Fuzzy Temporal Categorical and Intensity Information in Diagnosis

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Abstract. This paper proposes a way of incorporating fuzzy temporal reasoning within diagnostic reasoning. Disorders are described as an evolving set of necessary and possible manifestations. Fuzzy intervals are used to model ill-known moments in time (e.g. the beginning and end of a manifestation) and intensity of manifestations (e.g. "high" fever). The paper discusses several measures of consistency between a disorder model and the patient data, and defines when the manifestations presented by the patient can be explained by a disorder.

1 Introduction

Temporal information and temporal reasoning are important aspects of diagnostic reasoning [9, 8, 2, 13], specially in some domains, such as, for example, the diagnostics of infectious diseases.

That is the case, for instance, of the intoxication caused by ingestion of poisonous mushrooms of the species *Amanita phalloides, A. virosa*, and *A. verna* [10, Chap. 81]. It always causes abdominal cramps and nausea within 6 to 24 h from ingestion, lasting for up to 24 h, followed by a period of 1 to 2 days of no symptoms, finally followed by hepatic and renal failure (leading to death in 50% of the cases). This is called the model of the disease.

Faced with a case in which the patient has ingested mushrooms, felt abdominal cramps and nausea, but has not yet shown symptoms of renal and hepatic failure, one should not rule out intoxication by the *Amanita* family without verifying whether there has been enough time for those symptoms to develop.

The main goal of this paper is to answer the following questions:

- when is the model of a disease consistent with the case information and to what degree. For example, can we state that the model of intoxication with *Amanita* is consistent with the case of a patient who suffered from nausea and abdominal cramps for three days and then showed signs of renal failure and loss of sensation in the limbs two days later.

- when is the disease model categorically consistent the case information, that is, have all necessary symptoms in the model occurred (provided that they had had enough time to occur). If a patient had nausea and abdominal cramps for one day, then showed signs of renal failure two days later, but did not present signs of hepatic failure, can we consider that the model and the case are categorically consistent.
- when is a single disorder capable of explaining all symptoms of the present in the case. In the example of the patient that exhibited nausea and abdominal cramps, followed by renal failure and loss of sensation on the limbs, and given that the doctor had not made any test for hepatic failure, does poisoning by *Amanita* explain all symptoms, or in other words, is poisoning by *Amanita* a possible diagnostic.
- if we consider that a particular disease explains all the patient symptoms, what else do we know about other manifestations that the patient may have had, or will develop in the future.

In our model all temporal information is modeled by fuzzy sets, since most of the time we are dealing with information furnished by human beings, which are usually tainted with vagueness. For instance, a patient will normally tell that the interval between ingestion and cramps was "around 4 to 5.5 hours". On the other hand, a doctor would hardly discard the hypothesis of ingestion of *Amanita* if the patient has developed abdominal cramps exactly 25 hours after ingesting some kind of mushroom, instead of the expected 24 hours.

The use of fuzzy sets allow us moreover to obtain a degree of consistency between a model and a case. For instance, the 25 hours that elapsed between ingestion and cramps, although beyond the specified 24 hours, is not too far apart. But if it is known that this interval was precisely 36 hours, one is much more inclined to state the inconsistency. Thus one would like to have a concept of "degree of consistency," such that if the elapsed time between ingestion and cramps in the case is precisely known and is between 6 to 24 hours, then one can fully state the consistency of case and model, and this degree decreases the further away from the interval the case information is.

The next section brings definitions about fuzzy sets, defines what is the model of a disease and what is the information for a case. Section 3 provides answers to some of the questions raised above: when is the case temporally consistent with the model, when is the case categorically consistent with the model, and when is the model an explanation for the case. It also includes a subsection on intensity consistency. Section 4 brings some current research material about dealing with manifestations which have a known frequency of occurrence inside a given disorder. Finally section 5 discusses the limits of the model proposed and future work.

2 Basic Definitions

2.1 Fuzzy Intervals

A fuzzy set A in Θ [4] is characterized by a membership function $\mu_A : \Theta \to [0,1]$, such that $\exists x \in \Theta, \mu_A(x) = 1$. The height of a fuzzy set A is calculated as $h(A) = \sup_{x \in \Theta} \mu_A(x)$, and A is said to be normalized when $h(A) = 1$.

Let A and B be fuzzy sets in Θ. The sum $A \oplus B$, the subtraction $A \ominus B$ and the negation $-A$ are respectively characterized by membership functions [4]:

- $\mu_{A \oplus B}(z) = sup_{\{(x,y)/z=x+y\}} min(\mu_A(x), \mu_B(y))$,
- $\mu_{A \ominus B}(z) = sup_{\{(x,y)/z=x-y\}} min(\mu_A(x), \mu_B(y))$.
- $\mu_{-A}(z) = \mu_A(-z)$.

In this work, a fuzzy set A such that μ_A is convex will be called a fuzzy interval. An interval will be positive if Θ is the real line, and $\forall x < 0, \mu(x) = 0$.

In some cases we will assume that the fuzzy interval is trapezoidal, as in figure 1. In that case, the interval will be represented by a 4-tuple $\langle a, b, c, d \rangle$.

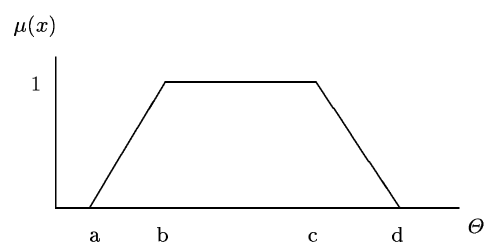

Fig. 1. A trapezoidal fuzzy interval

A fuzzy set $A = \langle a_1, a_2, a_3, a_4 \rangle$, such that $a_1 = a_2$ and $a_3 = a_4$ is a convex crisp set, and will sometimes be denoted by $A = \langle a_1, a_3 \rangle$, throughout this paper.

For a trapezoidal interval $A = \langle a_1, a_2, a_3, a_4 \rangle$ the range $[a_2, a_3]$, where $\mu_A(x) = 1$, will be called core. The range $[a_1, a_4]$, where $\mu_A(x) > 0$, will be called support.

For two trapezoidal intervals $A = \langle a_1, a_2, a_3, a_4 \rangle$, and $B = \langle b_1, b_2, b_3, b_4 \rangle$, the $\oplus$ and $\ominus$ operations are simply $A \oplus B = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4 \rangle$, and $A \ominus B = \langle a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1 \rangle$.

Throughout this paper, we shall make use of four particular fuzzy intervals. Let θ be a moment in Θ. The fuzzy intervals describing the possibility of an event occurring *at any time*, *exactly at* θ, *after* θ, and *before* θ are respectively defined as:

- $I_{\text{anytime}} = A$, such that $\forall x \in \Theta, \mu_A(x) = 1$,
- $I_{=\theta} = A$, such that $\mu_A(x) = 1$, in $x = \theta$, and $\mu_A(x) = 0$, otherwise.
- $I_{\geq\theta} = A$, such that $\forall x \in \Theta$, if $x \geq \theta, \mu_A(x) = 1$, and $\mu_A(x) = 0$, otherwise.
- $I_{\leq\theta} = A$, such that $\forall x \in \Theta$, if $x \leq \theta, \mu_A(x) = 1$, and $\mu_A(x) = 0$, otherwise.

We also use θ_0 to denote the present moment, and define $I_{beforenow} = I_{\leq\theta_0}$ and $I_{afternow} = I_{\geq\theta_0}$.

Finally, we will define that an interval A is tighter than an interval B (or, informally narrower) if $\mu_A(x) \leq \mu_B(x)$ for all $x \in \Theta$. If A and B are trapezoidal, then $A = \langle a_1, a_2, a_3, a_4\rangle$ is tighter than $B = \langle b_1, b_2, b_3, b_4\rangle$, iff $a_1 \geq b_1$, $a_2 \geq b_2$, $a_3 \leq b_3$, and $a_4 \leq b_4$.

2.2 The Knowledge Base

The knowledge base for a fuzzy temporal/categorical diagnostic problem is the information about disorders and how they evolve. The knowledge base is given by the tuple $\langle \Theta, D, M, N, P, V, T\rangle$ where:

- Θ is a time scale.
- D is the set of disorders.
- M the set of manifestations.
- N is the necessary effects function that associates to each disorder d_l a set $M_L \subseteq M$ of manifestations that d_l *necessarily* causes. That is, if $N(d_1) = \{m_4, m_5, m_7\}$ then it is not possible to have the disorder d_1 without having eventually the symptoms m_4, m_5 and m_7.
- P is the possible effects function that associates to each disorder d_l a set $M_L \subseteq M$ of manifestations that d_l *may* causes.
- We will define the derived function E, effects of a disorder, as $E(d) = N(d) \cup P(d)$.
- V associates to each disorder a set of (instantaneous) events. These events will be used to describe the evolution of the disorder. Among the events in $V(d_l)$ it must be included events that correspond to the beginning of all manifestations in $E(d_l)$. Furthermore, $V(d_l)$ can include events that correspond to the end of some of the manifestations in $E(d_l)$ and can also include other, non-observable events. For example, a common non-observable event in infectious diseases is the infection itself.
- $\mathcal{T}$ is a function that associates to *some* pairs of events $e_i, e_j \in V(d_l)$ a fuzzy temporal interval $\mathcal{T}(d_l)(e_i, e_j) = \pi$ which states that (according to the model for the disorder d_l) the elapsed time between the event represented by e_j and the event represented by e_i must be within the fuzzy temporal interval π.

Together, $V(d_l)$ and $\mathcal{T}(d_l)$ can be better understood in terms of a graph of events. The events in $V(d_l)$ are the nodes of the graph and if $\mathcal{T}(d_l)$ is defined for the pair of events (e_i, e_j) then there is a directed arc from e_j to e_i and the value in the arc is $\mathcal{T}(d_l)(e_i, e_j)$. We will call such interpretation as the temporal graph of the disorder.

2.3 Case Information

For a particular diagnostics problem, one needs, besides the knowledge base about the disorders, a particular case. The case information should describe the manifestations that the patient is suffering and have suffered from, temporal information about those when those symptoms started and ended, and information about manifestations that the diagnostician knows are not present in the patient.

Information about a given case is modeled by a tuple $Ca = \langle M^+, M^-, \mathrm{EV}^+, \mathrm{TIME}^+, \theta_0 \rangle$, where

- M^+ is the set of manifestations known to be or to have been present in the case.
- M^- is the set of manifestations known to be absent from the case.
- EV^+ is a set of events for which one has temporal information. Among the events in EV^+ are the ones that represent the beginning of each manifestation in M^+. Events representing the end of the manifestations in M^+ may also belong to the set EV^+.
- TIME^+ is a function that associates to each event $e \in \mathrm{EV}^+$ a fuzzy temporal interval that represents the possible moments in which that event happened.
- θ_0 is the moment of the diagnosis.

For instance, in our example, we could have a piece of information such as "the patient had nausea (m_2), starting 24 hours before the consultation, which lasted for about 2 or 3 hours, and he is sure he did not have abdominal cramps (m_1)". We would then have $M^+ = \{m_2\}$, $M^- = \{m_1\}$ and $\mathrm{EV}^+ = \{m_2^b, m_2^e\}$. If we consider that the consultation happened at time 0, the temporal information above would be translated as $\mathrm{TIME}^+(m_2^b) = \langle -24, -24 \rangle$ and $\mathrm{TIME}^+(m_2^e) = \langle -22, -21 \rangle$. Of course, all the temporal information can be given in terms of fuzzy intervals.

3 Consistency Between a Model and a Case

3.1 Minimal Network

Before discussing measures of consistency between a disorder model and a patient case we will present the concept of a minimal network [11]. Given a set of intervals among some events, the minimal network is way to compute the intervals between any two of those events, so that those computed intervals are as tight as possible.

The minimal network for each disorder d_l can be computed by the algorithm below, the Floyd-Warshall algorithm, which computes the shortest path for every pair of nodes [3, Chap.26]. We assume that the events in $V(d_l)$ are arbitrarily numbered, and that $|V(d_l)| = n$. The algorithm computes the values t_{ij} with is the interval between events e_i and e_j in the minimal network for a particular disorder d_l.

for i = 1 **to** n **do**
  for j = 1 **to** n **do**
    if i = j **then** $t_{ii} = I_{=0}$
    else if $\mathcal{T}(d_l)(e_i, e_j)$ is defined **then** $t_{ij} = \mathcal{T}(d_l)(e_i, e_j)$
    else if $\mathcal{T}(d_l)(e_j, e_i)$ is defined **then** $t_{ij} = -\mathcal{T}(d_l)(e_j, e_i)$

6 $\quad\quad\quad\quad$ **else** $t_{ij} = I_{\text{anytime}}$
7 $\quad$ **for** k = 1 **to** n **do**
8 $\quad\quad$ **for** i = 1 **to** n **do**
9 $\quad\quad\quad$ **for** j = 1 **to** n **do**
10 $\quad\quad\quad\quad$ $t_{ij} = t_{ij} \cap (t_{ik} \oplus t_{kj})$

We will define a function $\mathcal{T}^*(d_l)(e_i, e_i)$ which returns the value of t_{ij} in the minimal network for disorder d_l. We will abbreviate $\mathcal{T}^*(d_l)(e_i, e_i)$ as $\pi_l(e_i, e_j)$, and if the disorder is clear by the context, we will not use the superscript.

In terms of the graph analogy of V and $\mathcal{T}$, the minimal network computes the transitive closure of the graph, considering that if there is an arc from e_j to e_i with value $\pi_l(e_i, e_j)$, then there should be an arc from e_i to e_j with value $-\pi_l(e_i, e_j)$ (line 5 in the algorithm).

3.2 Temporal consistency

In evaluating the temporal consistency between the case and the model, one needs to compare the elapsed time between the events in the case (the events in EV^+) and the corresponding fuzzy intervals as specified in the model.

The fuzzy temporal distance between the real occurrences of any two events e_i and e_j is computed as $\text{DIST}^+(e_i, e_j) = \text{TIME}^+(e_j) \ominus \text{TIME}^+(e_i)$.

In order to verify how well these two events fit with the model of a particular disorder d_l we must compare $\text{DIST}^+(e_i, e_j)$ with $\pi_l(e_i, e_j)$ if both e_i and e_j belong to that disorder model. The temporal distance between two events e_i and e_j, taking into account both the model and the case information is given by $\text{DIST}^t(e_i, e_j) = \text{DIST}^+(e_i, e_j) \cap \pi_l(e_i, e_j)$. The degree of consistency between d_l and the case information in relation to the pair of events e_i and e_j is then the height of $\text{DIST}^t(e_i, e_j)$, i.e. $h(\text{DIST}^t(e_i, e_j))$.

Finally, the temporal consistency degree of the disorder d_l is defined as:

– $\alpha(d_l) = \inf\limits_{e_i, e_j \in \text{EV}^+, V(d_l)} h(\text{DIST}^t(e_i, e_j))$

3.3 Categorical Consistency

Categorical consistency between model and case refers to the fact that a necessary manifestation of a disorder must happen, if the patient is suffering from that disorder. If the case does not have a manifestation then no disorder that considers that manifestation necessary can be a possible diagnostic, or be part of a possible diagnostic. But categorical inconsistency is tightly bound with temporal reasoning. In fact we can only state that a case is categorically inconsistent with the model if a necessary manifestation has not occurred and there has been enough time for it to happen.

One can say that a manifestation m_i had had enough time to occur in d_l if

– there exists an event e_j, which was supposed to happen after the start of m_i, and that event has already occurred;

- or there exists an event e_j, which was supposed to happen before the start of m_i, and that event did happen as expected, but the expected elapsed time between the event and the start of m_i has already expired.

Categorical consistency can be calculated as temporal consistency if we assume that all necessary manifestations that have not yet occurred will start sometime after the moment of consultation. If, because of either the two reasons above, there is other temporal information that states that this event should have already started, the temporal consistency index of the disorder will reflect it. Thus, with the initialization

- $\forall m_i \in M^- \cap N(d_l), \mathrm{TIME}^+(m_i^b) = I_{\text{afternow}}$,

the temporal consistency index $\alpha(d_l)$, will reflect both the temporal and the categorical consistency. We will call this combined temporal and categorical index as $\alpha_{ct}(d_l)$.

3.4 Intensity Consistency

In some disorders, it is important to quantify the intensity with which some of its manifestations occur. For instance, let us suppose a given disease is characterized by strong fever at some time during its development; in this case, it is reasonable to suppose that that disorder will be the less plausible, the lower the temperature of the patient.

We use a fuzzy set S_{m_i/d_l} to model the intensity with which m_i is expected to occur in d_l. Each fuzzy set S_{m_i/d_l} is defined on its particular domain Ω_{m_i}. For manifestations m_i for which intensity is not a relevant matter in d_l, S_{m_i/d_l} is constructed as $\forall x \in \Omega_{m_i}$, $\mu_{S_{m_i/d_l}}(x) = 1$. When the intensity can be quantified by a precise constant x^* in Ω_{m_i}, S_{m_i/d_l} is constructed as $\mu_{S_{m_i/d_l}}(x) = 1$, if $x = x^*$, $\mu_{S_{m_i/d_l}}(x) = 0$, otherwise.

In the same way, the case information contains for each node $m_i \in M^+$ a fuzzy set $S^+_{m_i}$, defined in Ω_{m_i}, describing the intensity with which that manifestation occurred in that particular case.

The consistency of the intensity of a manifestation m_i, in relation to a disorder d_l, is quantified as follows: $\beta(m_i, d_l) = h(S_{m_i/d_l} \cap S^+_{m_i})$.

Finally, for a disorder d_l its intensity consistency is given by

- $\beta(d_l) = \inf\limits_{m_i \in E(d_l)} \beta(m_i, d_l)$.

3.5 Diagnostic Explanation

In this paper we assume that every explanation, or better diagnostic explanation, is a single disorder that is temporal, categorical, and intensity consistent with the symptoms and explains all symptoms present in the case. Thus d_l is a diagnostic for the case $Ca = \langle M^+, M^-, \mathrm{EV}^+, \mathrm{TIME}^+, \theta_0, \mathrm{INT}^+ \rangle$, if

- $\alpha_{ct}(d_l) > 0$
- $\beta(d_l) > 0$
- for all $m_i \in M^+$, $m_i \in E(d_l)$.

4 Possibilistic Vertices

So far, we have assumed that experts can give positive opinion about a manifestation m_i being necessary or only possible in the context of a given disorder, denoted as $m_i \in N$ and $m_i \in P$ respectively. Let us now suppose that for some of the possible, but not necessary manifestations, the expert is also capable of giving a frequency with which it occurs in the disorder.

For instance, he may say that "when the patient is suffering from d_l, m_i occurs 10% of the time", which is a way of saying that it is possible for m_i to occur in d_l, but that this is rarely the case. Or else, he may say that "m_i occurs 90% of the time", which means that m_i is not only possible but is very frequentely present in d_l. In the following we outline how our framework could be extended to deal with this kind of information.

Let $f_{m_i^+/d_l} \in [0,1]$ denote the frequency with which m_i is present when disorder d_l occurs. The frequency with which m_i is absent given that d_l occurs, is denoted by $f_{m_i^-/d_l} = 1 - f_{m_i^+/d_l}$.

When $m_i \in N$ for a given d_l, ie m_i is necessary in d_l, we have $f_{m_i^+/d_l} = 1$. On the other hand, when $m_i \in P$ for a given d_l, ie m_i is possible but not necessary in d_l, we have $0 < f_{m_i^+/d_l} < 1$. If a manifestation m_i is considered to be impossible to happen in d_l then we have $f_{m_i^-/d_l} = 1$.

Let $\pi_{m_i^+/d_l} \in [0,1]$ (respec. $\pi_{m_i^-/d_l} \in [0,1]$) denote the possibility degree that m_i is present (respec. absent) when disorder d_l occurs. These values are such that $\max(\pi_{m_i^+/d_l}, \pi_{m_i^-/d_l}) = 1$ and are obtained using the following procedure, adapted for the dichotomic case from [7]:

if $f_{m_i^+/d_l} > f_{m_i^-/d_l}$ **then** $\{\pi_{m_i^+/d_l} = 1, \pi_{m_i^-/d_l} = 2 * f_{m_i^-/d_l}\}$
else $\{\pi_{m_i^+/d_l} = 2 * f_{m_i^+/d_l}, \pi_{m_i^-/d_l} = 1\}$

For instance, when m_i is always present whenever d_l occurs, we have $\pi_{m_i^+/d_l} = 1$ and $\pi_{m_i^-/d_l} = 0$, whereas when m_i never happens in d_l, we have $\pi_{m_i^+/d_l} = 0$ and $\pi_{m_i^-/d_l} = 1$. In the example given above, in which m_i happens in 90% of the cases in d_l, we have $\pi_{m_i^+/d_l} = 1$ and $\pi_{m_i^-/d_l} = .2$. Alternatively, when m_i happens in 10% of the cases in d_l, we have $\pi_{m_i^+/d_l} = .2$ and $\pi_{m_i^-/d_l} = 1$.

Now, we have to compare this information with the patient data, given by M^+ and M^-. Also this kind of information could be uncertain, although not frequentist in nature. For instance, a patient may believe that m_i happened rather than the other way around. We will however assume here that the patient data is still supplied in terms of M^+ and M^-.

Using M^+ and M^- provided by the patient, for each m_i in d_l, we obtain the possibility of manifestation having or not occurred, denoted by $\pi_{m_i^+}$ and $\pi_{m_i^-}$, in the following manner:

if $m_i \in M^+$ **then** $\{\pi_{m_i^+} = 1, \pi_{m_i^-} = 0\}$
else if $m_i \in M^-$ **then** $\{\pi_{m_i^+} = 0, \pi_{m_i^-} = 1\}$

else $\{\pi_{m_i^+} = 1, \pi_{m_i^-} = 1\}$

The compatibility of the occurrence of m_i in d_l is given by

– $\psi(m_i, d_l) = \max[min(\pi_{m_i^+/d_l}, \pi_{m_i^+}), min(\pi_{m_i^-/d_l}, \pi_{m_i^-})]$

and the overall occurrence compatibility of d_l is given by:

– $\psi(d_l) = \inf_{m_i \in M} \psi(m_i, d_l)$

Therefore a disorder will have low occurrence compatibility degree when the patient presents manifestations considered to be rare in d_l, and also when if he does not present manifestations considered to be frequent in d_l. This scheme is equivalent to that presented in [6].

Using the original M^+, M^-, N and P, the other indices (temporal, categorical...) remain unchanged, and $\psi(d_l)$ can be seen as only an additional information for the physician. However, they could be used to affect the other indices, but this remain an issue for future investigation.

5 Conclusions and future work

This work presented a model to include fuzzy temporal information, categorical information, and (fuzzy) intensity information within a diagnostic framework. We provided answers to the following questions: when is the temporal information in the case consistent with a disorder model, when is the case categorically consistent with the model, and how information about intensity can be included. We have also outlined how to treat "fuzzy" categorical information, making it possible to model pieces of information furnished by a medical expert such as "in disorder d_l, manifestation m_i is very likely to occur" or "in disorder d_l, m_i will seldom occur".

In this paper we are not concerned on how the temporal information about the disorder model ($\mathcal{T}(d_l)$) is obtained (see [1] for details about this problem). Also, we are not concerned on how the temporal information about the case (TIME^+) is obtained, and assume here that the information about the case is internally consistent. [11] discusses this problem, and proposes a method to evaluate the consistency of the information and the most precise intervals for the occurrence of the events using minimal networks. Other researchers have also discussed similar issues [5]. The approach presented here yields only possibilistic compatibility degrees, but could be modified to obtain also entailment degrees, as in [11, 12].

This work extends a previous work by the authors [14]. In [14] it is assumed that the temporal graph of the disorder is a tree. In order to calculate the temporal consistency of the case and the model, the case information was propagated towards the root; any temporal inconsistency would result in conflicting intervals for the root. We have also been studying, once a diagnostic has been made, how can one make forecasts about future manifestations, past manifestations that have not been tested for, and so on, based not only on what the disorder model predicts but also based on how fast the case is progressing [15].

In the future, we intend to exploit the case in which the set of all manifestations presented by the patient can only be explained by a set of disorders, rather than by a single disorder, as addressed here. When all disorders in an explanation do not have any common manifestations, it seems that the theory above could be generalized by calculating the consistency indices for each disorder and attributing global consistency indices to the explanation as the minimum of the disorder's indices. However, it is not yet clear what should be done when a set of disorders explaining the manifestations have manifestations in common.

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