



Analysis of Variance—ANOVA

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Introduction



- *Analysis of variance* is called *ANOVA*.
- ANOVA is used to compare and statistically test the difference between at least two groups (i.e. at least two sample means).
 - ❖ Example: The average number of years of education is different for men than women.
 - ❖ Example: The average degree of racial tolerance is different for people of different religions (Catholic, Jewish, Protestant, other).

Introduction



- Groups in ANOVA are numbered from 1 to j where j is the number of groups.
- ANOVA tests the null hypothesis that all j sample means come from the same population and therefore are all equal to 1) each other, and 2) the mean without regard to group membership.

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_j$$

Introduction



- The alternative hypothesis is that at least one sample mean comes from a population that has a different mean than the population associated with the remaining sample means.
- Rejecting the null hypothesis implies one of the following:

1. Each population mean differs from every other mean:

$$H_1 : \mu_1 \neq \mu_2 \neq \dots \neq \mu_j$$

2. Some subsets among the population means differ from one another (e.g. μ_1 is different from μ_2 but is the same as μ_3 and μ_4).
3. Some combination of means are different from a single mean or combination of means from other groups (e.g. μ_2 differs from the average of μ_3 and μ_4).

Effects of Variables



- If the j group means all equal each other, then they also will equal the population mean, μ .
- We use this idea to measure the *effect* of being in a group on a dependent variable.

$$\alpha_j = \mu_j - \mu$$

where :

α_j = the effect

μ_j = the mean of group j

μ = the overall or grand mean

Effects Example



Average Sexual Frequency by Region

Means ($n=2,320$)	Effects
Grand Mean, $\mu_{\text{sexfreq}}=58.60$	
South, $\mu_S=58.91$	$58.91-58.60=0.31$
Midwest, $\mu_M=59.73$	$59.73-58.60=1.13$
Northeast, $\mu_N=52.84$	$52.84-58.60=-5.76$
West, $\mu_W=62.28$	$62.28-58.60=3.68$

Effects Example



Main Statistics		
Cells contain: -Means -N of cases		
REGION	1: Northeast	52.84 450
	2: South	58.91 830
	3: Midwest	59.73 590
	4: West	62.28 450
	COL TOTAL	58.60 2,320

$$H_0 : \mu_S = \mu_M = \mu_N = \mu_W$$

Analysis of Variance						
	SSQ	Eta_sq	df	MSQ	F	P
REGION	21,832.913	.002	3	7,277.638	1.650	.1759
Residual	10,218,179.844	.998	2,316	4,411.995		
Total	10,240,012.757	1.000	2,319			

The ANOVA Model



- ANOVA tries to report the proportion of the variation in a dependent variable that can be attributed to an observation (i) being in a group (j).
- Each observation can be broken down or decomposed into three components:

$$Y_{ij} = \mu + \alpha_j + e_{ij}$$

Where:

Y_{ij} = the score of the i^{th} observation in the j^{th} group

μ = the grand mean common to all groups

α_j = the effect of group j , common to every observation in that group

e_{ij} = the error score, unique to the i^{th} observation in the j^{th} group

The ANOVA Model



$$Y_{ij} = \mu + \alpha_j + e_{ij}$$

$$e_{ij} = Y_{ij} - \mu - \alpha_j$$

- Rearranging terms we see that the *error term*, or *residual*, is the part of the observed score that cannot be attributed to either the common component or the group component.
- Error terms are discrepancies between observed scores and those predicted by group membership.



The ANOVA Table: Sum of Squares

$$SS_{total} = \sum_{i=1}^n (Y_i - \bar{Y})^2 = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$$

	<i>Slow</i>	<i>Normal</i>	<i>Fast</i>
	<i>j=1</i>	<i>j=2</i>	<i>j=3</i>
<i>i=1</i>	23	27	23
<i>i=2</i>	22	28	24
<i>i=3</i>	18	33	21
	15	19	25
•	29	25	19
•	30	29	24
	23	36	22
•	16	30	17
	19	26	20
<i>i=10</i>	17	21	23

$$\begin{aligned} SS_{total} &= (23 - 24.47)^2 + (22 - 24.47)^2 + \dots + (17 - 24.47)^2 + \\ &\quad (27 - 24.47)^2 + (28 - 24.47)^2 + \dots + (21 - 24.47)^2 + \\ &\quad (23 - 24.47)^2 + (24 - 24.47)^2 + \dots + (23 - 24.47)^2 \\ &= 769.47 \end{aligned}$$

The ANOVA Table: Sum of Squares



$$SS_{total} = \sum_{i=1}^n (Y_i - \bar{Y})^2 = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$$

$$\begin{aligned} Y_{ij} - \bar{Y} &= Y_{ij} - \bar{Y} + \bar{Y}_j - \bar{Y}_j \\ &= \underbrace{(Y_{ij} - \bar{Y}_j)}_A + \underbrace{(\bar{Y}_j - \bar{Y})}_B \end{aligned}$$

$$\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2 = \underbrace{\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2}_A + \underbrace{\sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2}_B$$

Each observations deviation from the mean is a function of:

- A) The deviation of each observation from it's group mean, and
- B) the deviation of each group mean from the grand mean

The ANOVA Table: Sum of Squares



The total sum of squares SS_{total} may also be decomposed into two parts:

$$SS_{total} = \underbrace{SS_{within}}_A + \underbrace{SS_{between}}_B$$

$$SS_{within} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$$

$$SS_{between} = \sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2$$

- The within-group SS summarizes or reflects the operation of unmeasured or random factors.
- The between-group SS summarizes the effects of the independent classification variable under study.

The ANOVA Table: Sum of Squares

$$\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2 = \underbrace{\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2}_{\text{within}} + \underbrace{\sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2}_{\text{between}}$$

- If the null hypothesis is true, we would observe equal means in all j groups and the group means would equal the grand mean.
 - ❖ This means that the SS_{between} is equal to zero, and therefore SS_{within} is equal to SS_{total} .
- In other words, all of the observed variation in the dependent measure Y is due to *random error variance*.

$$SS_{\text{total}} = SS_{\text{within}} + SS_{\text{between}}$$

The ANOVA Table: Mean Squares

- After calculating the sum of squares, the next step is to calculate the *mean squares* corresponding to $SS_{between}$ and SS_{within} .
- Each mean square is an estimate of a variance.
 - ❖ The first due to group effects
 - ❖ The second due to error.
- If no group effect exists, the two estimates should be identical.
- If a significant effect exists, $SS_{between}$ will be larger than SS_{within} .

The ANOVA Table: Mean Squares

- *Mean squares* are calculated by dividing the sum of squares by the appropriate degrees of freedom (*df*).
 - ❖ SS_{within} is divided by $n-j$ and is called *mean square within*, or MS_{within} .
 - ❖ $SS_{between}$ is divided by $j-1$ and is called *mean square between*, or $MS_{between}$.

$$MS_{within} = \frac{\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2}{n - j}$$
$$= \frac{SS_{within}}{n - j}$$

$$MS_{between} = \frac{\sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2}{j - 1}$$
$$= \frac{SS_{between}}{j - 1}$$

The ANOVA Table: F -Ratio



- Logically, we can examine the ratio of the two mean squares, $MS_{between}$ to MS_{within} .
 - ❖ If the categorization or independent measure has no effect, than this ratio will be small.
 - ❖ If the categorization or independent measure has an effect, the ratio will be large.
- The ratio of two mean squares is distributed as F .

$$F_{j-1, n-j} = \frac{MS_{between}}{MS_{within}} = \frac{\frac{SS_{between}}{j-1}}{\frac{SS_{within}}{n-j}} = \frac{\frac{\sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2}{j-1}}{\frac{\sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2}{n-j}}$$

ANOVA Table



ANOVA Table

Source of Variation	SS	df	MS	F
Between	$SS_{between} = \sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2$	$j-1$	$MS_{between} = \frac{SS_{between}}{j-1}$	$\frac{MS_{between}}{MS_{within}}$
Within	$SS_{within} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$	$n-j$	$MS_{within} = \frac{SS_{within}}{n-j}$	
Total	$SS_{total} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$	$n-1$		

ANOVA Table



$$\mu_{South} = \mu_{Midwest} = \mu_{Northeast} = \mu_{West}$$

$$\mu_{South} = 58.91$$

$$\mu_{Midwest} = 59.73 \quad 1998 \text{ GSS}$$

$$\mu_{Northeast} = 52.84$$

$$\mu_{West} = 62.28$$

$$\overline{sexfreq} = 58.60$$

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ANOVA Table

Source of Variation	SS	df	MS	F
Between	21,832.913	3	7,277.638	1.650
Within	10,218,179.844	2,316	4,411.995	
Total	10,240,012.757	2,319		

$\alpha = .05$

v_1/v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: Adapted from E. S. Pearson and H. O. Hartley: *Biometrika Tables for Statisticians*, 2nd ed. (Cambridge: Cambridge University Press, 1962).

ANOVA Example



- A study was done to understand the influence of certain auditory and visual stimuli on the ability of people to recall spoken materials.
 - ❖ Specifically, what is the effect of lack of synchrony between auditory and visual cues upon retention of spoken words?
- The study:
 - ❖ Thirty students randomly assigned to three groups. Each group was shown a movie of someone reciting a list of fifty words.
 - Group 1 (Fast): sound preceded lip movements.
 - Group 2 (Normal): sound and lip movement in normal synchrony.
 - Group 3 (Slow): lip movements precede sound.
 - ❖ Participants were asked to recall as many of the fifty words as possible
- Research question: Is there a difference in the average word recall between these three groups?

<i>Fast</i>	<i>Normal</i>	<i>Slow</i>
23	27	23
22	28	24
18	33	21
15	19	25
29	25	19
30	29	24
23	36	22
16	30	17
19	26	20
17	21	23

Steps in Hypothesis Testing



- There are five basic steps in hypothesis testing:
 - 1) Assume the *null* hypothesis of no difference
 - 2) We have to have an idea about the range of outcomes if the null hypothesis is true. We obtain this from an appropriate *sampling distribution*.
 - 3) We have to decide or set a criterion for enough evidence to be convinced that the null hypothesis is false. This is a significance level called *alpha* or α .
 - 4) We have to go to the real world and collect data. That is determine some sample statistic.
 - 5) We compare 4 with 3 and reject or fail to reject the null hypothesis. If the value we calculate falls in the critical region or exceeds the critical value associated with α , we must reject the null hypothesis; otherwise we fail to reject it.



Steps in Hypothesis Testing

- There are five basic steps in hypothesis testing:
- 1) Assume the null hypothesis of no difference

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_1 : \mu_1 \neq \mu_2 \neq \mu_3$$

Steps in Hypothesis Testing



- There are five basic steps in hypothesis testing:
 - 2) We have to have an idea about the range of outcomes if the null hypothesis is true. We obtain this from an appropriate sampling distribution.

When comparing more than two or more means we can use an F -test. Therefore, we use the F sampling distribution.

Steps in Hypothesis Testing



- There are five basic steps in hypothesis testing:
 - 3) We have to decide or set a criterion for enough evidence to be convinced that the null hypothesis is false. This is a significance level called alpha or α .
- Using $\alpha=0.05$ with 2 and 27 degrees of freedom ($j-1$ and $n-j$, respectively, $\alpha_{critical}$ is equal to 3.35.
 - ❖ If our calculated F exceeds this value, we reject the null hypothesis, otherwise we fail to reject the null hypothesis.

$\alpha = .05$

v_1/v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: Adapted from E. S. Pearson and H. O. Hartley: *Biometrika Tables for Statisticians*, 2nd ed. (Cambridge: Cambridge University Press, 1962).

Steps in Hypothesis Testing



- There are five basic steps in hypothesis testing:
 - 4) We have to go to the real world and collect data.
That is determine some sample statistic.
- In this case we need to calculate the within and between mean squares.

ANOVA Example



$$SS_{total} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$$

$$\overline{\text{recall}} = 23.467$$

Slow	23	0.22
Slow	22	2.15
Slow	18	29.88
Slow	15	71.68
Slow	29	30.62
Slow	30	42.68
Slow	23	0.22
Slow	16	55.75
Slow	19	19.95
Slow	17	41.82

Normal	27	12.48
Normal	28	20.55
Normal	33	90.88
Normal	19	19.95
Normal	25	2.35
Normal	29	30.62
Normal	36	157.08
Normal	30	42.68
Normal	26	6.42
Normal	21	6.08

Fast	23	0.22
Fast	24	0.28
Fast	21	6.08
Fast	25	2.35
Fast	19	19.95
Fast	24	0.28
Fast	22	2.15
Fast	17	41.82
Fast	20	12.02
Fast	23	0.22

$$SS_{total} = (23_{1,1} - 23.467)^2 + (22_{2,1} - 23.467)^2 + \dots + (23_{10,3} - 23.467)^2$$

$$= 769.467$$

ANOVA Example



$$SS_{within} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$$

$$\begin{aligned}\overline{Fast} &= 21.8 \\ \overline{Normal} &= 27.4 \\ \overline{Slow} &= 21.2\end{aligned}$$

$$\begin{aligned}SS_{within} &= (23_{1,1} - 21.8)^2 + (22_{2,1} - 21.8)^2 + \dots \\ &\quad + (27_{1,2} - 27.4)^2 + (28_{2,2} - 27.4)^2 + \dots \\ &\quad + (23_{1,3} - 21.2)^2 + (24_{2,3} - 21.2)^2 \\ &= 535.6\end{aligned}$$

ANOVA Example



$$SS_{between} = \sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2$$

$$\overline{Fast} = 21.800$$

$$\overline{Normal} = 27.400$$

$$\overline{Slow} = 21.200$$

$$\overline{Recall} = 23.467$$

$$\begin{aligned} SS_{between} &= 10(21.8 - 23.467)^2 + 10(27.4 - 23.467)^2 + 10(21.2 - 23.467)^2 \\ &= 233.867 \end{aligned}$$

$$\begin{aligned} SS_{total} &= SS_{within} + SS_{between} \\ 769.467 &= 535.600 + 233.867 \end{aligned}$$

ANOVA Example



ANOVA Table

Source of Variation	SS	df	MS	F
Between	$SS_{between} = \sum_{j=1}^j n_j (\bar{Y}_j - \bar{Y})^2$	$j-1$	$MS_{between} = \frac{SS_{between}}{j-1}$	$\frac{MS_{between}}{MS_{within}}$
Within	$SS_{within} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_j)^2$	$n-j$	$MS_{within} = \frac{SS_{within}}{n-j}$	
Total	$SS_{total} = \sum_{j=1}^j \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y})^2$	$n-1$		

ANOVA Example



ANOVA Table

Source of Variation	SS	<i>df</i>	MS	<i>F</i>
Between	233.867	2	116.933	5.895
Within	535.600	27	19.837	
Total	769.467	29		

Steps in Hypothesis Testing



- There are five basic steps in hypothesis testing:
 - 5) We compare 4 with 3 and reject or fail to reject the null hypothesis. If the value we calculate falls in the critical region or exceeds the critical value associated with α , we must reject the null hypothesis; otherwise we fail to reject it.
- $F_{achieved}$ is greater than $F_{critical}$ ($5.895 > 3.35$), so we reject the null hypothesis and conclude that at least one group mean statistically significantly differs from one of the other means.