LaserTank is NP-Complete

Per Alexandersson and Petter Restadh[(✉)]

KTH The Royal Institute of Technology, 100 44 Stockholm, Sweden
`per.w.alexandersson@gmail.com`, `petterre@kth.se`

Abstract. We show that the classical game *LaserTank* is NP-complete, even when the tank movement is restricted to a single column and the only blocks appearing on the board are mirrors and solid blocks. We show this by reducing 3-SAT instances to LaserTank puzzles.

Keywords: NP-completeness · LaserTank · 3-SAT

1   Introduction

From Wikipedia: "*LaserTank (also known as Laser Tank) is a computer puzzle game requiring logical thinking to solve a variety of levels*".[1] It was first released on the Windows platform in 1995, and a similar game was released in 1998 for the graphing calculator Texas Instruments Ti-83, under the name *Laser Mayhem.*[2] To our knowledge, the complexity of LaserTank has not been studied before, while several other classical games have been shown to be (co)NP-complete, NP-hard or PSPACE-complete. For example, *Sokoban* [3], *Tetris* [2], *Rush Hour* [4], and *Minesweeper* [5,6] to list a few.

   In this short note, we prove the following.

Theorem 1. *LaserTank is* NP-*complete.*

It should be noted that one can perhaps apply more general meta-theoretical approaches for puzzle games and planning games in particular, to prove NP-completeness. It is likely that the framework by Viglietta [7]—which can be applied to games such as *Boulder Dash*, *Pipe Mania* and *Starcraft*—can successfully be applied to LaserTank as well. We opted for a self-contained hands-on approach where 3-SAT is reduced to LaserTank. Furthermore, we only use a small subset of the available pieces in the original game, as well as restrict the movement of the tank in two directions. These restrictions have the benefit that they imply that the *Laser Mayhem* variant is also NP-complete.

1.1   Short Background on 3-SAT

A 3-*SAT expression E* is a conjunction of clauses, where each clause involves exactly three distinct literals. A literal is either a boolean variable, or its

---

[1] https://en.wikipedia.org/wiki/LaserTank.
[2] https://www.ticalc.org/archives/files/fileinfo/95/9532.html.

negation. The 3-*SAT problem* states: Determine if E is *satisfiable*—that is, there is an assignment of truth values to the variables that makes E true. For example, $E = (x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee \neg z) \wedge (x \vee \neg y \vee z)$ is such a conjunction, and the assignment $x, z = \texttt{true}$, $y = \texttt{false}$ shows that E is satisfiable. Determining satisfiability of a 3-SAT expression is an NP-complete problem [1].

2   LaserTank

LaserTank is a turn-based single-player puzzle game played on a 2-dimensional grid (the *board*), where in each turn, the player either moves the tank, or fires a laser from the tank. The laser interacts with different *pieces* on the board, and the goal is to hit a certain piece with the laser. The pieces[3] we use are *mirrors* $\{\triangleright, \triangledown, \triangle, \triangledown\}$, *solid blocks* ■, *movable blocks* ▢, the *tank* ▷, and the *goal* ⊕. The tank is the only piece directly controlled by the player, and the laser exits the tank from the front, which is the pointy end of ▷. In our version, the tank is restricted to sideways movement only, see Example 1. In our final notes we explain how we take care of the case where the movement of the tank is not restricted. The tank can fire a laser from the front. If the laser hits a mirror on a slanted edge it is reflected. When a mirror is hit on one of the two (non-reflective) short edges by the laser, the mirror is pushed in the direction of the laser. A movable block is pushed one step if it is hit by the laser. A movable block or a mirror is only pushed if the tile directly behind it is empty. The aim of the puzzle is to hit the goal piece with the laser. The solid blocks do not allow lasers or the tank to pass through and they do not move when hit by the laser. The following example shows all game mechanics in action.

Example 1. Here is a small instance of the problem, with a step-by-step solution. The tank fires a laser which moves a mirror (1), then takes one step sideways, (2). It then shoots a laser at the movable block (3), and finally moves in position to have a clear shot of the goal (4).
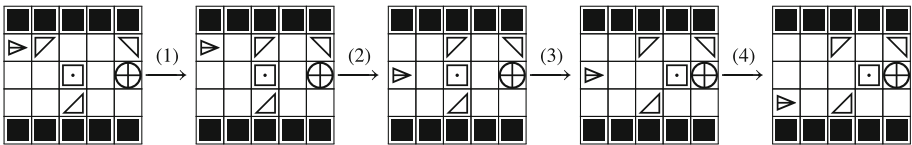
   Our goal is now to construct puzzles which imitates an instance of 3-SAT. We employ so called *gadgets* that emulate boolean operations. Below, we let ⇢, ↓ indicate the inputs to the gadgets (considered as boolean variables), and $\{\overset{}{\dashrightarrow} *, \overset{\vdots}{\downarrow}\}$ indicate inputs that are always available as clear shots from the tank. The latter are used for producing the output of the gadget.

---

[3] For a complete list of pieces available in the official game, see https://lasertan ksolutions.blogspot.com/p/in-my-opinionlaser-tank-is-best-logic.html.
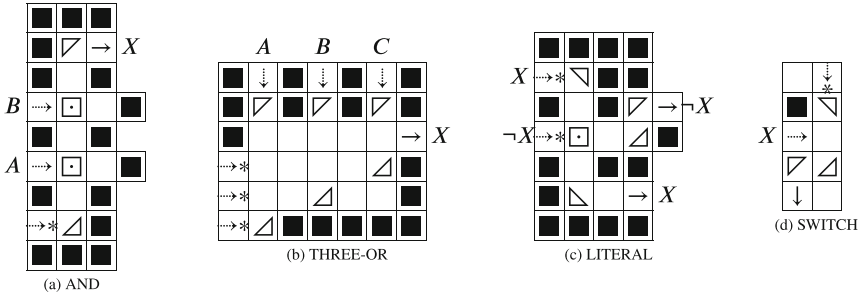
Fig. 1. The and-gadget, three-or-gadget, literal-gadget, and switch-gadget.

The and Gadget. The configuration in Fig. 1a serves as our **and**-gadget. We need to shoot through both A and B in order to allow for $A \wedge B = X$ as output. Notice that the two movable blocks can only be moved up, right and down. If we want the gadget to produce an output through X, all movable blocks must be moved out of the way. This can only be accomplished if the movable block must have been moved to the right via activation from both A and B, which shows that the gadget is indeed an **and**-gadget. The **and**-gadget can easily be generalized to more than two inputs.

The **three-or** gadget is depicted in Fig. 1b. If either of the inputs A, B or C are available, then X allows for output. The only way to produce output from X is to move a $\triangledown$ to the same row as X. The $\triangledown$ can only be moved into that row from above and thus we must have some input from A, B or C in order for a laser to pass out through X. Thus the **three-or** gadget works the way intended.

The **literal** gadget is depicted in Fig. 1c. This gadget emulates a literal, with two different mutually exclusive outputs depending on the choice of value of the literal. To unlock X as output, fire once through $\neg X$ first. This moves the movable block out of the way but prevents $\neg X$ from being available as output. Similarly for $\neg X$.

The **switch** gadget is depicted in Fig. 1d. The **switch**-gadget is our main building block for encoding an instance of a 3-SAT problem. It allows for the input X to be available first as output to the right, then redirected down. This allows X to be used in multiple **or**-clauses.

Example 2. In the puzzle in Fig. 2, only a single "input", X, is available. However, with the **switches** we can redirect input X to activate the **and** gadget. Notice the two $\triangledown$ pieces that are required to activate the **switches** and that the rightmost switch gadget must be used first in order to solve the puzzle. This is also true in the general setup, where switches should be used from right to left.

2.1   The Reduction

A 3-SAT expression may now be encoded as a LaserTank puzzle as follows. There is one **literal**-gadget for each variable appearing in the expression, a
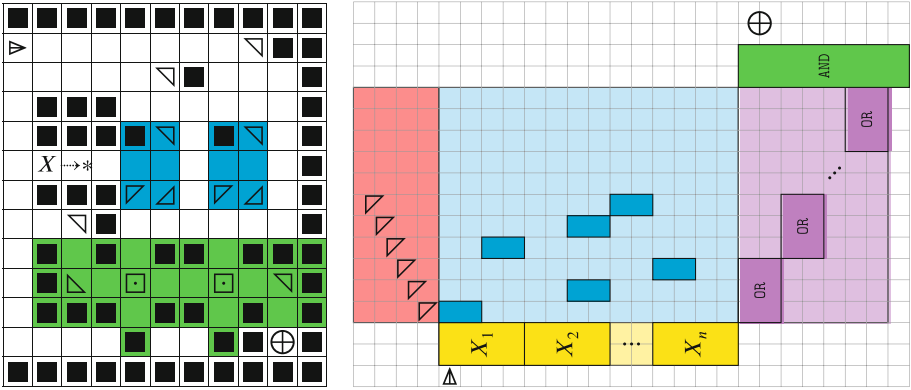
Fig. 2. Left: A small puzzle showing the use of two `switch`-gadgets and one `and`-gadget. Right: Layout of a general 3-SAT puzzle. Above each `or`-gadget are three switches, corresponding to the three literals involved on the or-clause.

`three-or-gadget` for each `or`-clause, and a single `and`-gadget with multiple inputs is used to bind all together. The puzzle is designed such that the output of the `and`-gadget is the only way to hit the goal. The general layout of such a puzzle is shown in Fig. 2. For each three-or-clause in the 3-SAT expression, three switches are placed on the board corresponding to the three literals involved. In other words, the clauses of the 3-SAT expression are encoded via `switch`-gadgets. The switches can always be activated via the ◸ pieces at the top of the board as in Fig. 2. As a concrete example, the expression $(A \vee B \vee \neg C) \wedge (A \vee \neg B \vee C)$ is encoded as the puzzle shown in Fig. 3.

The following lemma shows that solving LaserTank puzzles can be done in polynomial time with a non-deterministic Turing machine. Hence LaserTank is in NP.

Lemma 1. *A solution consisting of k steps to a LaserTank puzzle on a board of size n can be verified in time $O(kn)$.*

Proof. It is straightforward to show that the laser movement is time-reversible. This implies that it is impossible for a laser shot by the tank to end up in an "infinite loop" while being reflected by mirrors. Remember also that the laser stops as soon as it hits a solid block, a movable block, or moves a mirror. It follows that after firing the laser, it takes less than $4n$ steps before the laser finds its final destination, where n is the number of tiles on the board. Simulating a sequence of k moves thus requires $O(kn)$ time.

From our construction, it is a straightforward calculation to see that given a 3-SAT expression with V variables and C clauses gives a puzzle contained on a board with size $(7V + 9C + 4)(7C + 10)$. This is evidently polynomial in the size of the expression.
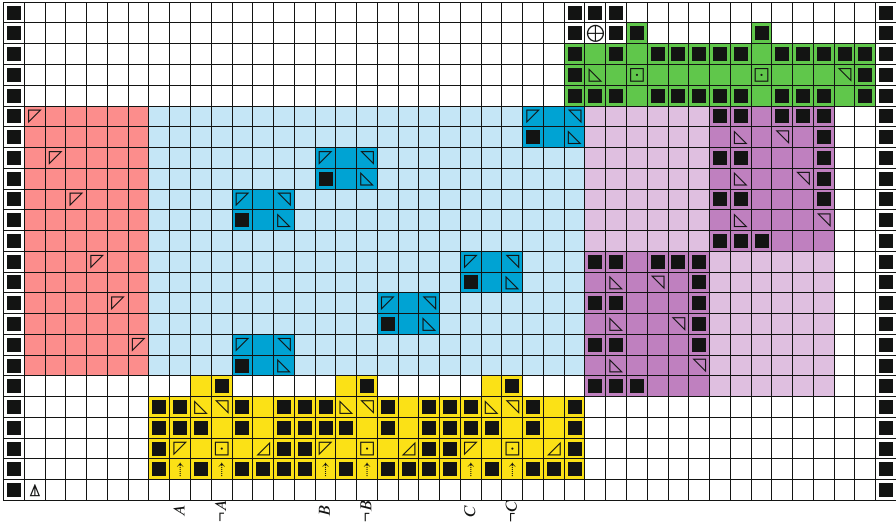
Fig. 3. The puzzle corresponding to the expression $(A \lor B \lor \neg C) \land (A \lor \neg B \lor C)$. Note that if $A = \texttt{true}$, the expression is satisfied. Thus this particular puzzle can be solved without deciding truth values for the variables B and C, and the movable blocks in the B and C variable gadgets do not need to be moved.

Proof (of Theorem 1). A 3-SAT problem can be converted to a LaserTank puzzle in polynomial time since the board size is a polynomial in the number of variables and clauses. Furthermore, a solution to such a LaserTank puzzle can easily be translated back to a solution of the original 3-SAT problem in polynomial time, by simply performing all the steps and reading how the `variable`-gadgets have been used. Note that a LaserTank puzzle solution might not decide the truth value of some variables (see caption of Fig. 3), in which case, one may simply let these values be `true`. Given a solution to the 3-SAT problem we can easily construct a solution to the corresponding LaserTank instance. Thus the 3-SAT problem is satisfiable if and only if the corresponding LaserTank puzzle has a solution. According to Lemma 1, the translation of a puzzle solution to a 3-SAT solution only requires a polynomial time in the input size (number of steps). This shows that LaserTank is at least as hard as 3-SAT. Finally, Lemma 1 shows that a solution can be verified in polynomial time and hence LaserTank is NP-complete.

Notice that in both the `and`– and `literal`-gadget, each movable block can be replaced with a $\triangledown$-mirror without changing the behavior of the gadget. Thus Theorem 1 is valid even in the case when restricting to puzzles without movable blocks. Furthermore, we can extend Theorem 1 to the case where the tank can turn and move in all four directions. To do so, we need to make sure the tank only has access to the same inputs as in the previous setup. This can be done by inserting additional rows in the puzzle such that every other row is empty, and

then inserting two columns in with the pattern between the initial position of the tank and the rest of the board. We leave the details to the reader.

Finally, we note that in some variants of LaserTank where the tank may turn and move in all directions, the goal is not to shoot at a goal but rather move the tank to the goal. Our construction can also be adapted to this situation, by blocking off a path (using solid blocks) from the first column to the `and`-gadget at the bottom of the instance and remove the mirrors in the `and`-gadget. Then the tank can only move through the `and`-gadget if it has been cleared of movable blocks, and the setup is now equivalent to our previous setup.

References

1. Cook, S.A.: The complexity of theorem-proving procedures. In: Proceedings of the Third Annual ACM Symposium on Theory of Computing, STOC 1971, pp. 151–158. ACM, New York (1971). https://doi.org/10.1145/800157.805047
2. Demaine, E.D., Hohenberger, S., Liben-Nowell, D.: Tetris is hard, even to approximate. Technical report, MIT-LCS-TR-865. MIT, Cambridge (2002). https://arxiv.org/abs/cs/0210020
3. Dor, D., Zwick, U.: SOKOBAN and other motion planning problems. Comput. Geom. **13**(4), 215–228 (1999). https://doi.org/10.1016/s0925-7721(99)00017-6
4. Flake, G.W., Baum, E.B.: Rush Hour is PSPACE-complete, or "Why you should generously tip parking lot attendants". Theor. Comput. Sci. **270**(1–2), 895–911 (2002). https://doi.org/10.1016/s0304-3975(01)00173-6
5. Kaye, R.: Minesweeper is NP-complete. Math. Intell. **22**(2), 9–15 (2000). https://doi.org/10.1007/bf03025367
6. Scott, A., Stege, U., van Rooij, I.: Minesweeper may not be NP-complete but is hard nonetheless. Math. Intell. **33**(4), 5–17 (2011). https://doi.org/10.1007/s00283-011-9256-x
7. Viglietta, G.: Gaming is a hard job, but someone has to do it!. Theory Comput. Syst. **54**(4), 595–621 (2013). https://doi.org/10.1007/s00224-013-9497-5