Epistemic Conditional Logics

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Abstract. This paper develops two conditional logics that allows one to reason about the belief of another agent (which reasons in conditional logic himself). The desirable properties of such logics that reason about the beliefs of another non-monotonic agent are discussed and the two logics proposed here are shown to have those properties.

1 Introduction

An intelligent system sometimes must interact with other intelligent systems to accomplish a task or a goal. If that is the case, it may be important for the system to represent the knowledge of the other agents with whom it interacts. For example, a tutoring system may find it is important to explicitly represent and reason about the knowledge of agent being taught. Or, an intelligent system may find it necessary to reason about the knowledge of the other agents in order to effectively communicate with them.

On the other hand, it is widely assumed that intelligent reasoning must include a component of non-monotonicity. Thus many non-monotonic logics have been proposed to model different aspects of intelligent reasoning. This paper deals with the intersection of the two issues above: the proposal of a non-monotonic logic that not only models the agent reasoning (non-monotonically) about the world, but also reasoning about the knowledge of other agents.

There are in principle two approaches to developing multi-agent non-monotonic logics: to develop new logics or to adapt some of the existing ones. This paper follows the latter approach: we will propose an extension of two conditional logics of normality proposed in [Bou94], so that the extended logics can represent and reason about formulas that contain reference to the knowledge of another agent. This is only a first step in developing full multi-agent conditional logics. The logic presented here will be called epistemically extended conditional logic (following [Wai93]): the logic allows the system being modeled to reason about the knowledge of just another agent, but that agent does not reason about the system's knowledge. Furthermore, the logic assumes the system's point of view (what [Lev90] calls an internal perspective) since that is the approach taken by the majority of the non-monotonic logics (with the exception of [Lev90, Lak93]).

We will first present an an overview of the two conditional logics on which we base this work.

2 Conditional Logics of Normality

Recent work in non-monotonic reasoning has led to the development of some conditional theories of default inferencing such as the logic N [Del87], ϵ-entailment [Pea88], preferential entailment [KLM90], CT4O and CO [Bou94] among others. These conditional logics have a minimal set of properties that ought to be common to all non-monotonic inference systems and that constitue, as has been suggested by Pearl [Pea89], a "conservative core" of non-monotonic reasoning. The properties are related to the fact that these approaches:

- provide a more natural representation of defaults than the traditional formalisms, such as default logic [Rei80], autoepistemic logic [Moo85] or circumscription [McC80],
- do not have problems with fixed points or multiple extensions,
- deal satisfactorily with the specificity of defaults.

Because of their clear semantics, we chose CT4O and CTO [Bou94] as the base of our work.

2.1 The CT4O logic

The language of CT4O, $\mathcal{L}_{\mathcal{C}}$, is that of the classical propositional logic (CPL) formed from a set of atomic sentences **P**, augmented with two unary modal operators $\Box$ and $\overleftarrow{\Box}$. Furthermore, four modal operators are defined as abbreviations:

Definition 1 – $\Diamond A \equiv_{def} \neg\Box\neg A.$
- $\overleftarrow{\Diamond} A \equiv_{def} \neg \overleftarrow{\Box} \neg A.$
- $\overleftrightarrow{\Box} A \equiv_{def} \Box A \wedge \overleftarrow{\Box} A.$
- $\overleftrightarrow{\Diamond} A \equiv_{def} \Diamond A \vee \overleftarrow{\Diamond} A.$

The semantics of CT4O of normality is based on Kripkean possible world structures, where a *possible world* is any subset of **P**. The sentences of $\mathcal{L}_{\mathcal{C}}$ are interpreted in terms of a *CT4O*-model:

Definition 2 *A CT4O-model is a pair $M = \langle W, \geq \rangle$, where W is a set of possible worlds and $\geq$ is a reflexive, transitive binary relation on W.*

The satisfiability of a sentence $A \in \mathcal{L}_{\mathcal{C}}$, at a world w in a CT4O-model M is defined as:

Definition 3 *Let $M = \langle W, \geq \rangle$ a CT4O-model, the satisfability at a world w, in a CT4O-model M is defined by:*

- $M, w \models A$ *iff* $A \in w$ *for* A *a atomic formula*
- $M, w \models \neg A$ *iff* $M, w \not\models A$
- $M, w \models A \supset B$ *iff* $M, w \not\models A$ *or* $M, w \models B$.

– $M, w \models \Box A$ *iff* $M, v \models A$, *for each* v *such that* $w \geq v$.
– $M, w \models \overleftarrow{\Box} A$ *iff* $M, v \models A$, *for each* v *such that* $w \not\geq v$.

The accessibility relation $\geq$ among possible worlds in a $CT4O$-model reflects some measure of normality on all states of affairs, and it should be interpreted as follows: for possible worlds w and v, $w \geq v$ states that v is at least as normal as w. Thus from a particular world w, following the $\geq$ relation, one sees a sequence of successively "less exceptional" worlds.

The logic provides a uniform way to deal with defaults. It uses a conditional connective $\Rightarrow$ to represent default rules (or statements of normality): $A \Rightarrow B$ is read as "If A then normally B". $A \Rightarrow B$ is defined to be true just when A is false at all accessible worlds or when there is a world in which $A \wedge B$ is true and $A \supset B$ is true at all equally or less exceptional worlds. Formally:

Definition 4 $A \Rightarrow B =_{def} \overleftrightarrow{\Box} (\Box\neg A \vee \Diamond(A \wedge \Box(A \supset B)))$

We introduce, also, the following abbreviation:

Definition 5 $A \not\Rightarrow B =_{def} \neg(A \Rightarrow B)$

In the CT4O logic, Modus Ponens is not valid, one can not infer B from A and $A \Rightarrow B$, but it is reasonable to conclude that normally B would hold. Formally:

Theorem 1 $\overset{CT4O}{\models} (A \wedge (A \Rightarrow B)) \Rightarrow B$

Hence, in the framework of the CT4O logic, default reasoning can be naturally modeled as the process of asking what normally follows form a set of statements, based on the validity of the weaker version of Modus Ponens.

CT4O is a system of default reasoning that can deal with the specificity of defaults and simple cases of inheritance with exceptions. For for example, from the fact that birds (normally) fly, penguins necessarily are birds, penguins do not fly, from the fact that something is a penguin, one can conclude (by default) that it normally will not fly. Formally:

$$\overset{CT4O}{\models} \begin{pmatrix} bird \Rightarrow fly \wedge \\ \Box(penguin \supset bird) \wedge \\ penguin \Rightarrow \neg fly \wedge \\ penguin \end{pmatrix} \Rightarrow \neg fly \qquad (1)$$

2.2 The CO logic

The CT4O structures reflect the minimal conditions that can be imposed on a plausibility ordering over a set of possible worlds. To obtain a logic with some further properties, more restrictions must be placed on $\geq$. The logic CO is defined when the $\geq$, besides being reflexive and transitive, is also totally connected.

The language of CO is the same of CT4O, only the definition of a CO-model changes:

Definition 6 *A CO-model is a pair $M = \langle W, \geq \rangle$, where W is a set of possible worlds, and $\geq$ is reflexive, transitive, and totally connected relation over W*[1].

The restriction imposed on $\geq$ enforces the intuitive notion that all the worlds are comparable on the scale of normality. Differently as in CT4O, the CO-models have only one path of normality. Hence, $\overleftarrow{\Diamond} A$ is read as " A holds at some less normal world". Because there are no inaccessible worlds, the definition of $\Rightarrow$ can be reformulated as:

$$A \Rightarrow B =_{def} \overleftrightarrow{\Box} \neg A \vee \overleftrightarrow{\Diamond} (A \wedge B \wedge \Box(A \supset B))$$

3 Epistemic Extension of a Logic

An epistemic extension is a syntactic and semantic extension of a non-monotonic logic in such a way that the extended logic is able to deal with formulas that contain a belief operator. The extended logic represents the point of view of a particular agent (called the system) reasoning about reality and about the knowledge (or belief) of other agent. In an epistemically extended logic, the formula $p \wedge \mathcal{B}q$ should be interpreted as "the system believes p and it believes that the other agent believes q."

Let us use $\mathcal{L}^*$ to denote the epistemic extension of the non-monotonic logic $\mathcal{L}$, and the symbol $\rightsquigarrow$ to represent the way default rules are represented in both logics. [Wai93] describes some of the properties that the logic $\mathcal{L}^*$ should have, among them:

- The logic $\mathcal{L}^*$ should be a conservative extension of $\mathcal{L}$, that is $\mathcal{L}^*$ has the same deductive power as $\mathcal{L}$ when dealing with formulas without the $\mathcal{B}$ operator:
$$\alpha \overset{\mathcal{L}}{\models} \beta \quad \text{iff} \quad \alpha \overset{\mathcal{L}^*}{\models} \beta \tag{2}$$
- The logic $\mathcal{L}^*$ should also model the behavior of the $\mathcal{B}$ operator. If we assume that the modal logic KD45 is a good model for the belief operator, then
$$\text{if} \quad \overset{KD45}{\models} \alpha \quad \text{then} \quad \overset{\mathcal{L}^*}{\models} \alpha \tag{3}$$
- The system and the other agent should have the same non-monotonic reasoning abilities, or in formal notation
$$\alpha \overset{\mathcal{L}}{\models} \beta \quad \text{iff} \quad \mathcal{B}\alpha \overset{\mathcal{L}^*}{\models} \mathcal{B}\beta \tag{4}$$
- The logic $\mathcal{L}^*$ must allow for default formulas that combine the $\mathcal{B}$ operator with sub-formulas without it, and derive the "correct" conclusions from them. For example:
$$\begin{gathered} \mathcal{B}\alpha \wedge (\mathcal{B}\alpha \rightsquigarrow \beta) \overset{\mathcal{L}^*}{\models} \beta \\ \text{and} \\ \alpha \wedge (\alpha \rightsquigarrow \mathcal{B}\beta) \overset{\mathcal{L}^*}{\models} \mathcal{B}\beta \end{gathered} \tag{5}$$

[1] A relation R on a set A is totally connected if for all x and y in A, xRy or yRx.

In terms of a conditional logic, $\mathcal{L}_\mathcal{C}$, these requirements can be stated as:

- $\overset{\mathcal{L}_C}{\models} \alpha \Rightarrow \beta$ iff $\overset{\mathcal{L}_C^*}{\models} \alpha \Rightarrow \beta$
- if $\overset{KD45}{\models} \alpha$ then $\overset{\mathcal{L}_C^*}{\models} \alpha$
- $\overset{\mathcal{L}_C}{\models} \alpha \Rightarrow \beta$ iff $\overset{\mathcal{L}_C^*}{\models} \mathcal{B}\alpha \Rightarrow \mathcal{B}\beta$
- $\overset{\mathcal{L}_C^*}{\models} (\mathcal{B}\alpha \wedge (\mathcal{B}\alpha \Rightarrow \beta)) \Rightarrow \beta$ and $\overset{\mathcal{L}_C^*}{\models} (\alpha \wedge (\alpha \Rightarrow \mathcal{B}\beta)) \Rightarrow \mathcal{B}\beta$

4 The Extension of CT4O

We propose an extension of the conditional logic of normality $CT4O$, which we call $CT4O^e$. The language of $CT4O^e$ ($\mathcal{L}_{\mathcal{C}^*}$) is that of $CT4O$ with a $\mathcal{B}$ modal operator. If A is a well formed formula (wff) of $\mathcal{L}_{\mathcal{C}^*}$ and it does not contain a $\mathcal{B}$ operator, then $\mathcal{B}A$ is also a wff. Thus, the language $\mathcal{L}_{\mathcal{C}^*}$ does not allow for the belief operator within the scope of another belief operator.

To extend the semantics, we adapt the classical Kripke structures to deal with the belief of the two agents. We use essentially the concept of possible worlds but in a somewhat different way. The possible worlds we use have two components, one to model the beliefs of the system and the other to model the possible beliefs of the system's view of the beliefs of the other agent.

We begin introducing the augmented worlds.

Definition 7 *An augmented world (A-world) is a pair $\langle w, \langle W, \geq \rangle \rangle$, where W is a set of propositional worlds, $w \in W$, and $\geq$ is a transitive, reflective relation over W.*

A-worlds are $CT4O$-models with a distinguished world. The intuition is that A-worlds are structures that allow one to interpret formulas of the type α and $\alpha \Rightarrow \beta$, where both α and β are formulas without $\mathcal{B}$.

We will now define an auxiliary relation $\geq_\triangle$ among a set of augmented worlds.

Definition 8 *$\langle w_1, M_1 \rangle \geq_\triangle \langle w_2, M_2 \rangle$ iff*

- *$M_1 = M_2$ and*
- *$w_1 \geq w_2$, where $\geq$ is the binary relation of the M_1 (and M_2).*

That is, two A-worlds can be compared under the $\geq_\triangle$ relation if their $CT4O$-models are the same and their distinguished worlds can be compared under the normality relation $\geq$ of those $CT4O$-models.

Definition 9 *A belief world (B-world) is a pair $\langle w, \mathcal{A} \rangle$, where w is a propositional world and $\mathcal{A}$ is a set of augmented worlds.*

The first component of the belief worlds allows one to interpret formulas related to the beliefs of the system, formulas of the form α, where α does not have any ocorrence of the operator $\mathcal{B}$. The second component of the belief worlds allows one to interpret formulas related to the belief of the other agent, formulas of the form $\mathcal{B}\alpha$ and $\mathcal{B}(\alpha \Rightarrow \beta)$. Thus, what still needs to be done is to define a relation of "normality" among B-worlds, so that formula that contains $\Rightarrow$ at the top level can be interpreted. We define this relation in such a way that the resulting logic has the "correct" properties.

We will define a relation among B-worlds based on a transitive, reflexive relation among possible worlds $\geq$.

Definition 10 $\langle w_1, \mathcal{A}_1 \rangle \geq_\Box \langle w_2, \mathcal{A}_2 \rangle$ *iff*

- $w_1 \geq w_2$ *and*
- *for all* $a \in \mathcal{A}_1$ *there is* $b \in \mathcal{A}_2$ *such that* $a \geq_\triangle b$ *and for all* $b \in \mathcal{A}_2$ *there is* $a \in \mathcal{A}_1$ *such that* $a \geq_\triangle b$

We finish by defining the central concept that captures the semantics of our logic. The definition of a $CT4O^e$-model is:

Definition 11 *A* $CT4O^e$*-model is a pair* $\langle X, \geq \rangle$*, where* X *is a set of B-worlds,* $\geq$ *is a reflexive and transitive binary relation defined on the set of the first component of each B-world in* X *and the relation* $\geq_\Box$*, based in the relation* $\geq$ *defined as in 10, is the relation of normality among the B-worlds of* X*.*

Furthermore we require X *should not have "holes" in the following sense: if* $\langle w, \{\langle w_i, M_i \rangle\}_{\{i \in I\}}\} \rangle \in X$ *and if* $\{\langle w'_i, M_i \rangle\}_{\{i \in I\}}\} \rangle$ *is a set of augmented worlds, such that for all* $i \in I$, $\langle w_i, M_i \rangle \geq_\triangle \langle w'_i, M_i \rangle$*, then there is* w_0 *such that* $w \geq w_0$ *and* $\langle w_0, \{\langle w'_i, M_i \rangle\}_{\{i \in I\}}\} \rangle \in X$*.*

Satisfiability of formula α in a $CT4O^e$-model is defined as expected. Given that $M = \langle X, \geq \rangle$ is a $CT4O^e$-model, $\geq_\Box$ is the relation defined on the set X, based in the relation $\geq$, x is a particular B-world of X, Π_1 and Π_2 are the first component and second component functions for pairs (conveniently extended for sets of pairs[2]), then we define satisfiability of a formula α at a B-world x of a $CT4O^e$-model as:

Definition 12

- $M, x \models \alpha$ *iff* $\alpha \in \Pi_1(x)$ *for* α *an atomic formula.*
- $M, x \models \neg\alpha$ *iff* $M, x \not\models \alpha$*.*
- $M, x \models \alpha \supset \beta$ *iff* $M, x \models \beta$ *or* $M, x \not\models \alpha$*.*
- $M, x \models \Box\alpha$ *iff for all* y *such that* $x \geq_\Box y$, $M, y \models \alpha$*.*
- $M, x \models \overleftarrow{\Box}\, \alpha$ *iff for all* y *such that* $x \not\geq_\Box y$, $M, y \models \alpha$*.*
- $M, x \models \mathcal{B}\alpha$ *iff* $\Pi_2(a), \Pi_1(a) \models \alpha$*, for all* $a \in \Pi_2(x)$*.*

Finally, validity in a model and validity are defined as usual:

Definition 13 *A wff* α *of* $\mathcal{L}_{C^e}$ *is valid on a* $CT4O^e$*-model* $M = \langle X, \geq \rangle$ *(written* $M \models \alpha$*) iff* $M, x \models \alpha$ *for each* $x \in X$*,*

[2] $\Pi_1(A) = \{x | \langle x, y \rangle \in A\}$

Definition 14 *α is $CT4O^e$-valid (written $\overset{CT4O^e}{\models} \alpha$) just when $M \models \alpha$ for every $CT4O^e$-model M.*

4.1 Results

The structure defined gives to the logic $CT4O^e$ a semantics, such that all the requirements of an epistemic extension of a logic mentioned above are satisfied. Thus we have the following results:

Theorem 2 *If $\alpha, \beta \in \mathcal{L}_{\mathcal{C}}$ then: $\overset{CT4O}{\models} \alpha \Rightarrow \beta$ iff $\overset{CT4O^e}{\models} \alpha \Rightarrow \beta$.*[3]

Theorem 3 *If $\overset{KD}{\models} \alpha$ then $\overset{CT4O^e}{\models} \alpha$.*

Theorem 4 *If $\alpha, \beta \in \mathcal{L}_{\mathcal{C}}$ then $\overset{CT4O}{\models} \alpha \Rightarrow \beta$ iff $\overset{CT4O^e}{\models} \mathcal{B}\alpha \Rightarrow \mathcal{B}\beta$.*

Theorem 5 – $\overset{CT4O^e}{\models} (\alpha \Rightarrow \mathcal{B}\beta \wedge \alpha) \Rightarrow \mathcal{B}\beta$
– $\overset{CT4O^e}{\models} (\mathcal{B}\alpha \Rightarrow \beta \wedge \mathcal{B}\alpha) \Rightarrow \beta$

The proof of these results can be found in [Mont96].

The logic just defined provide us with a framework where the knowledge of the another agent can be represented and reasoned about, and the essential properties of specificity of CT4O remain true when the system reason about the knowledge of the other agent. Thus according to theorem 4, if the system believes initially that if there are birds the other agent believes that they fly and there are birds then, by default, the system believes that the other agent believes that they fly. Formaly:

$$\overset{CT4O^e}{\models} ((bird \Rightarrow \mathcal{B}\, fly) \wedge bird) \Rightarrow \mathcal{B}\, fly$$

But the logic goes even further, taking into consideration the logic of the $\mathcal{B}$ operator itself. If to the initial assertion we add $\mathcal{B}\,\neg fly$ then we cannot infer by default that $\mathcal{B}\, fly$. In the same way, we have:

$$\overset{CT4O^e}{\not\models} (bird \Rightarrow \mathcal{B}\, fly \wedge bird \wedge \neg\mathcal{B}\, fly) \Rightarrow \mathcal{B}\, fly$$

Similarly, from $\mathcal{B}bird \Rightarrow \mathcal{B}fly$, $\Box(\mathcal{B}penguin \supset \mathcal{B}bird)$, $\mathcal{B}penguin \Rightarrow \mathcal{B}\neg fly$, and $\mathcal{B}\, penguin$, the more specific default applys and cancels the more generic one, and one could infer that $\mathcal{B}\neg fly$. Notice that this is not similar to example 1, since in this case, the fly and $\neg fly$ are within the scope of the belief operator.

And also from the above assertions we can derive the default $\mathcal{B}\, bird \Rightarrow \neg\mathcal{B}\, penguin$, but not $\mathcal{B}(bird \Rightarrow \neg penguin)$. This last inference would be an

[3] In fact we can prove a stronger result: $\overset{CT4O}{\models} \alpha$ iff $\overset{CT4O^e}{\models} \alpha$.

attribution of some belief to the other agent based on the system's beliefs, which is incorrect.

An unexpected mode of inference that is allowed in $CT4O^e$ is demonstrated by the following exemple. If the system believes that: if there are birds then the other agent, normally, believes that; the other believes that the birds normally fly; and given that in the most normal states of affaires there are birds, then one can conclude by default that the other agent believes that they fly. Formaly:

$$\overset{CT4O^e}{\models} \quad \Diamond\Box bird \wedge (bird \Rightarrow \mathcal{B}\, bird) \wedge \mathcal{B}(bird \Rightarrow fly) \quad \Rightarrow \quad \mathcal{B}\, fly$$

This inference shows that $CT4O^e$ allows for some chaining across contexts: the system's defaults can be composed with the other agent's defaults.

5 The extension of CO

We can extend the logic CO, in the same way that $CT4O$ was extended. Let us remind that the difference between $CT4O$ and CO is that the normality relation in CO is totally connected.

Thus, in the definition of a A-world, $\langle w, \langle W, \geq \rangle \rangle$ we also demand $\geq$ to be totally connected. The relations $\geq_\triangle$ and B-worlds are defined in a similar way, and for the relation $\geq_\Box$ we require the base relation to be totally connected.

Definition 15 *A CO^e-model is a pair $\langle X, \geq \rangle$, where X is a set of B-worlds such that $\Pi_1(X) = W$, and $\geq$ is a reflexive, transitive and totally connected binary relation defined on W, and $\geq_\Box$ is a relation totally connected on X as defined for $CT4O^e$.*

Furthermore, if $\langle w, \{\langle w_i, M_i \rangle\}_{\{i \in I\}}\}\rangle \in X$ and if $\{\langle w'_i, M_i \rangle\}_{\{i \in I\}}\}\rangle$ is a set of augmented worlds, such that for all $i \in I$, $\langle w_i, M_i \rangle \geq_\triangle \langle w'_i, M_i \rangle$, then there is w_0 such that $w \geq w_0$ and $\langle w_0, \{\langle w'_i, M_i \rangle\}_{\{i \in I\}}\}\rangle \in X$ and the models of the augmented worlds are CO-models..

The logic CO^e keeps all the properties of $CT4O^e$ and have some new ones. For example, from $\mathcal{B}(bird) \not\Rightarrow \mathcal{B}(penguin)$ and $\mathcal{B}(bird) \Rightarrow \mathcal{B}(fly)$ we can deduce in CO^e:

$$\mathcal{B}(bird) \wedge \mathcal{B}(\neg penguin) \Rightarrow \mathcal{B}(fly)$$

This deduction cannot be done in $CT4O^e$.

6 Conclusions

What has been accomplished? The epistemic extension of both CT4O and CO proposed in this paper extend both logics so that they can deal with formulas that contain a belief operator in such a way that all properties (and short comings) of the original logics are retained and the desired properties of epistemic extended logics (2-5) are satisfied (the property (3) is equivalent to theorem 3 given that the language does not allow for nested $\mathcal{B}$). Furthermore, we believe that the methodology used to develop the logics $CT4O^e$ and CO^e (that is, the use of augmented worlds, B-worlds, and the definition of the $\geq_\Box$ relations based on the $\geq$ relation) can be used to extend some of the other semantically defined conditional logics.

But such approach has some limitations. [Bou94] proposes the logic CO^* which offers a limited solution for the problem of relevance in conditional logics. The logic is a CO logic where all propositional valuations are among the possible worlds, and the solution for the irrelevance problems depends both on the fact that the normality relation is totally connected and that all valuations are among the possible worlds. But following the methodology proposed herein it is not possible to develop an epistemic extension of the logic CO^*, since it is not possible to construct a structure where all possible B-worlds are present and where the $\geq_\Box$ relation is totally connected.

Conditional logics can be seen as a core non-monotonic logic, to which some extra logic features are added to strengthen the logic. For example, [Del87] adds to a conditional theory a set of formulas that cannot be proven false in ordered to deal with the irrelevance problem. Similar extra logic extensions are proposed in [Bel90, Del94]. We believe that the two epistemically extended conditional logics developed here could be amenable for such extra-logical extension in order to strengthen them, but with some caveats: such extra-logical strengthenings must work also within the $\mathcal{B}$ operator. That may pose some further complications.

Finally, we leave a syntactic characterization of the extended logics for future work. Since both $CT4O$ and CO have sound and complete syntactic characterizations, and since the epistemic part of these extended logics can be characterized by the standard modal logic KD, we have hopes that a sound and complete descriptions of both $CT4O^e$ and CO^e can be obtained.

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