Baltimore}})$
- $(\text{label}_{\text{Washington}}, \overline{\text{index}}_{\text{Washington}})$



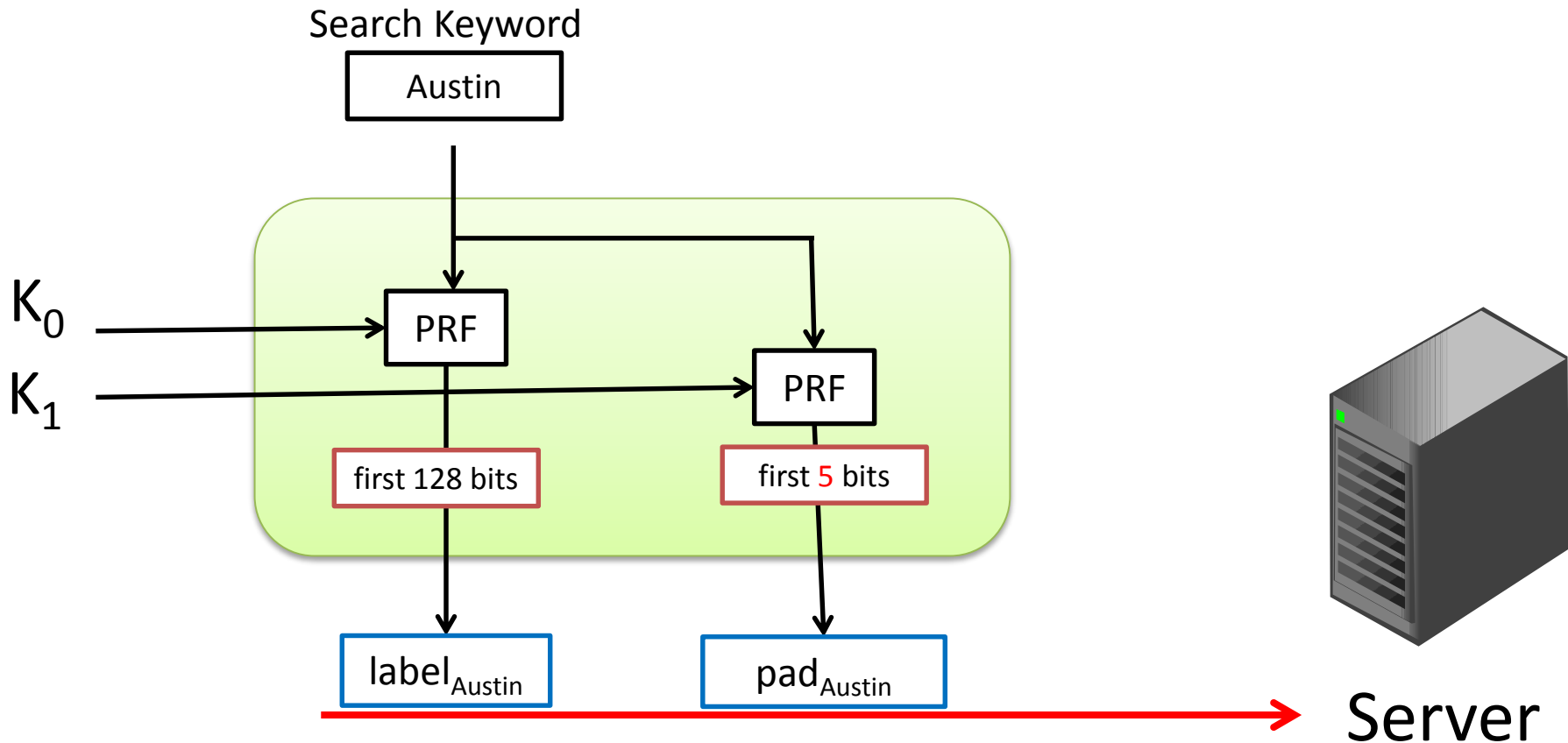
Client



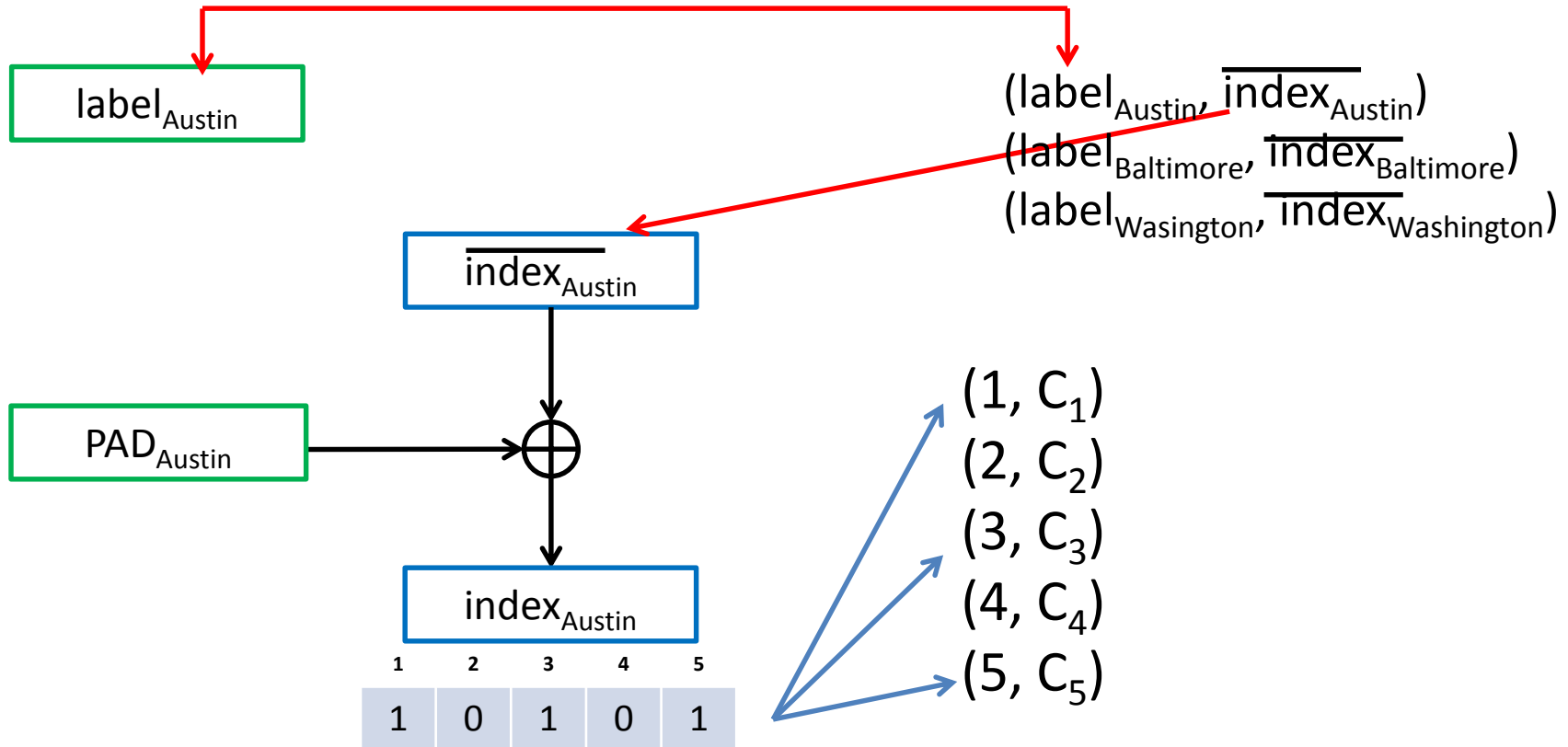
Server

# Search phase: Keyword encryption

The client computes the label and pad for the keyword.

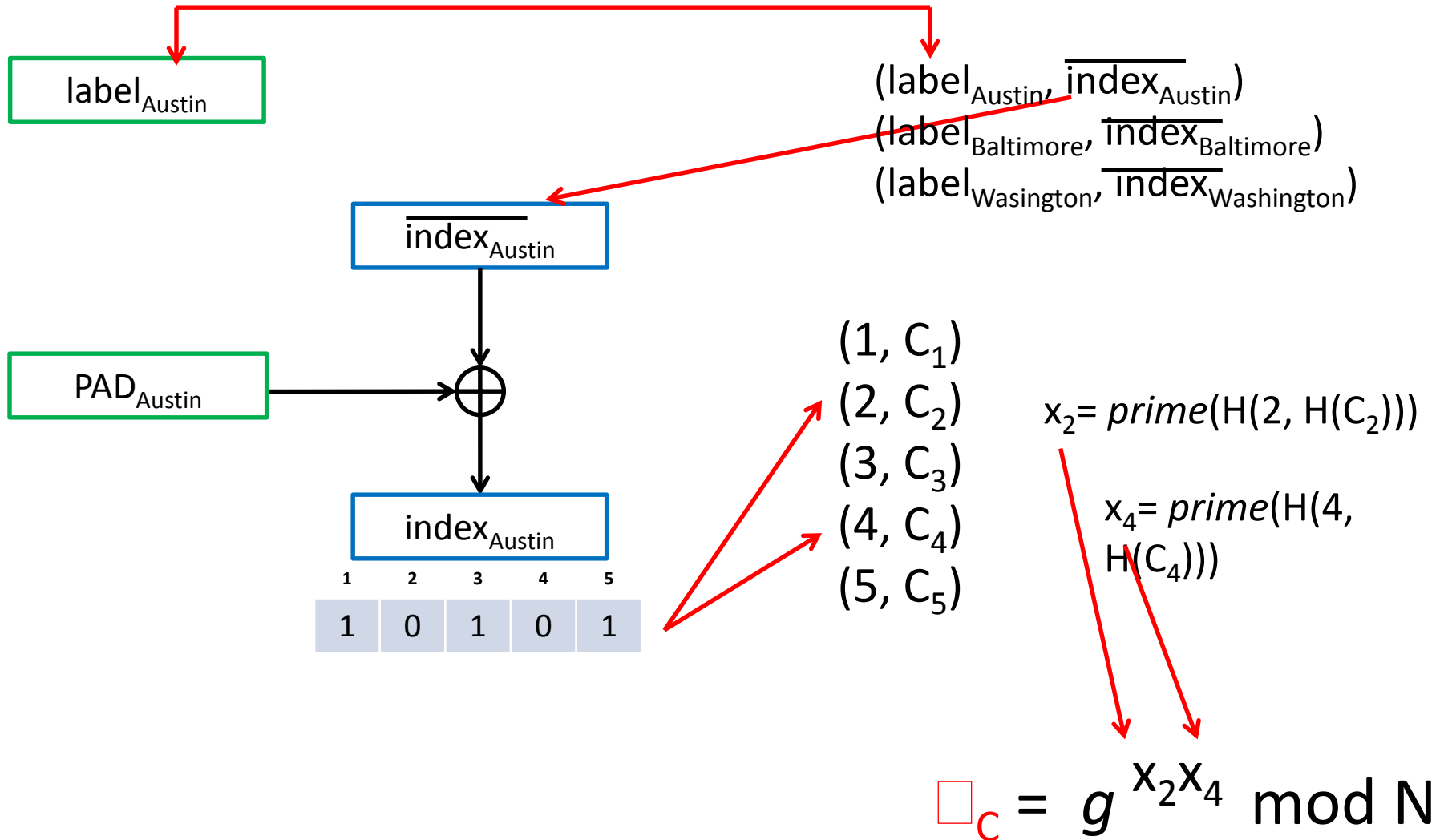


# Search phase: Find $C(\text{austin})$



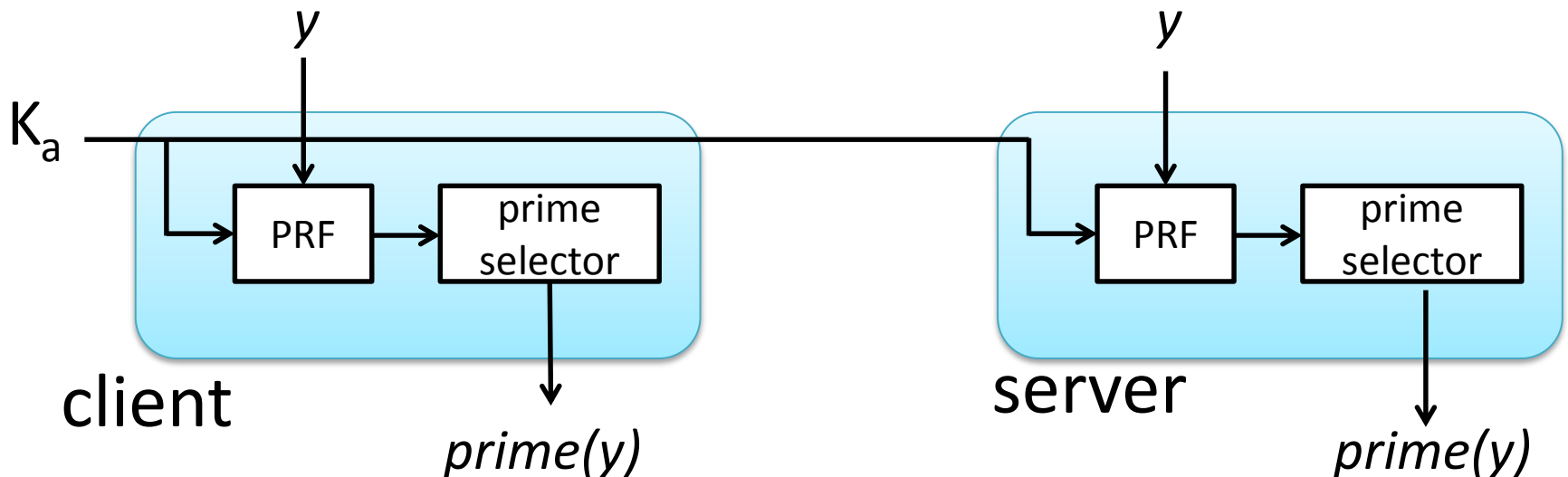
$$C(\text{Austin}) = (1, C_1), (3, C_3), (5, C_5)$$

# Search phase: Compute $\square_C$



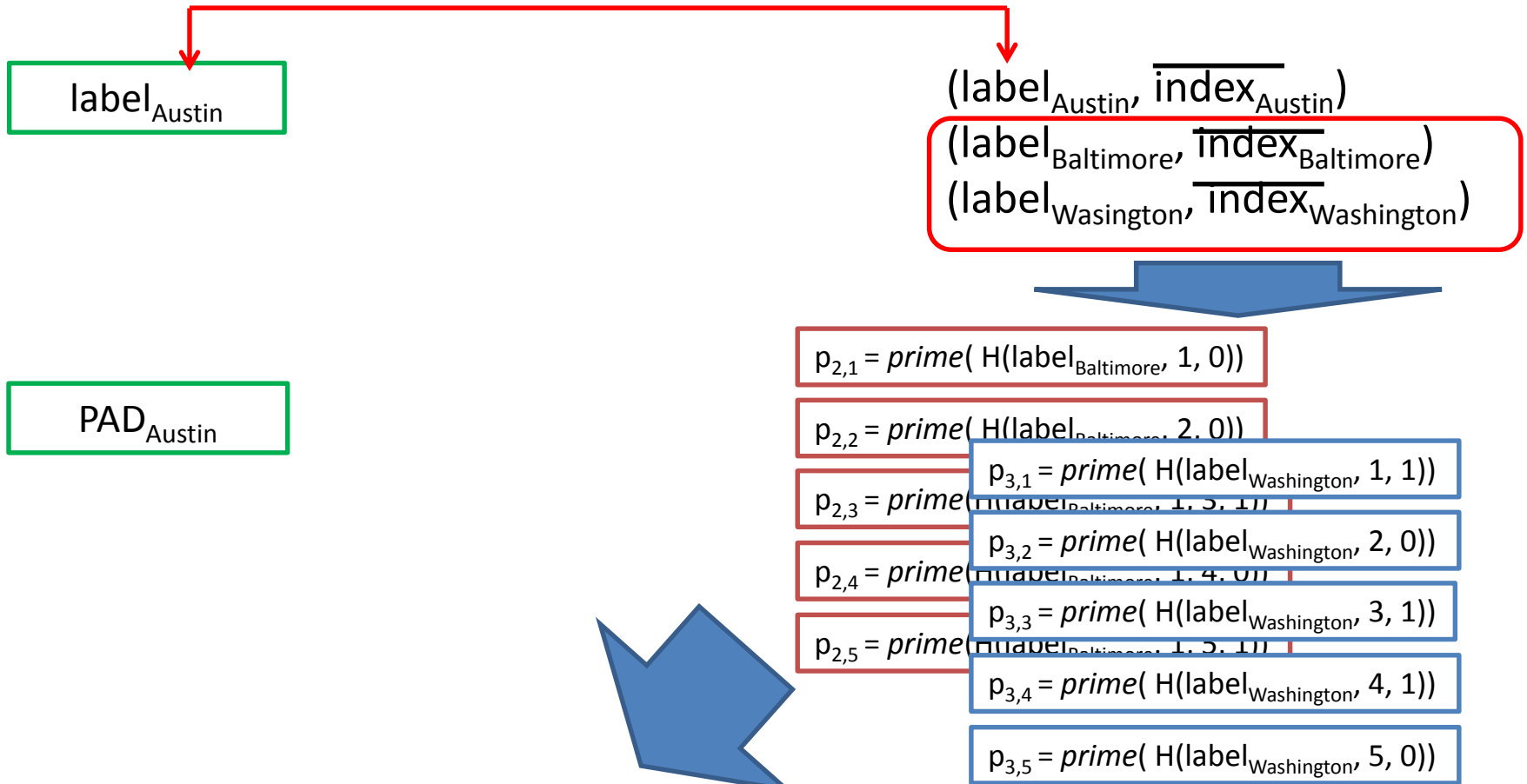
# How can the server calculate *prime()* locally?

- *prime(y)* should be a prime number chosen *randomly* by the client.
- In our scheme, we use PRF with a key  $K_a$  chosen randomly by the client .





# Search phase : Compute $\square_I$



$$\square_I = \text{gg } p_{2,1} \cdots p_{2,5} p_{3,1} \cdots p_{3,5} \text{ mod } N$$

# Search phase : Sending the results

The server send the encrypted documents and two witnesses to the client.

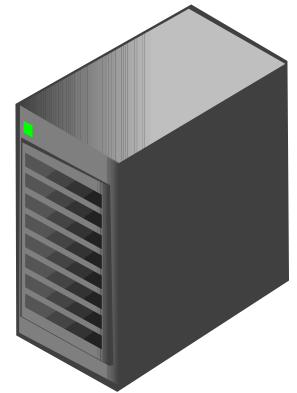


Client

label<sub>Austin</sub>    PAD<sub>Austin</sub>

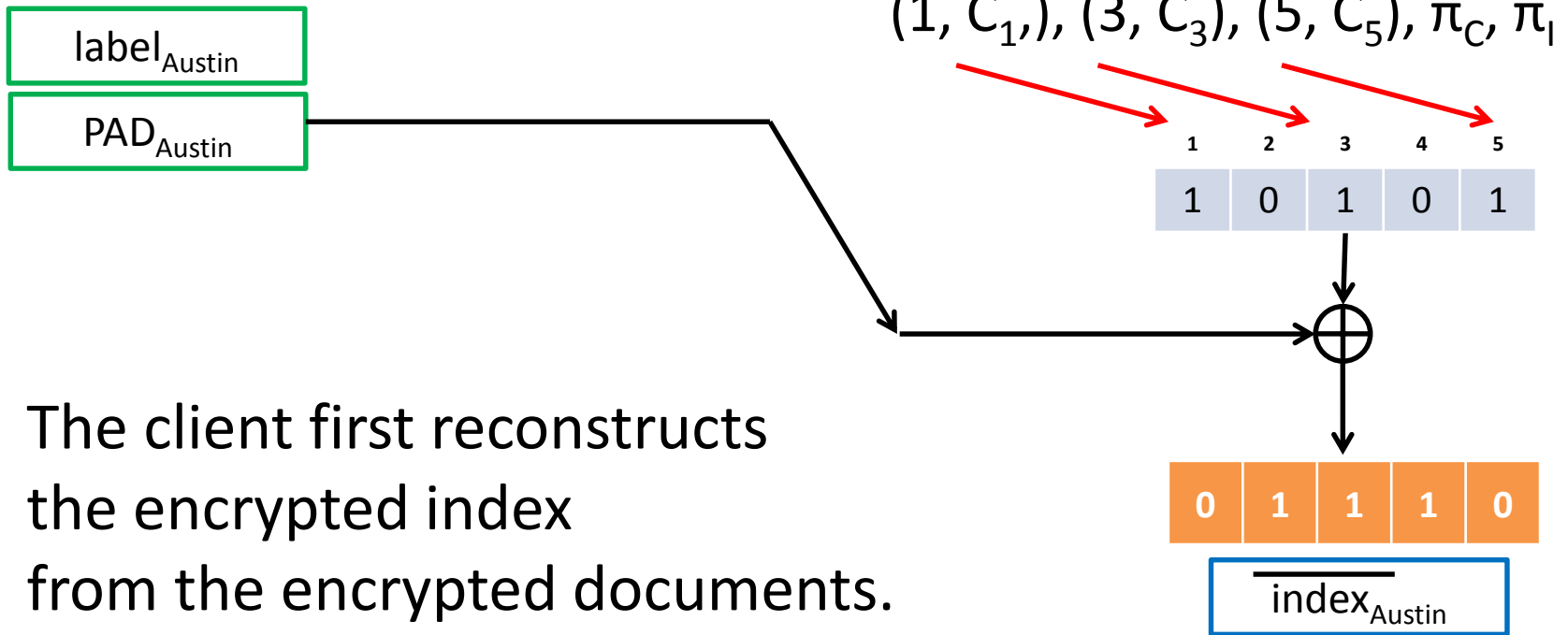


$(1, C_1), (3, C_3), (5, C_5), \square_C, \square_I$



Server

# Search phase: Verify (1/4)

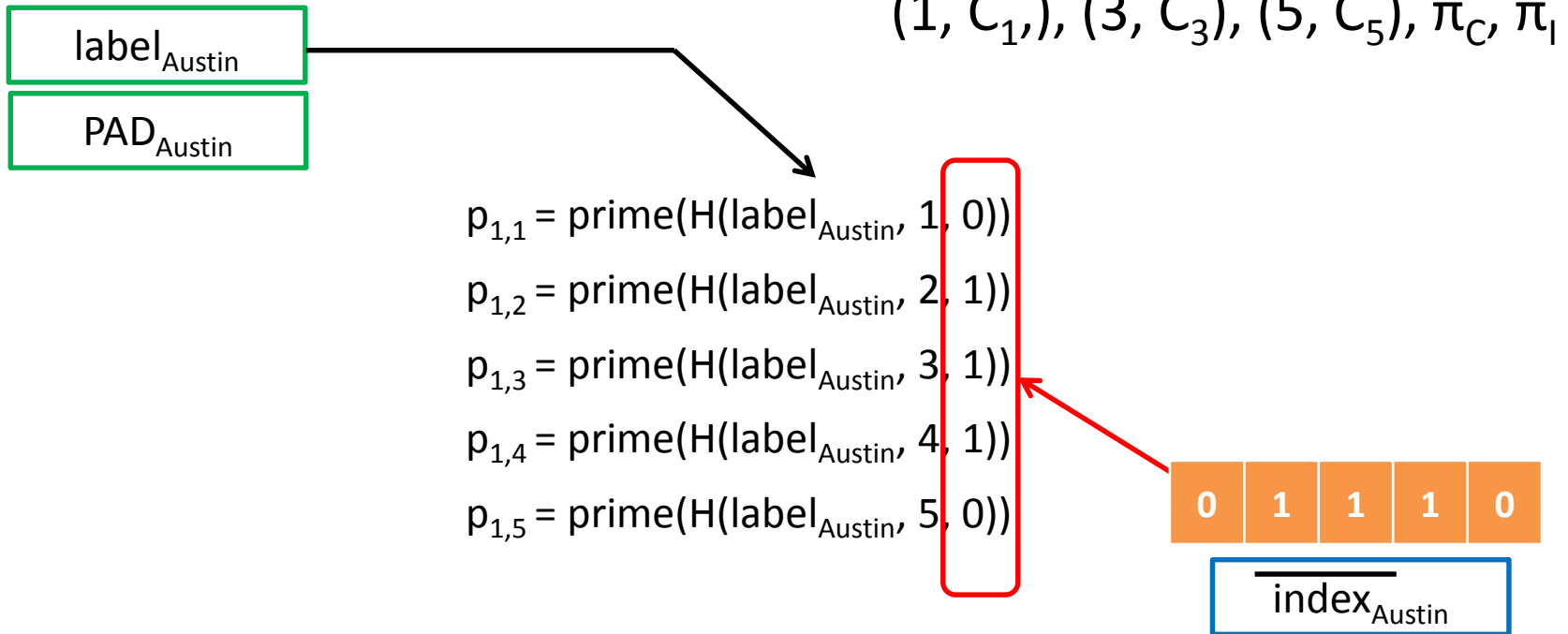


The client first reconstructs the encrypted index from the encrypted documents.

$A_C$

$A_I$

# Search phase: Verify (2/4)



$A_C$

$A_I$

Then he can compute the primes for the index elements related to the keyword.

# Search phase: Verify (3/4)

label<sub>Austin</sub>

PAD<sub>Austin</sub>

$(1, C_1), (3, C_3), (5, C_5), \pi_C, \pi_I$

$$p_{1,1} = \text{prime}(H(\text{label}_{\text{Austin}}, 1, 1))$$

$$p_{1,2} = \text{prime}(H(\text{label}_{\text{Austin}}, 2, 0))$$

$$p_{1,3} = \text{prime}(H(\text{label}_{\text{Austin}}, 3, 1))$$

$$p_{1,4} = \text{prime}(H(\text{label}_{\text{Austin}}, 4, 0))$$

$$p_{1,5} = \text{prime}(H(\text{label}_{\text{Austin}}, 5, 1))$$

0 1 1 1 0

index<sub>Austin</sub>

$A_C$

$A_I$

$$A_I = \square \mid p_{1,1} \cdots p_{1,5}$$

mod N



# Search phase: Verify (4/4)

label<sub>Austin</sub>

PAD<sub>Austin</sub>

$(1, C_1), (3, C_3), (5, C_5), \pi_C, \pi_I$

$x_1 = \text{prime}(H(1, H(C_1)))$

$x_3 = \text{prime}(H(3, H(C_3)))$

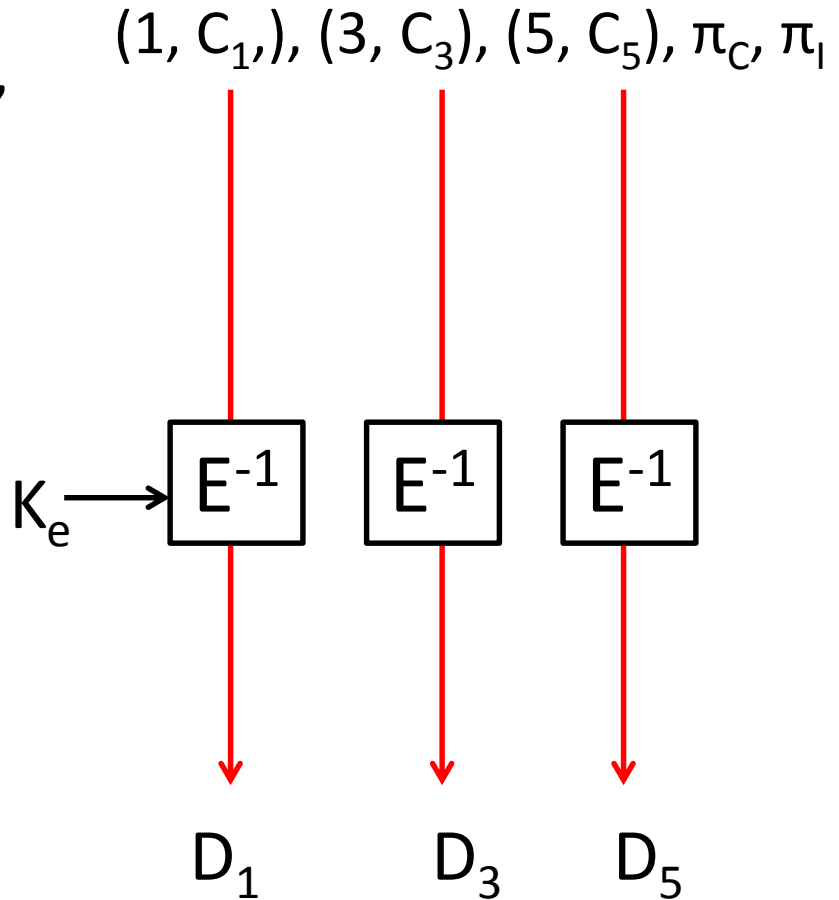
$x_5 = \text{prime}(H(5, H(C_5)))$

$$A_C \leftarrow A_C = \pi_C^{x_1 x_3 x_5} \pmod N$$

$A_I$

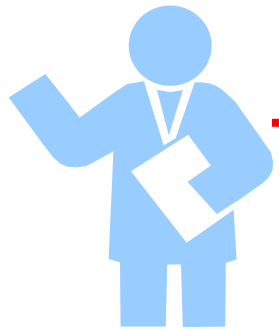
# Search phase : Decrypt

- If all the checks succeed, then the client decrypts and outputs the documents.
- otherwise outputs *reject*.



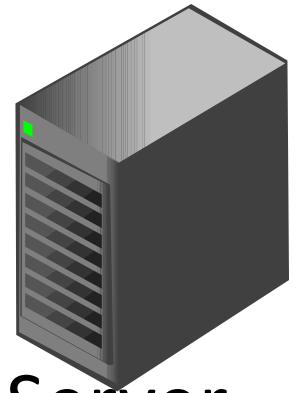
# How to modify $C_2$ to $C'_2$ (1/5)

- the client sends  $(2, C'_2)$  to the server.



Client

$(2, C'_2)$



Server



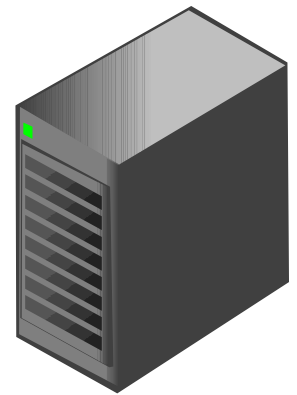
# How to modify $C_2$ to $C'_2$ (2/5)

- the server computes  $\pi_C$ .



Client

$(2, C'_2)$



Server

$$x_1 = \text{prime}(H(1, H(C_1)))$$

$$x_3 = \text{prime}(H(3, H(C_3)))$$

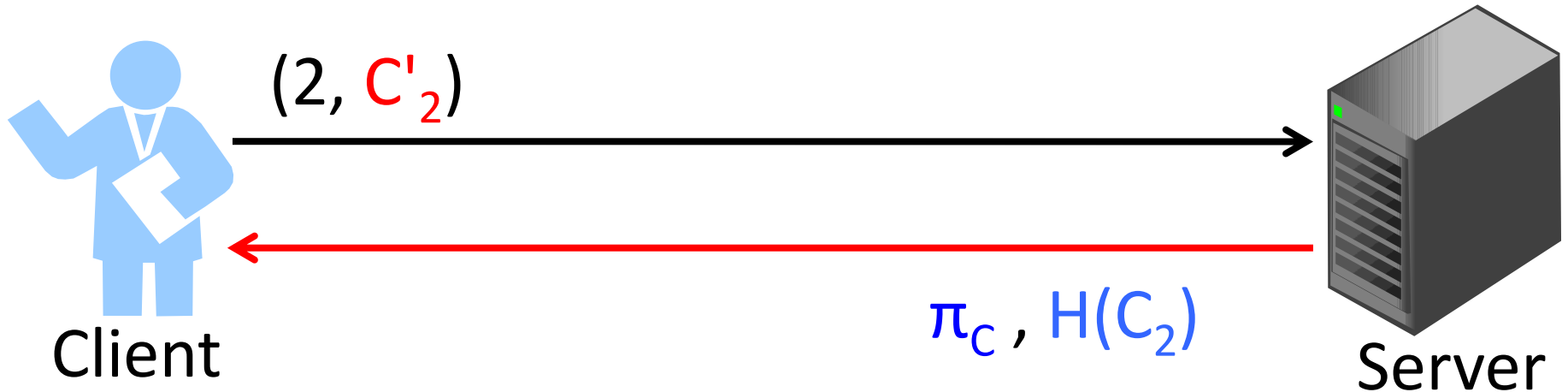
$$x_4 = \text{prime}(H(4, H(C_4)))$$

$$x_5 = \text{prime}(H(5, H(C_5)))$$

$$\pi_C = g^{x_1 x_3 x_4 x_5} \pmod N$$

# How to modify $C_2$ to $C'_2$ (3/5)

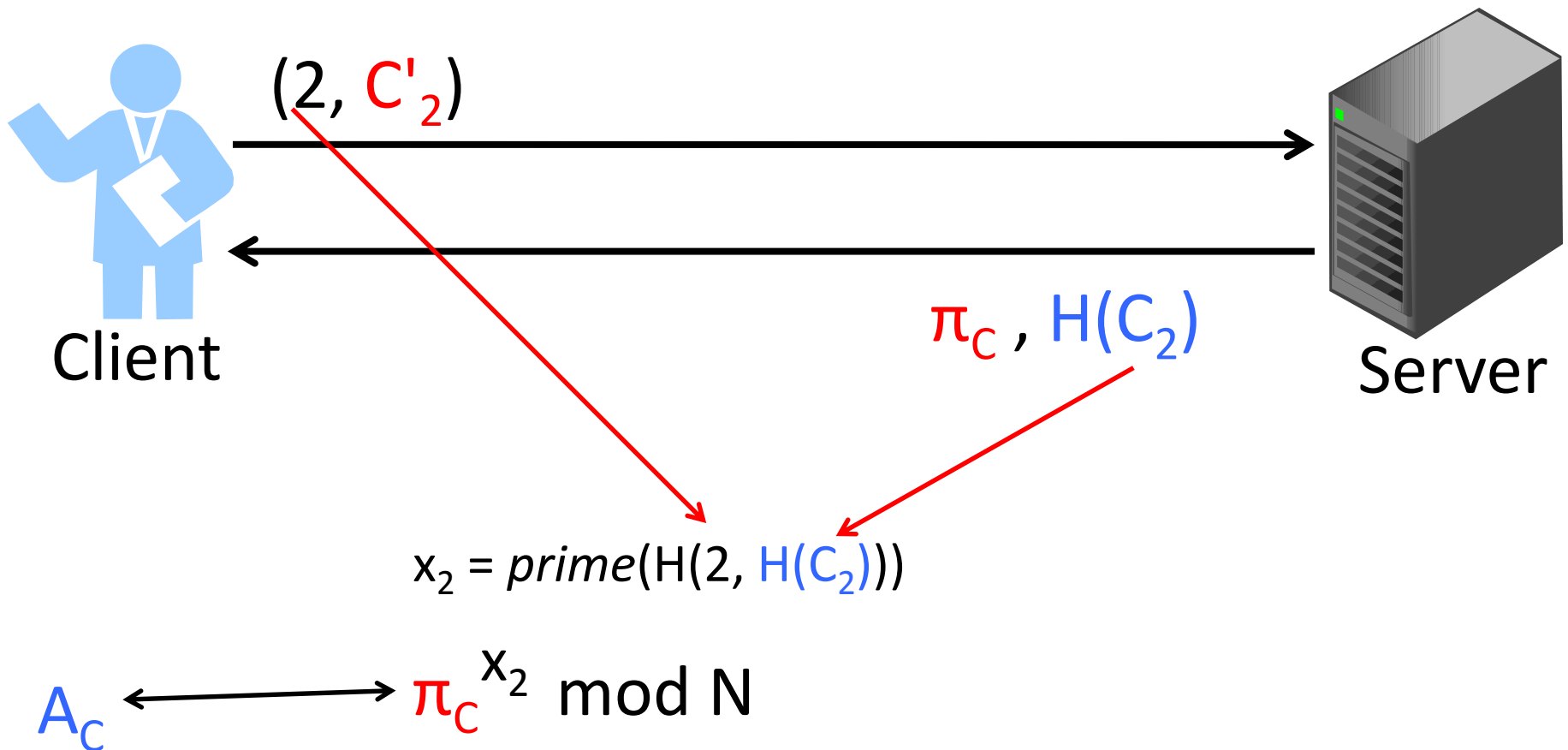
- the server returns  $\pi_C$  and  $H(C_2)$  and then updates  $(2, C_2)$  to  $(2, C'_2)$ .



$$\pi_C = g^{x_1 x_3 x_4 x_5} \text{ mod } N$$

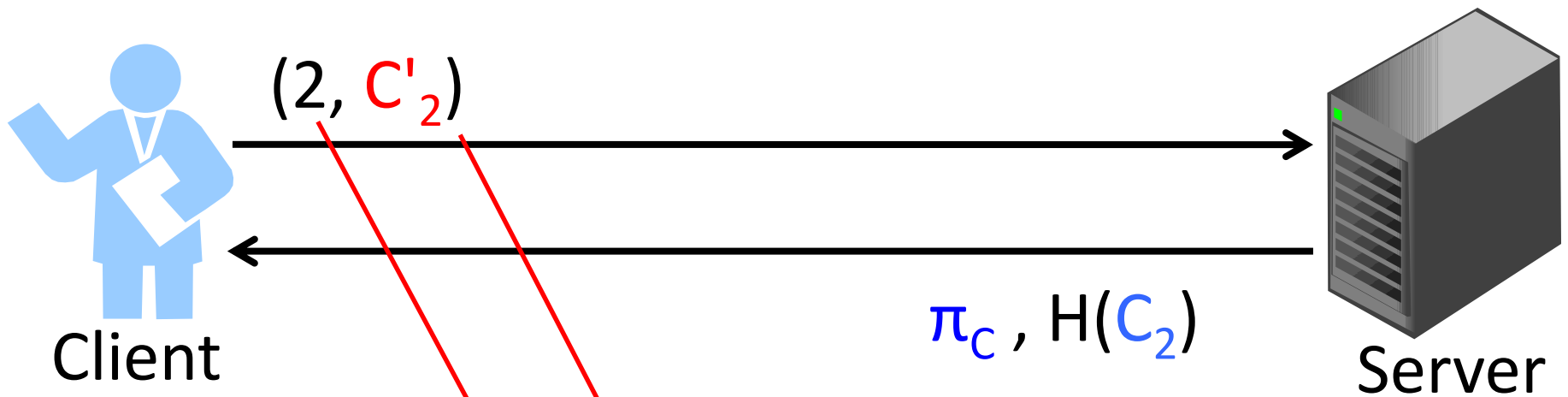
# How to modify $C_2$ to $C'_2$ (4/5)

- the client verifies the latest  $C_2$  the server had



# How to modify $C_2$ to $C'_2$ (5/5)

- and then updates  $A_c$  to  $A'_c$

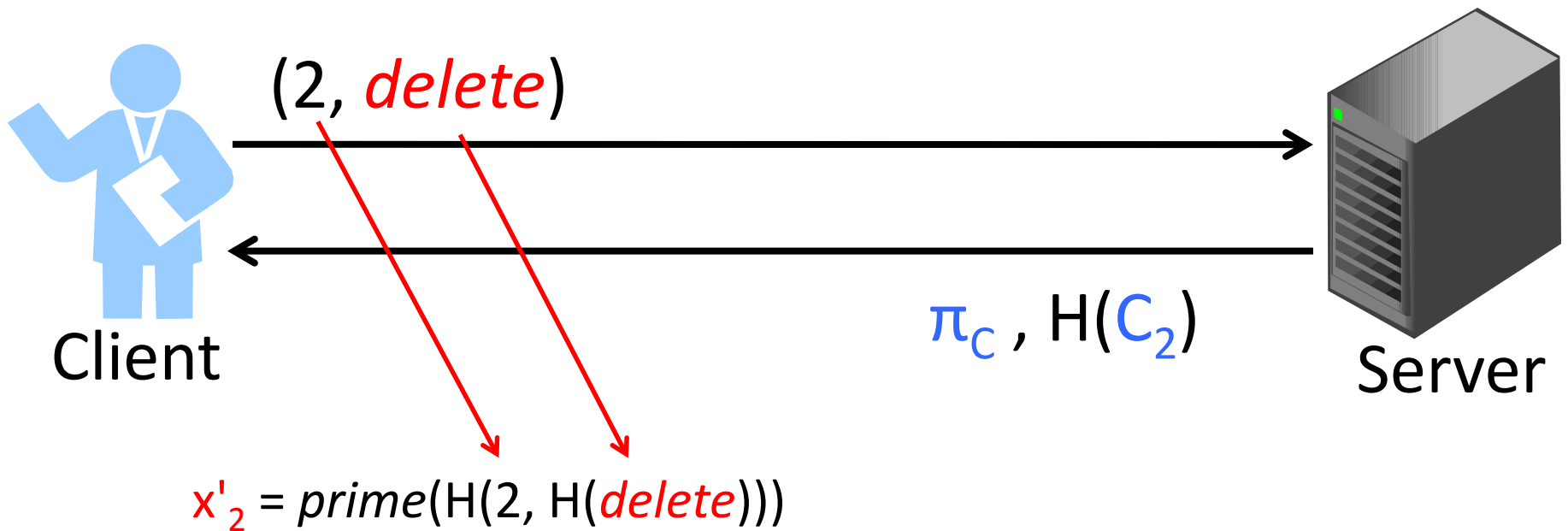


$$x'_2 = \text{prime}(H(2, H(C'_2)))$$

$$A'_c \leftarrow \begin{aligned} A'_c &= \pi_c^{x'_2} \text{ mod } \\ &= g^{x_1 x'_2 x_3 x_4 x_5} \text{ mod } N \end{aligned}$$

# How to *delete* $C_2$

- Do the *modify* with  $C'_2 = \textit{delete}$ .



$$A'_C \longleftarrow A'_C = \pi_C x'_2 \bmod N$$

# To *add* a new document $D_6$ (1/5)

$$C_6 = E_{Ke}(D_6)$$

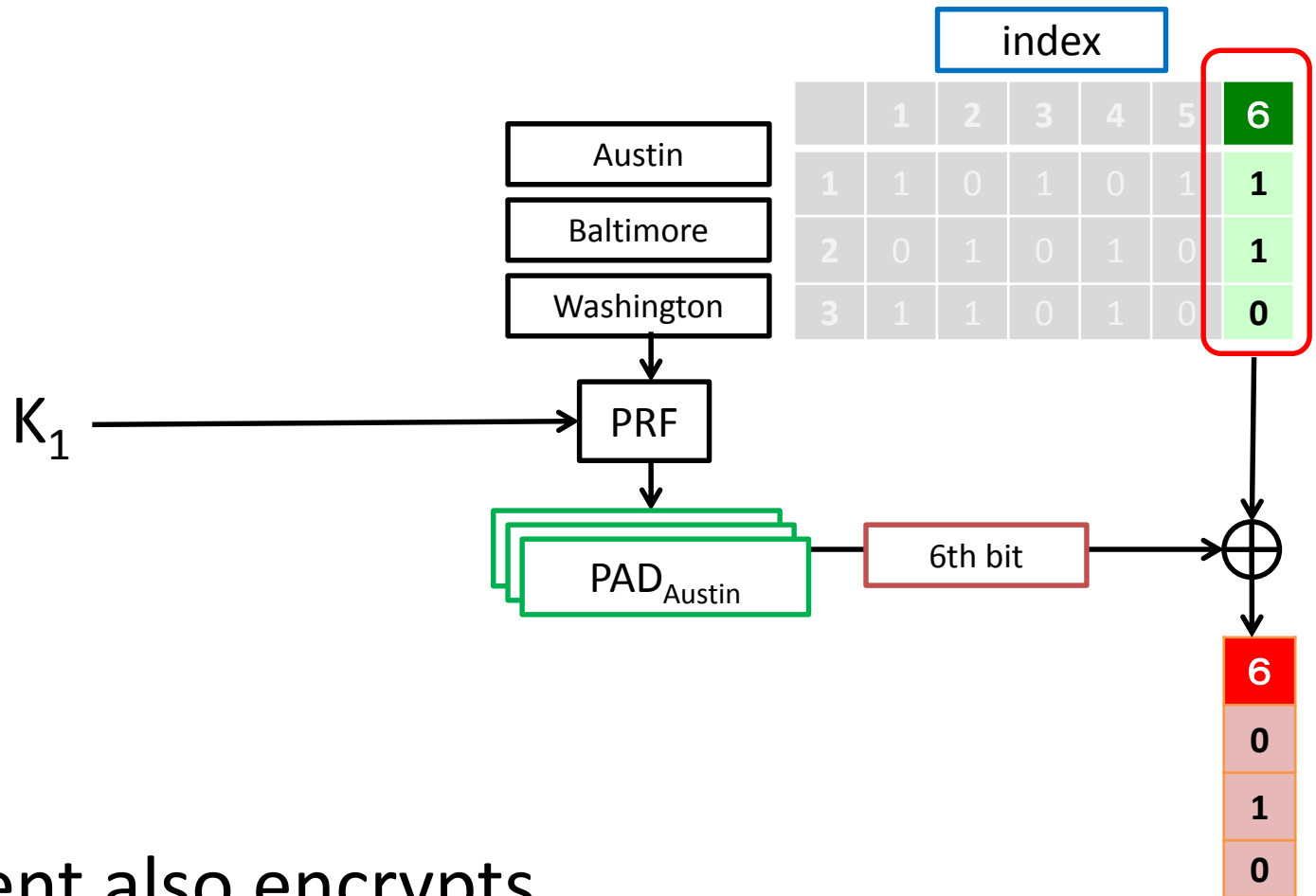
	index					
	1	2	3	4	5	6
Austin	1	0	1	0	1	1
Baltimore	2	0	1	0	1	1
Washington	3	1	1	0	1	0

$$x_6 = \text{prime}(H(6, H(C_6)))$$

$$A'_c \longleftarrow A'_c = A_c^{x_6} \pmod N$$

- The client encrypts the document and assign a new prime  $x_6$  and updates  $A_c$  to  $A'_c$ .

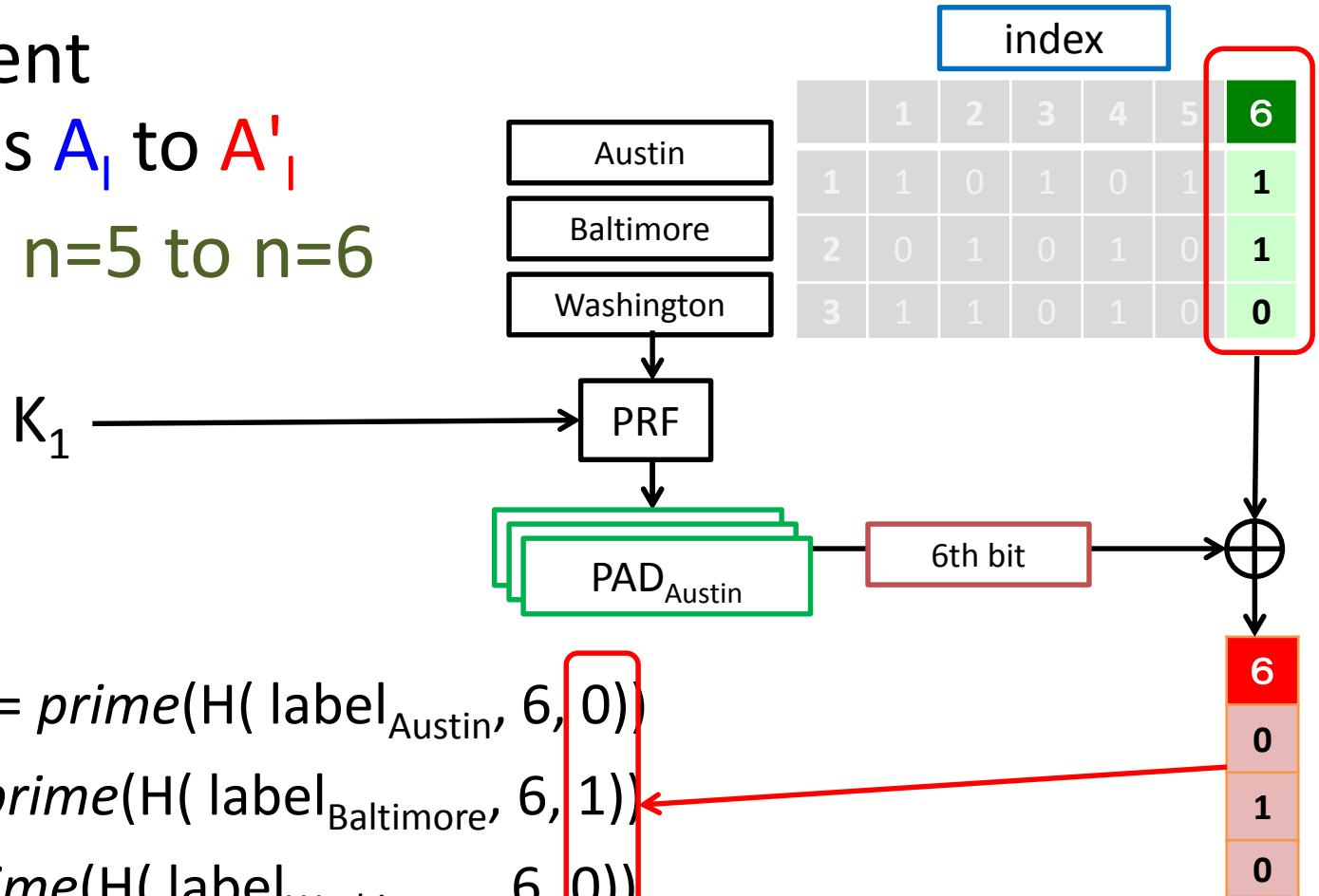
# To *add* a new document $D_6$ (2/5)



- The client also encrypts the index elements related to the new document.

# To *add* a new document $D_6$ (3/5)

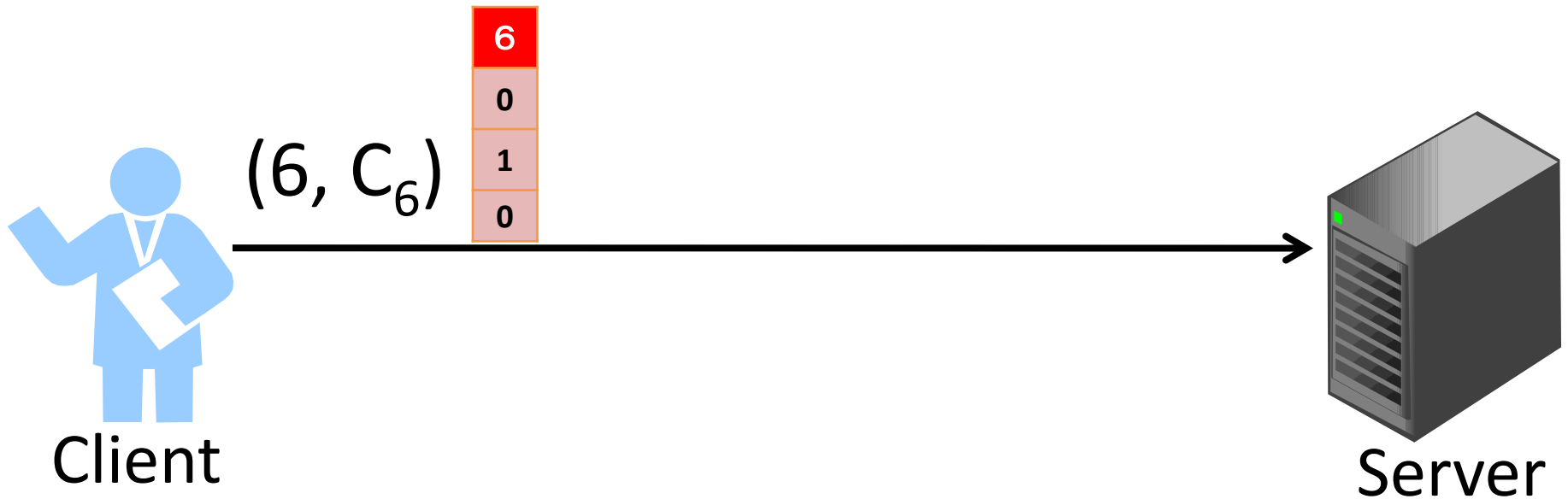
- The client updates  $A_i$  to  $A'_i$
- update  $n=5$  to  $n=6$



$$A'_i = A_i \cdot p_{1,6} p_{2,6} p_{3,6} \pmod N$$



# To *add* a new document $D_6$ (4/5)



- Finally the client sends the encrypted document and the corresponding index.

	1	2	3	4	5
1	0	1	1	1	0
2	1	0	0	1	1
3	1	0	1	0	0

index

# To *add* a new document $D_6$ (5/5)



- The server stores the encrypted document and append the index to the 6<sup>th</sup>-column.

	1	2	3	4	5	6
1	0	1	1	1	0	0
2	1	0	0	1	1	1
3	1	0	1	0	0	0

index

# Security

- The proposed scheme is **UC-secure** against non-adaptive adversaries.
- Assumptions
  - the strong RSA assumption
  - SKE is CPA-secure
  - PRF is a pseudo-random function
  - H is a collision-resistant hash function

Please refer to the paper for the details.

# Efficiency

	search	modify	delete	add
communication overhead	$n+O(\lambda)$	$O(\lambda)$	$O(\lambda)$	$m$
computation cost of the Server	$O(nm)$	$O(n)$	$O(n)$	$O(m)$
computation cost of the Client	$O(n)$	$O(1)$	$O(1)$	$O(m)$

$\lambda$  : security parameter  
 $n$  : number of Documents  
 $m$  : number of Keywords

# Summary

- Proposed a Verifiable Dynamic SSE scheme
- Construction based on RSA Accumulator
- UC-Secure

Thank you