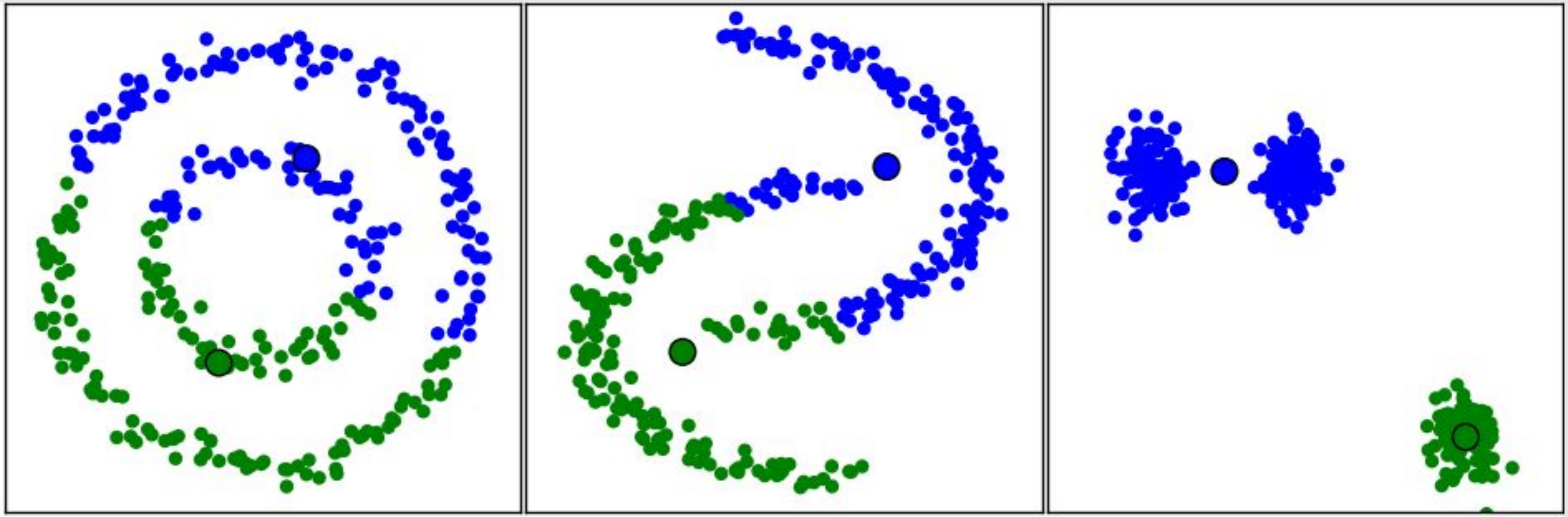
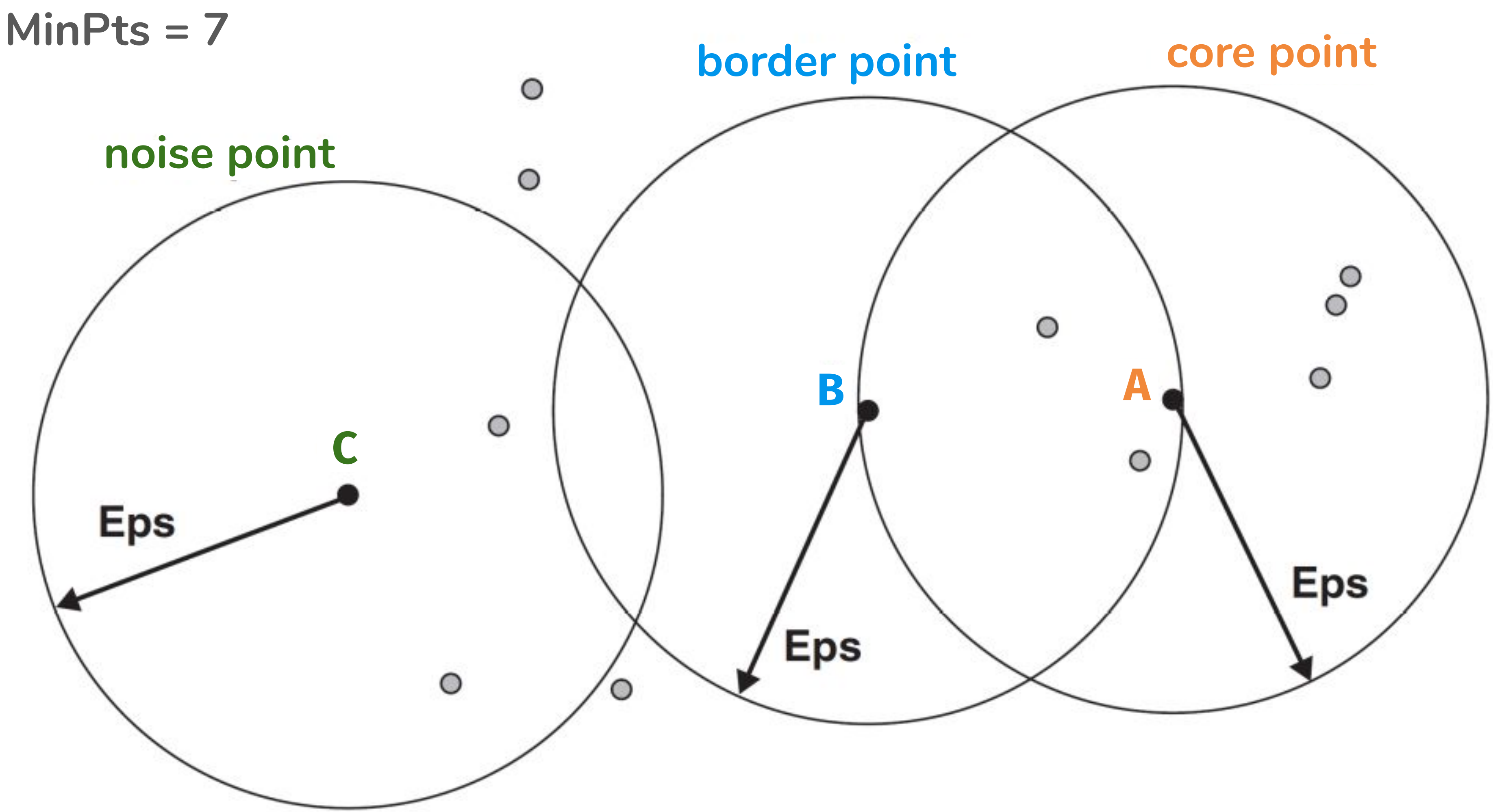
Recall from last time ...

DBSCAN

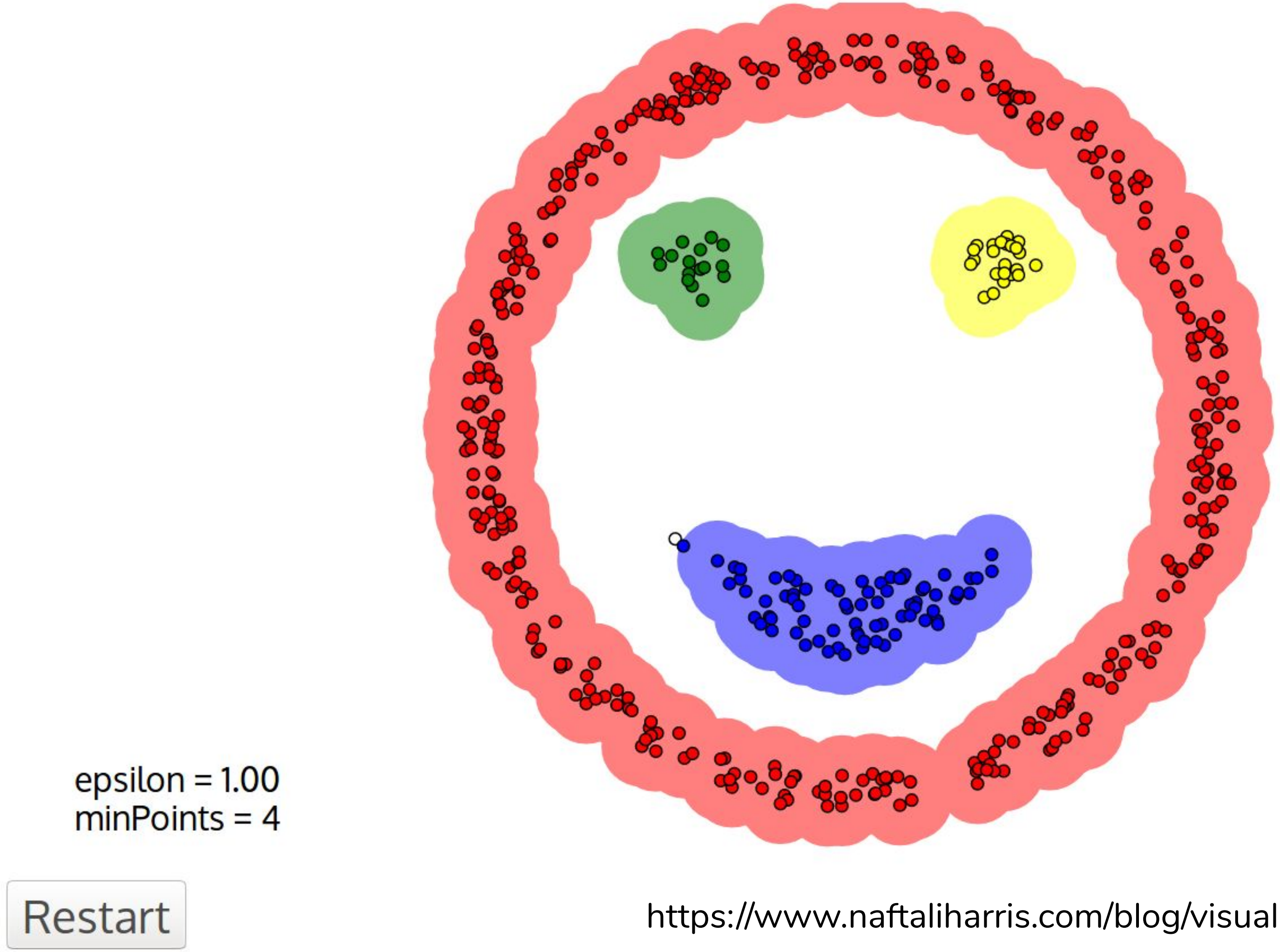


DBSCAN Clustering

- **Core points**: A point is a core point if there are at least ***MinPts*** within a distance of ***Eps***, where *MinPts* and *Eps* are user-specified parameters.
- **Border points**: A border point is not a core point, but falls within the neighborhood of a core point.
- **Noise points**: A noise point is any point that is neither a core point nor a border point.



MinPts = 7
border point
core point
noise point
B
A
C
Eps
Eps
Eps

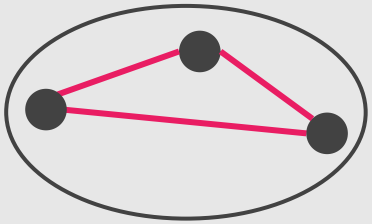


https://www.naftaliharris.com/blog/visualizing-dbscan-clustering

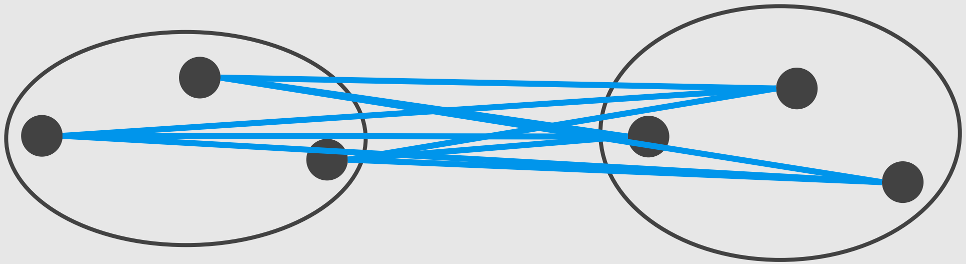
Clustering Performance Evaluation

Silhouette Coefficient

- The silhouette value is a measure of how similar a sample is to its own cluster (**cohesion**) compared to other clusters (**separation**).



Cohesion



Separation

Silhouette Coefficient

- The silhouette value is a measure of how similar a sample is to its own cluster (**cohesion**) compared to other clusters (**separation**).

- The silhouette ranges from −1 to +1.
    - High value = the clustering configuration is appropriate.
    - Low value = the clustering configuration may have too many or too few clusters.

Silhouette Coefficient

- The Silhouette Coefficient is defined **for each sample** and is composed of two scores:
  - a: The mean distance between a sample and all other points **in the same cluster**.
  - b: The mean distance between a sample and all other points **in the next nearest cluster**.

Silhouette Coefficient

- The Silhouette Coefficient s for **a single sample** is given as:

$$s = \frac{b - a}{max(a,b)}$$

- The score is bounded between -1 for incorrect clustering and +1 for highly dense clustering ($a \ll b$). Scores around zero indicate overlapping clusters.

http://scikit-learn.org/stable/modules/clustering.html#clustering

scikit-learn v0.19.0
Other versions

Please **cite us** if you use the software.

2.3. Clustering

Clustering of unlabeled data can be performed with the module `sklearn.cluster`.

Each clustering algorithm comes in two variants: a class, that implements the `fit` method to learn the clusters on train data, and a function, that, given train data, returns an array of integer labels corresponding to the different clusters. For the class, the labels over the training data can be found in the `labels_` attribute.

Input data

One important thing to note is that the algorithms implemented in this module can take different kinds of matrix as input. All the methods accept standard data matrices of shape `[n_samples, n_features]`. These can be obtained from the classes in the `sklearn.feature_extraction` module. For `AffinityPropagation`, `SpectralClustering` and `DBSCAN` one can also input similarity matrices of shape `[n_samples, n_samples]`. These can be obtained from the functions in the `sklearn.metrics.pairwise` module.

2.3.1. Overview of clustering methods

Dimensionality Reduction

Machine Learning

Prof. Sandra Avila
Institute of Computing (IC/Unicamp)

MC886, September 30, 2019

Why is Dimensionality Reduction useful?

Why is Dimensionality Reduction useful?

- Data Compression
  - Reduce **time complexity**: less computation required
  - Reduce **space complexity**: less number of features
  - **More interpretable**: it removes noise

Why is Dimensionality Reduction useful?

- Data Compression
  - Reduce **time complexity**: less computation required
  - Reduce **space complexity**: less number of features
  - **More interpretable**: it removes noise
- Data Visualization

Why is Dimensionality Reduction useful?

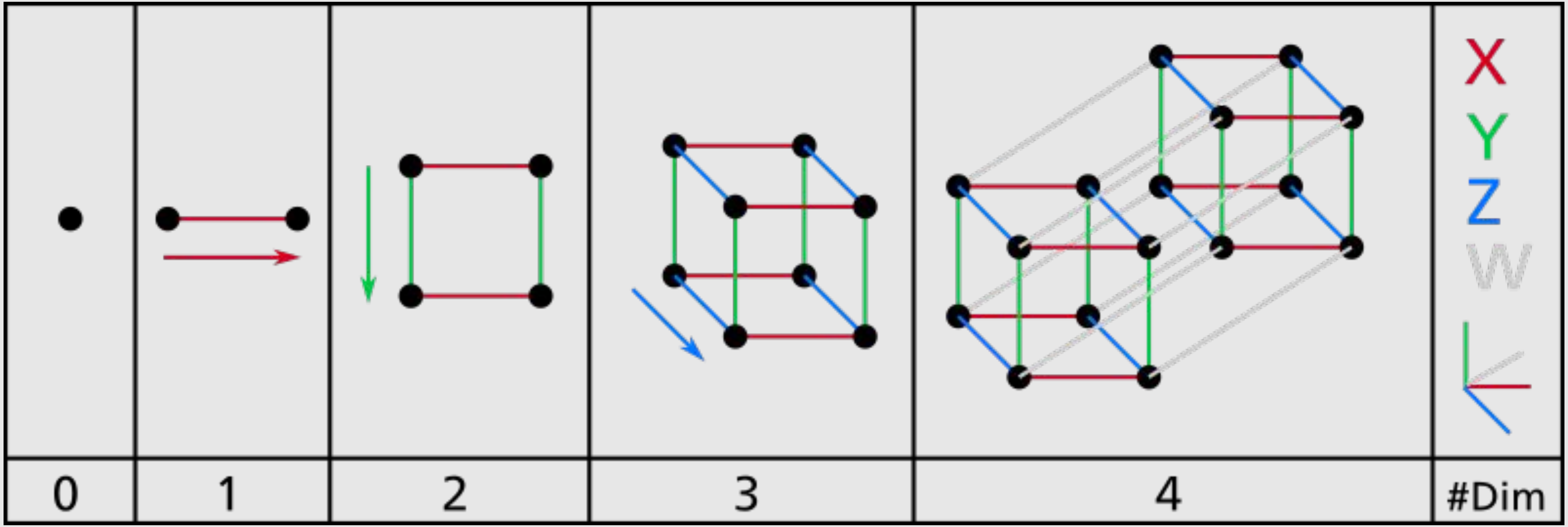
- Data Compression
    - Reduce **time complexity**: less computation required
    - Reduce **space complexity**: less number of features
    - **More interpretable**: it removes noise
- Data Visualization
- To mitigate “**the curse of dimensionality**”

Today's Agenda

- The Curse of Dimensionality
- PCA (Principal Component Analysis)
    - PCA Formulation
    - PCA Algorithm
    - Choosing k

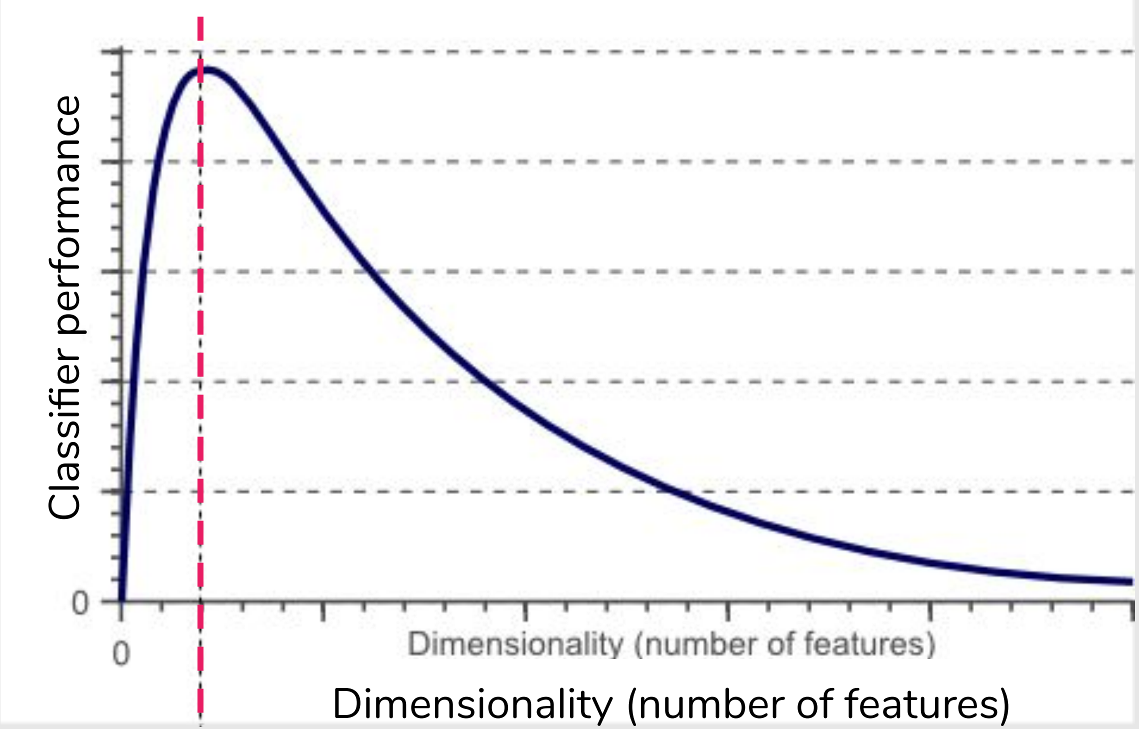
The Curse of Dimensionality

The Curse of Dimensionality



Even a basic 4D hypercube is incredibly hard to picture in our mind.

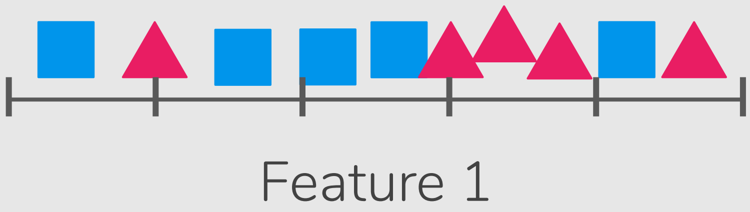
The Curse of Dimensionality



Optimal number of features

The Curse of Dimensionality

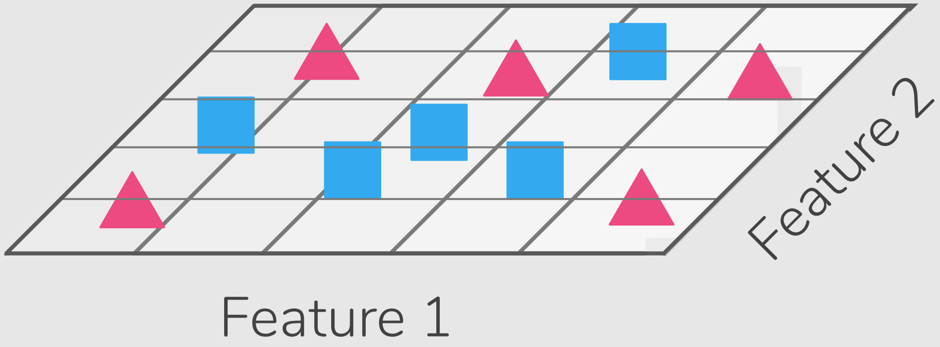
As the dimensionality of data grows, the density of observations becomes lower and lower and lower.



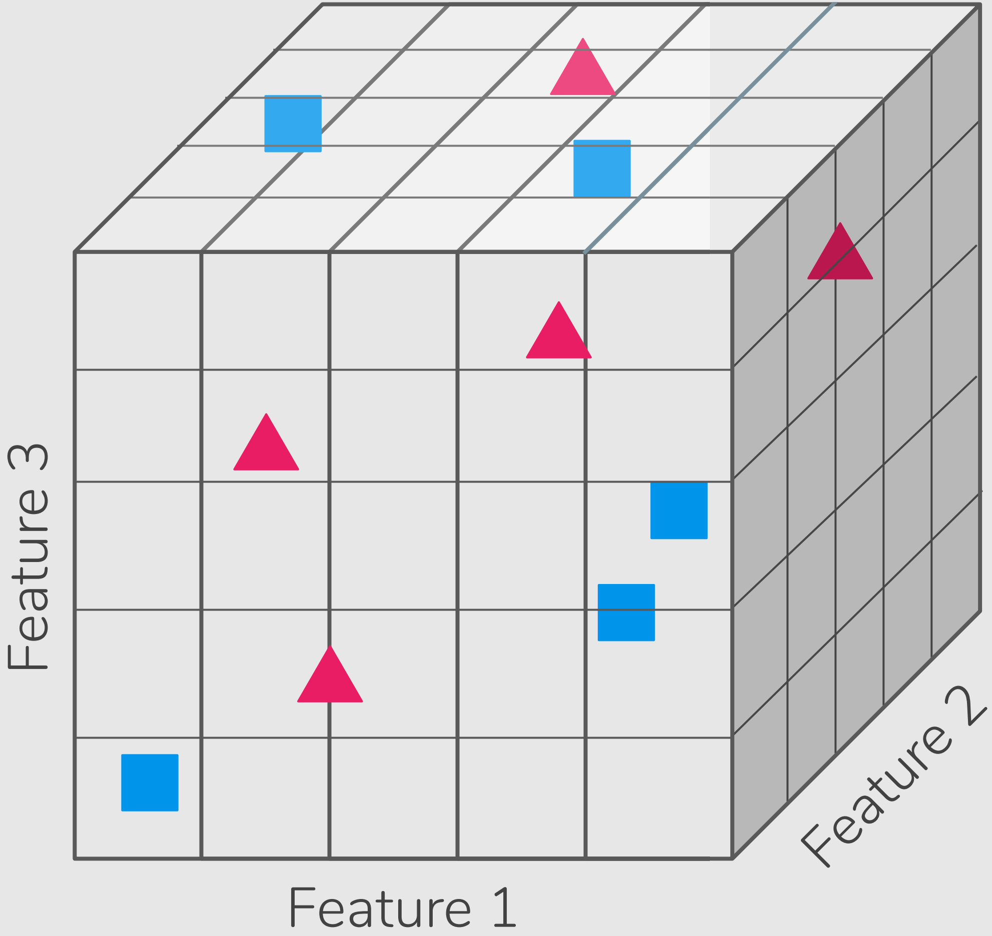
10 samples
1 dimension: 5 regions

The Curse of Dimensionality

As the dimensionality of data grows, the density of observations becomes lower and lower and lower.



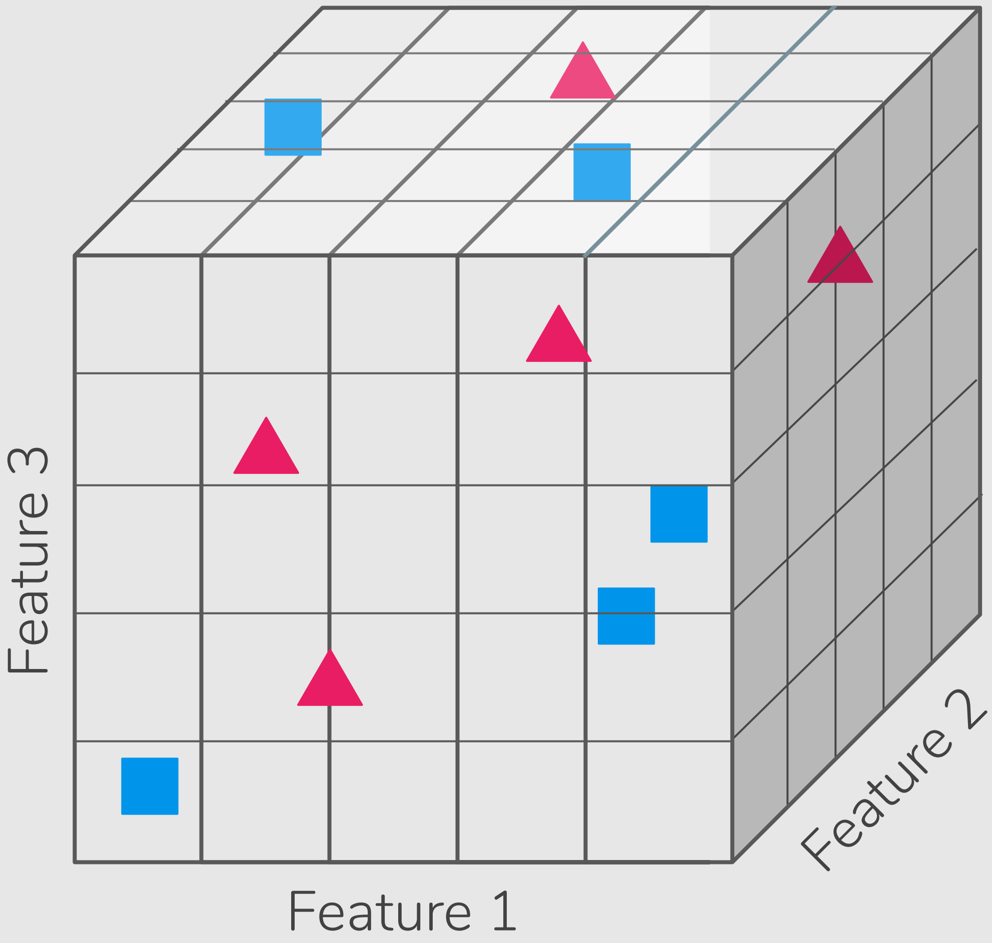
10 samples
2 dimensions: 25 regions



As the dimensionality of data grows, the density of observations becomes lower and lower and lower.

10 samples

3 dimensions: 125 regions



- 1 dimension: the sample density is 10/5 = 2 samples/interval
- 2 dimensions: the sample density is 10/25 = 0.4 samples/interval
- 3 dimensions: the sample density is 10/125 = 0.08 samples/interval

The Curse of Dimensionality: Solution?

The Curse of Dimensionality: Solution?

- **Increase the size of the training set** to reach a sufficient density of training instances.

The Curse of Dimensionality: Solution?

- **Increase the size of the training set** to reach a sufficient density of training instances.
- Unfortunately, the number of training instances required to reach a given density grows exponentially with the number of dimensions.

How to reduce dimensionality?

How to reduce dimensionality?

- Feature Selection

- Feature Extraction

How to reduce dimensionality?

- **Feature Selection:** choosing a subset of all the features (the ones more informative).
    - $\mathbf{x_1}$, x_2, $\mathbf{x_3}$, x_4, $\mathbf{x_5}$
- **Feature Extraction**

How to reduce dimensionality?

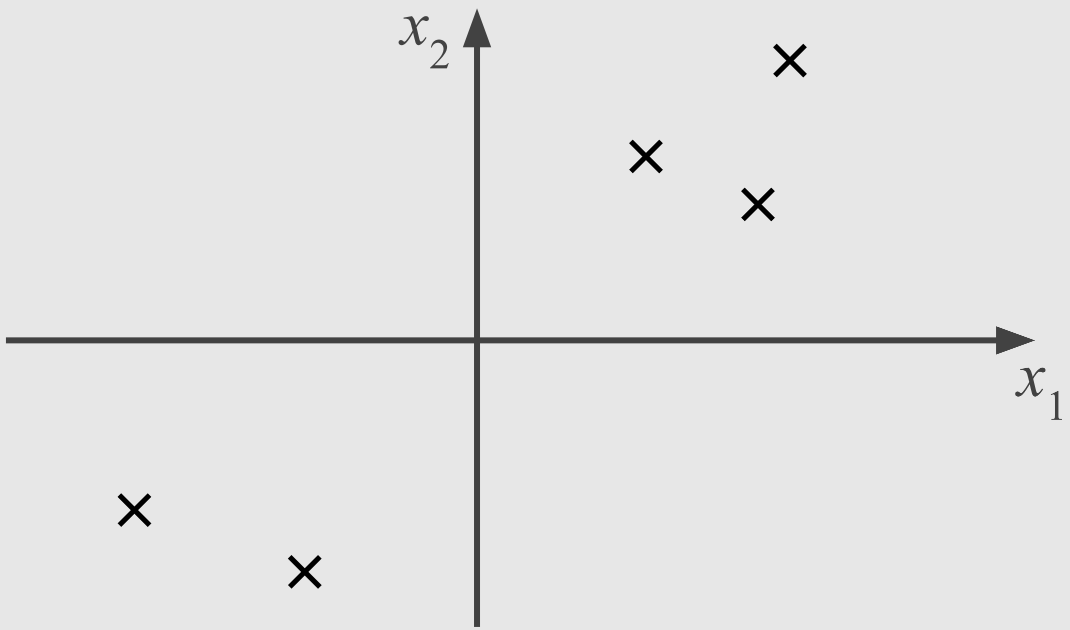
- **Feature Selection:** choosing a subset of all the features (the ones more informative).
  - $\mathbf{x_1}$, x_2, $\mathbf{x_3}$, x_4, $\mathbf{x_5}$
- **Feature Extraction:** create a subset of new features by combining the existing ones.
  - $z = f(x_1, x_2, x_3, x_4, x_5)$

PCA: Principal Component Analysis

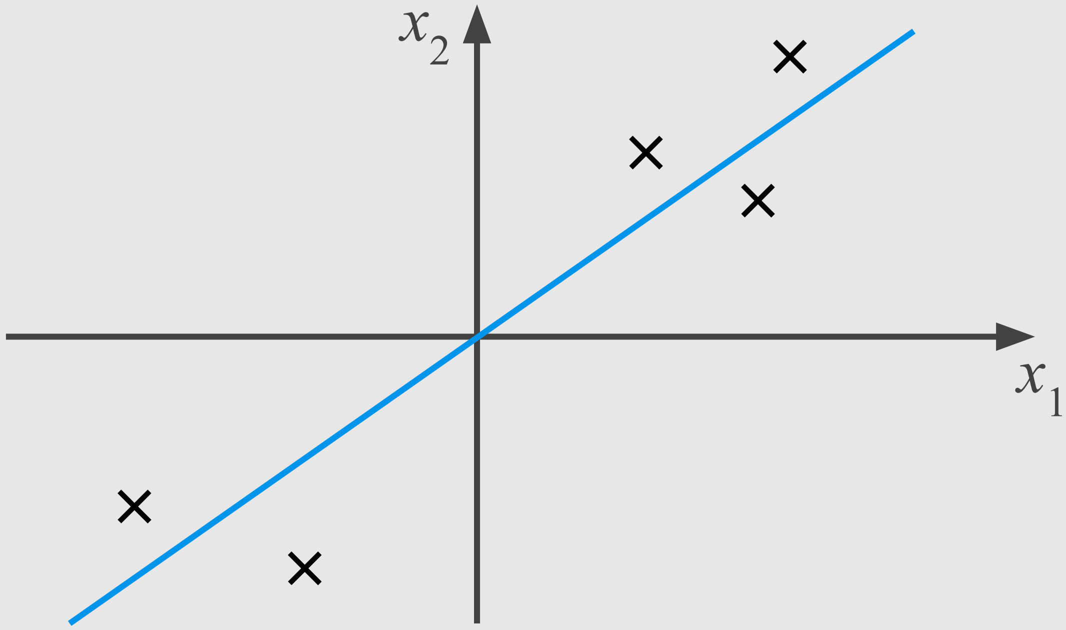
Principal Component Analysis (PCA)

- The most popular dimensionality reduction algorithm.
- PCA have two steps:
  - It **identifies the hyperplane** that lies closest to the data.
  - It **projects** the data onto it.

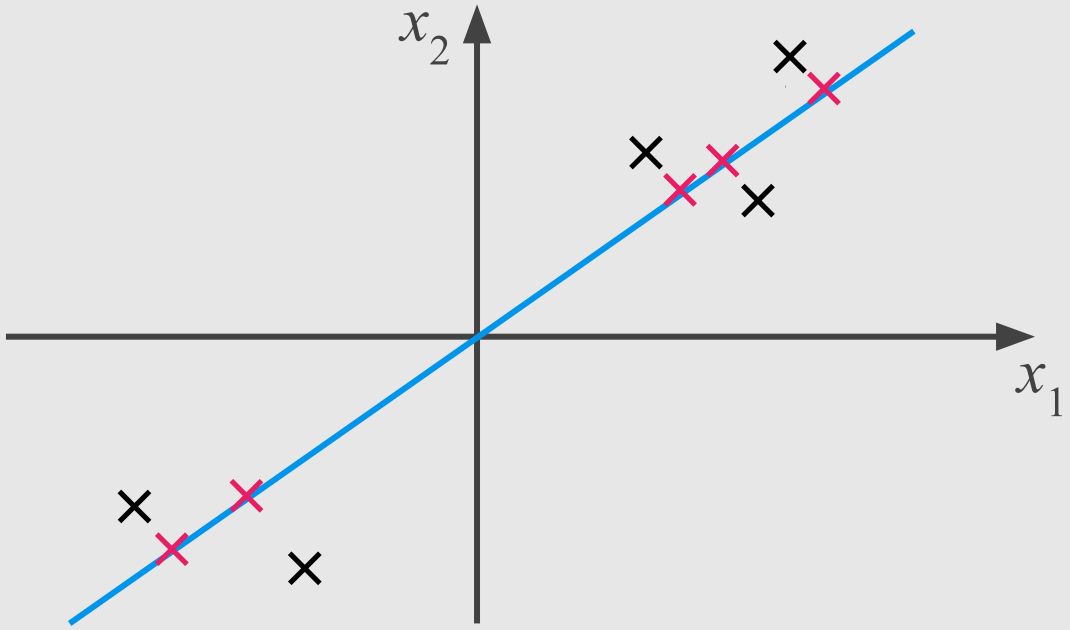
Problem Formulation (PCA)



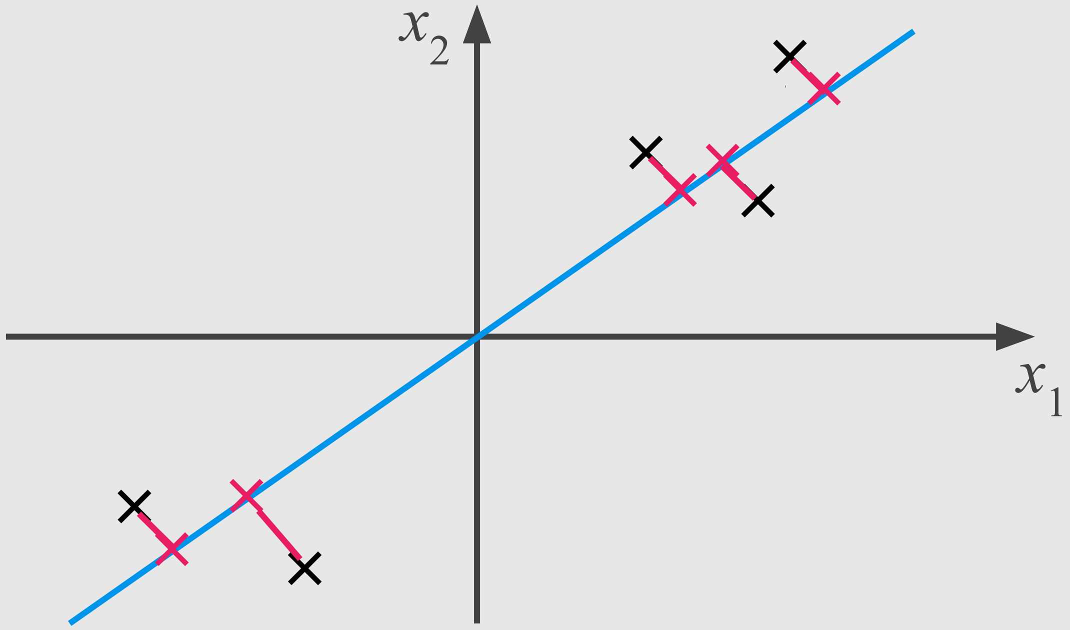
Problem Formulation (PCA)



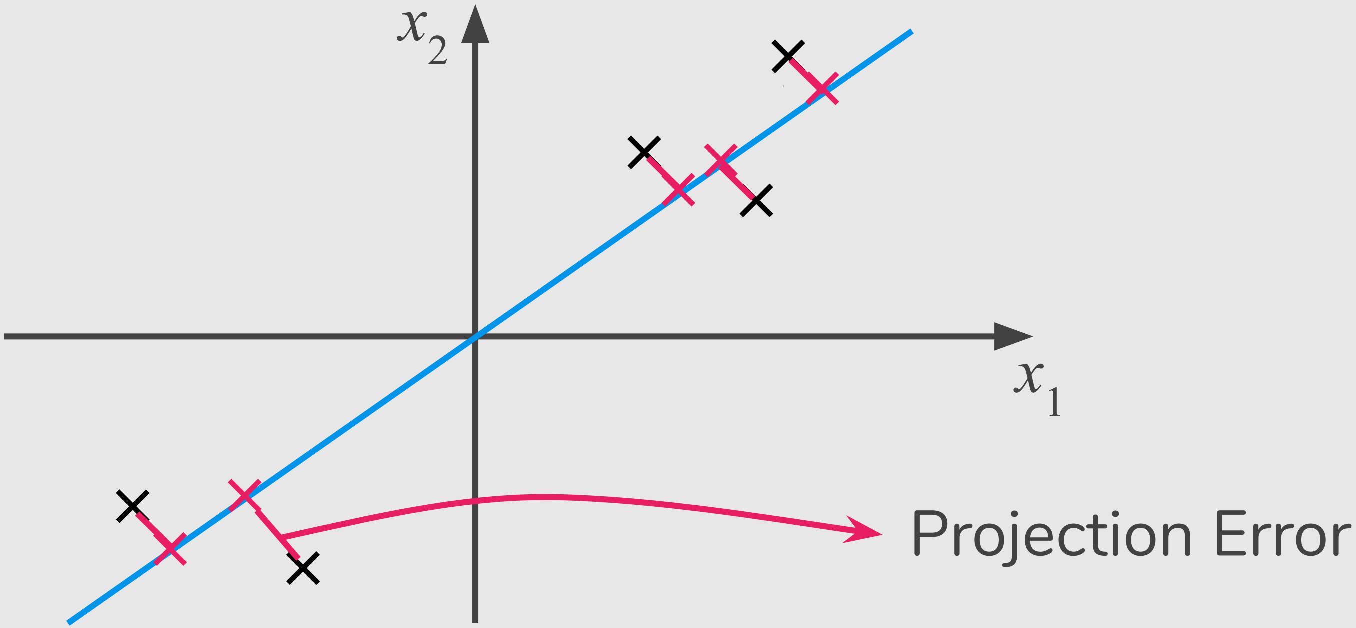
Problem Formulation (PCA)



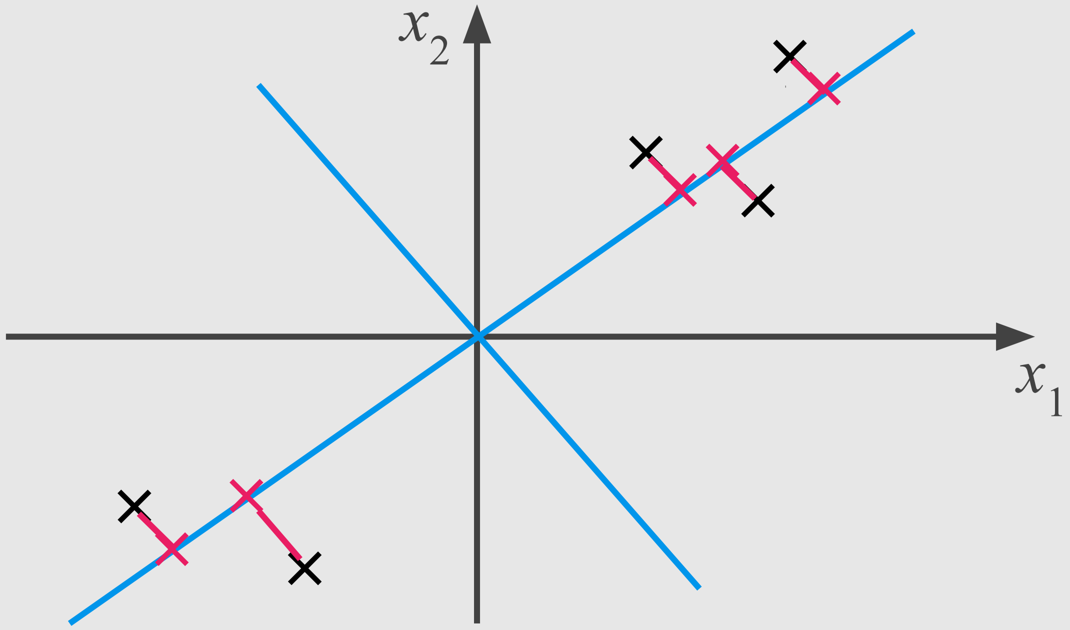
Problem Formulation (PCA)



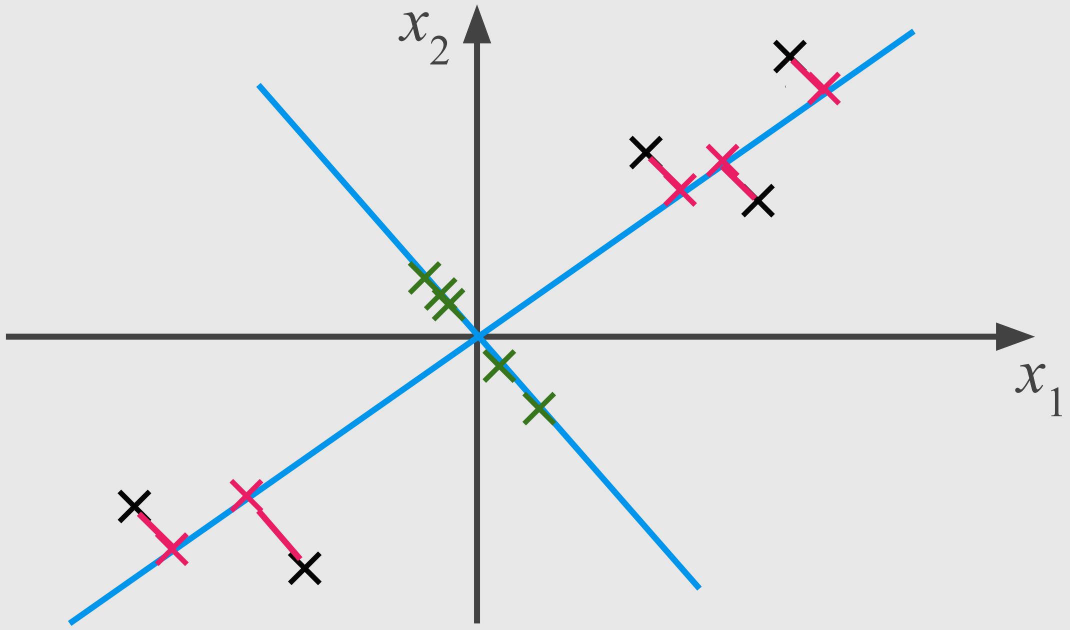
Problem Formulation (PCA)



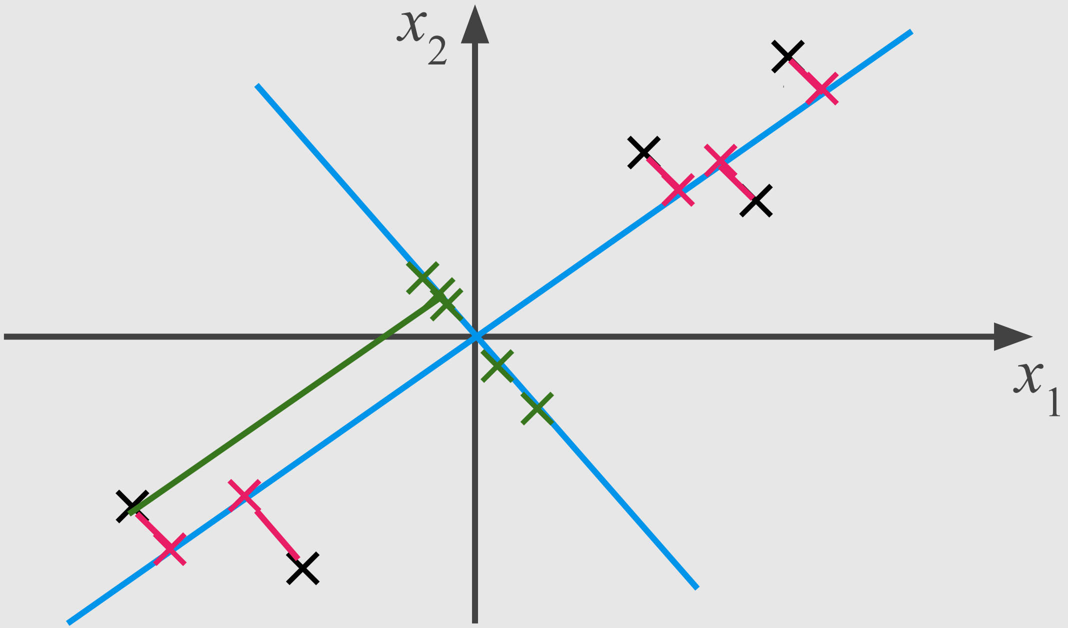
Problem Formulation (PCA)



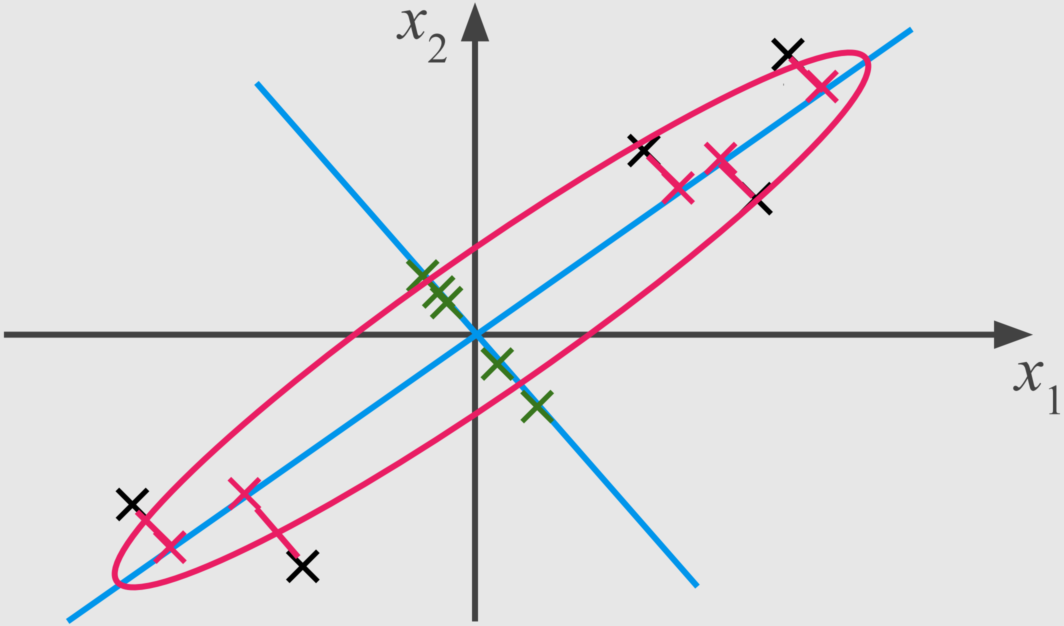
Problem Formulation (PCA)



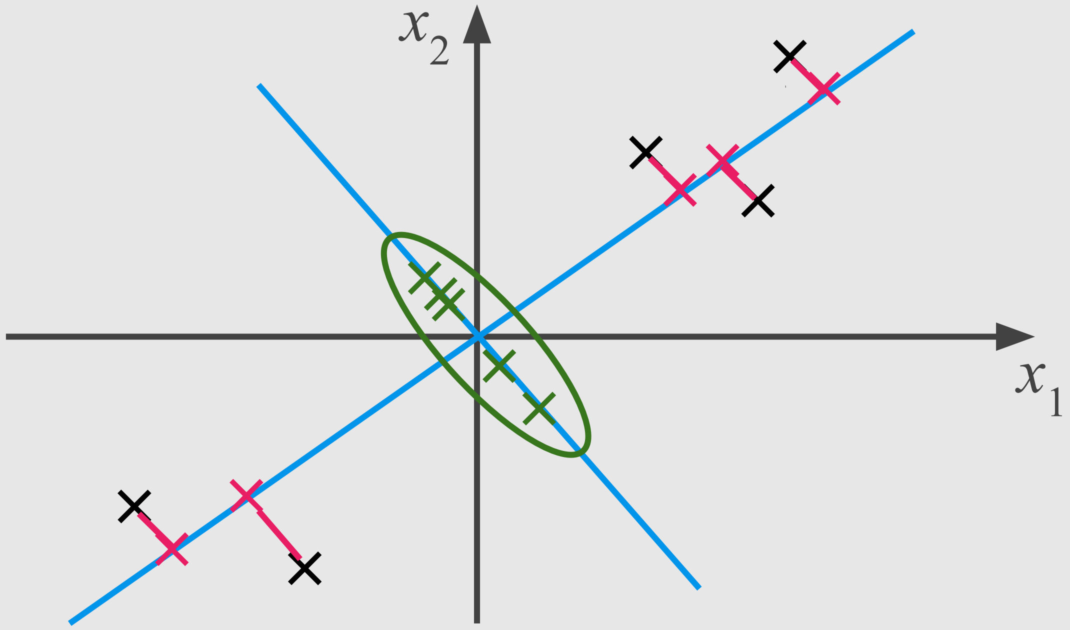
Problem Formulation (PCA)



Problem Formulation (PCA)

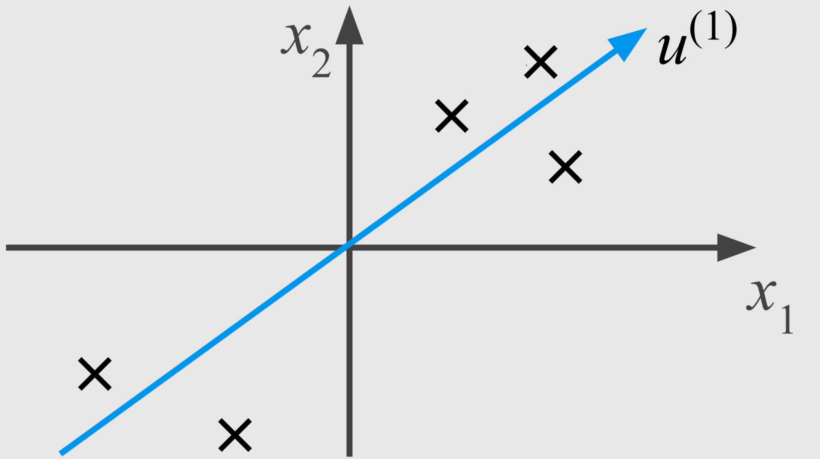


Problem Formulation (PCA)



Problem Formulation (PCA)

- Reduce from 2-dimension to 1-dimension: Find a direction (a vector $u^{(1)} \in \mathbb{R}^n$) onto which to project the data so as to minimize the projection error.



Problem Formulation (PCA)

- Reduce from n-dimension to k-dimension: Find k vectors $u^{(1)}, u^{(2)}, \ldots, u^{(k)}$ onto which to project the data, so as to minimize the projection error.

PCA Algorithm
By Eigen Decomposition

PCA in a Nutshell (Eigen Decomposition)

1. Center the data (and normalize)
2. Compute covariance matrix $\mathbf{\Sigma}$
3. Find eigenvectors u and eigenvalues λ
4. Sort eigenvalues and pick first k eigenvectors
5. Project data to k eigenvectors

PCA in a Nutshell (Eigen Decomposition)

1. **Center the data (and normalize)**
2. Compute covariance matrix $\boldsymbol{\Sigma}$
3. Find eigenvectors u and eigenvalues λ
4. Sort eigenvalues and pick first k eigenvectors
5. Project data to k eigenvectors

Data Preprocessing

Training set: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$

Preprocessing (feature scaling/mean normalization):

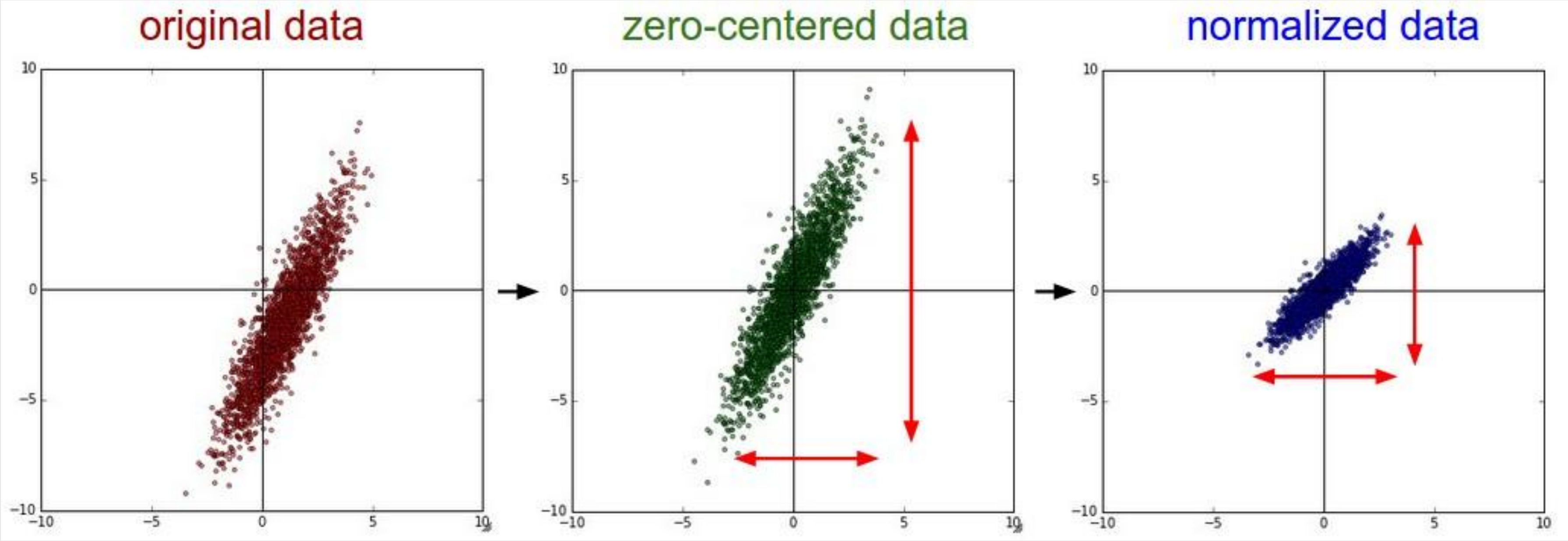
$$\mu_j = \frac{1}{m} \sum_{i=1}^{m} x_j^{(i)}$$

Center the data

Replace each $x_j^{(i)}$ with $x_j - \mu_j$.

If different features on different scales, scale features to have comparable range of values.

Data Preprocessing



Credit: http://cs231n.github.io/neural-networks-2/

PCA in a Nutshell (Eigen Decomposition)

1. Center the data (and normalize)
2. **Compute covariance matrix $\boldsymbol{\Sigma}$**
3. Find eigenvectors u and eigenvalues λ
4. Sort eigenvalues and pick first k eigenvectors
5. Project data to k eigenvectors

PCA Algorithm

Reduce data from n-dimensions to k-dimensions

Compute "covariance matrix":

$$\Sigma = \frac{1}{m} \sum_{i=1}^{n} (x^{(i)})(x^{(i)})^{\mathrm{T}} \Rightarrow n \times n \text{ matrix}$$

PCA Algorithm

Reduce data from n-dimensions to k-dimensions

Compute “covariance matrix”:

$$\Sigma = \frac{1}{m} \sum_{i=1}^{n} (x^{(i)})(x^{(i)})^{\mathrm{T}}$$ → $n \times n$ matrix

Covariance of dimensions x_1 and x_2:

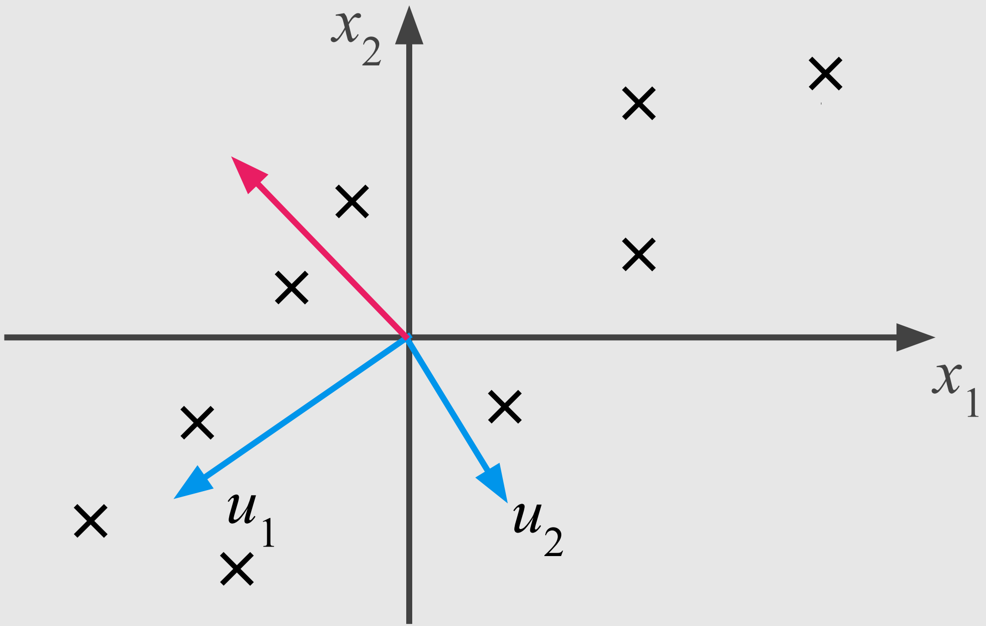
- Do x_1 and x_2 tend to increase together?
- or does x_2 decrease as x_1 increases?

$$\begin{array}{cc} & \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \end{array}$$

PCA Algorithm

Multiple a vector by $\mathbf{\Sigma}$:

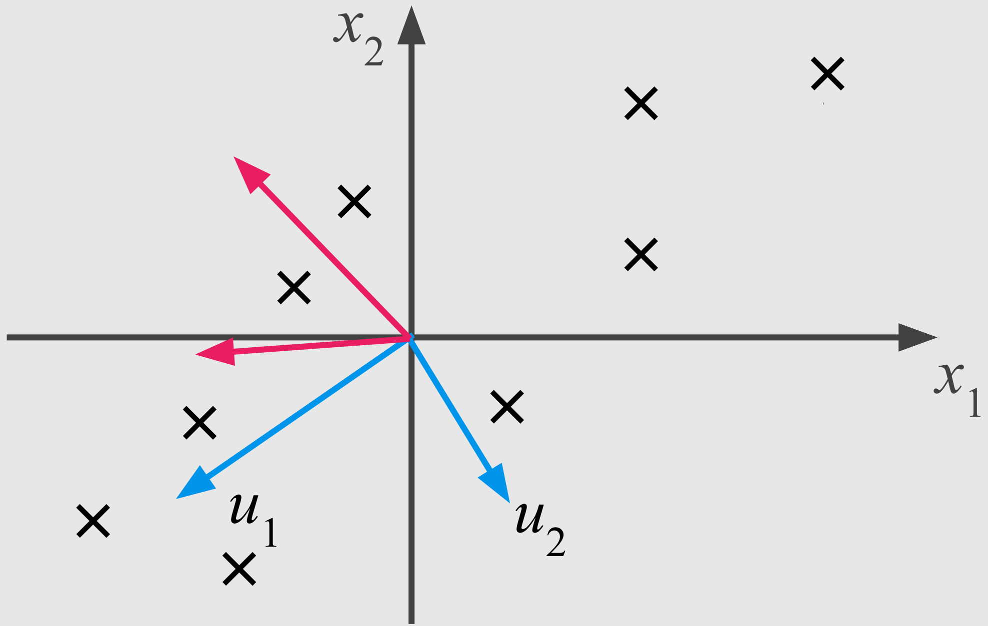
$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



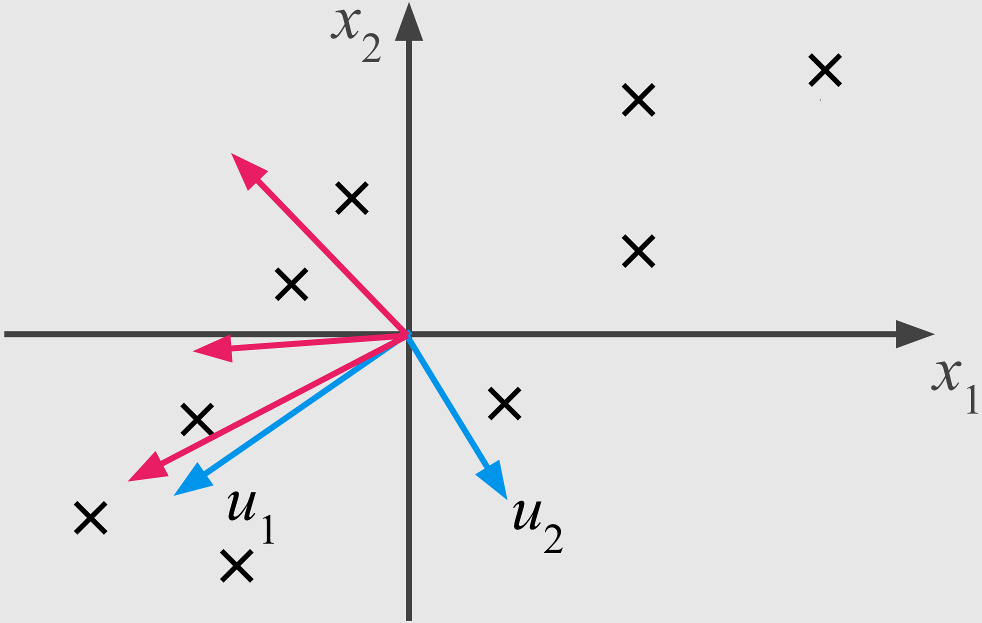
PCA Algorithm

Multiple a vector by $\boldsymbol{\Sigma}$:

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$$



PCA Algorithm

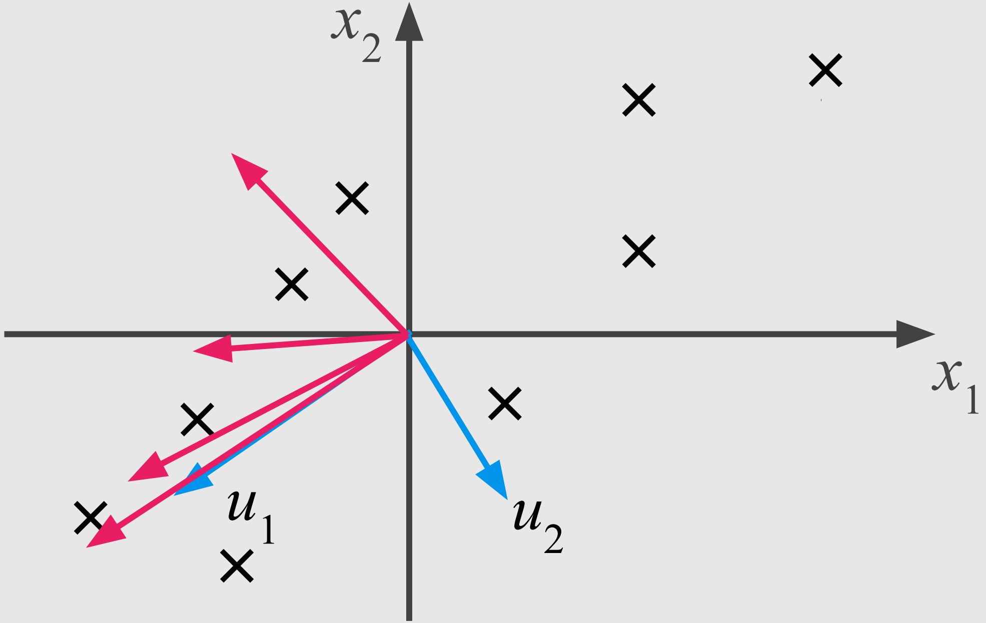


Multiple a vector by $\boldsymbol{\Sigma}$:

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix}$$

PCA Algorithm



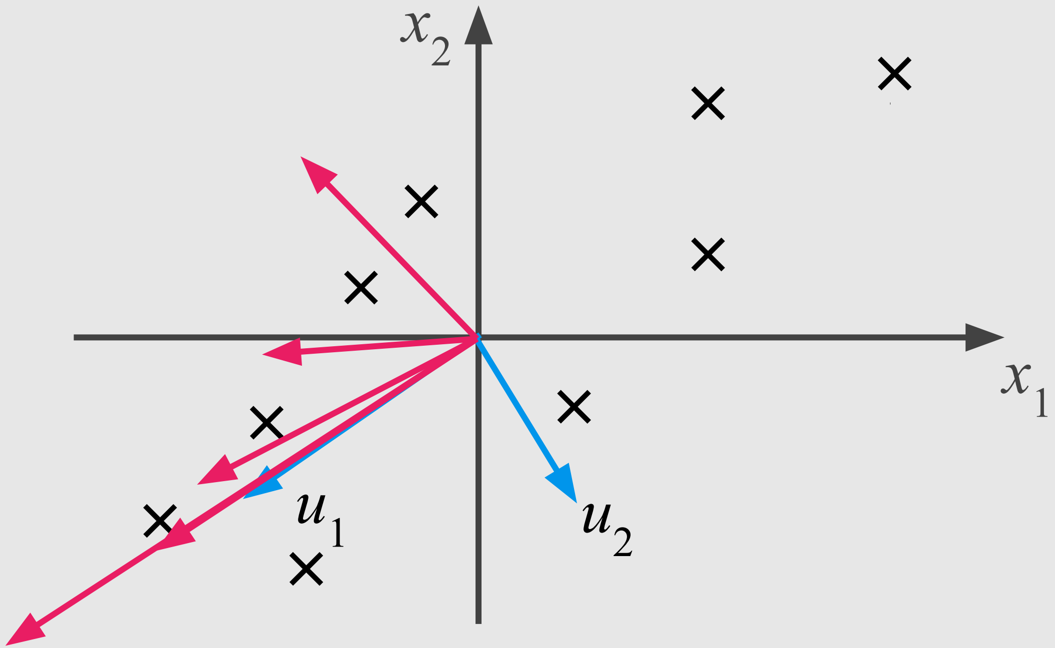
Multiple a vector by $\mathbf{\Sigma}$:

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix} = \begin{bmatrix} -6.0 \\ -2.7 \end{bmatrix}$$

PCA Algorithm



Multiple a vector by $\mathbf{\Sigma}$:

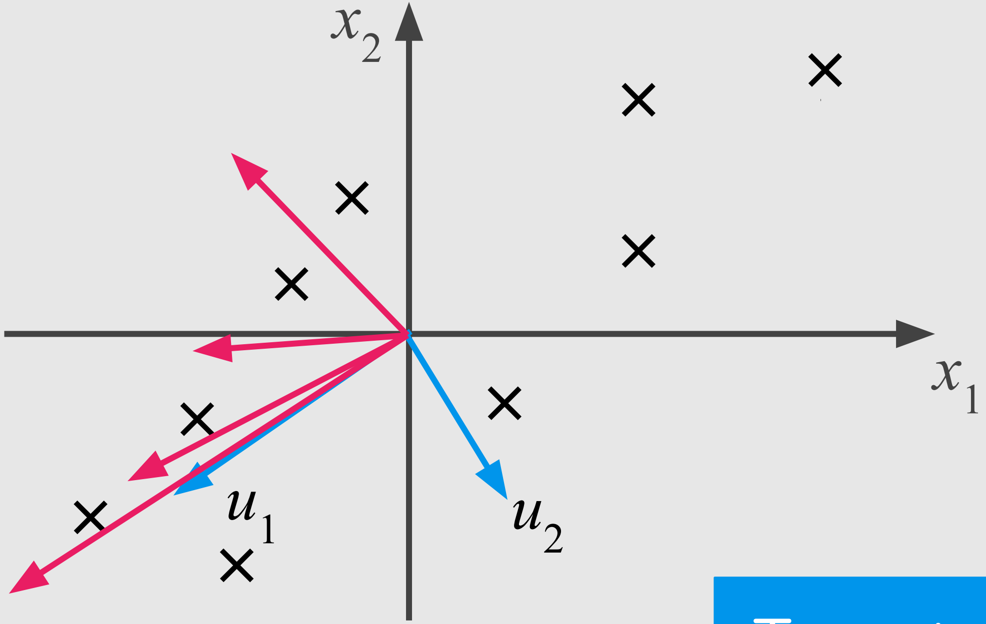
$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix} = \begin{bmatrix} -6.0 \\ -2.7 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -6.0 \\ -2.7 \end{bmatrix} = \begin{bmatrix} -14.1 \\ -6.4 \end{bmatrix}$$

PCA Algorithm



Multiple a vector by $\boldsymbol{\Sigma}$:

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$$

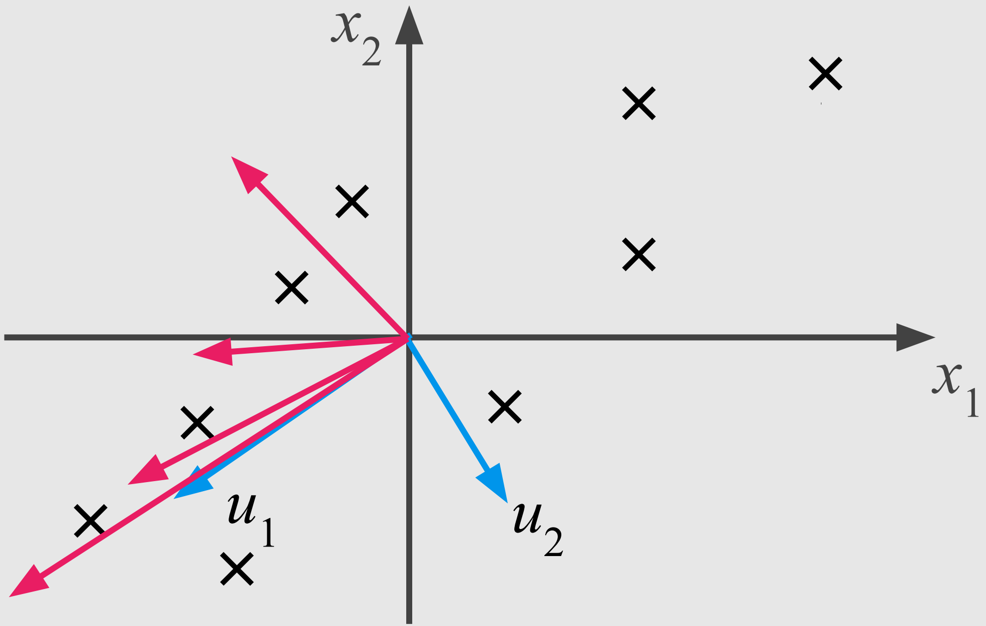
$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -2.5 \\ -1.0 \end{bmatrix} = \begin{bmatrix} -6.0 \\ -2.7 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \times \begin{bmatrix} -6.0 \\ -2.7 \end{bmatrix} = \begin{bmatrix} -14.1 \\ -6.4 \end{bmatrix}$$

Turns towards direction of variation

PCA Algorithm

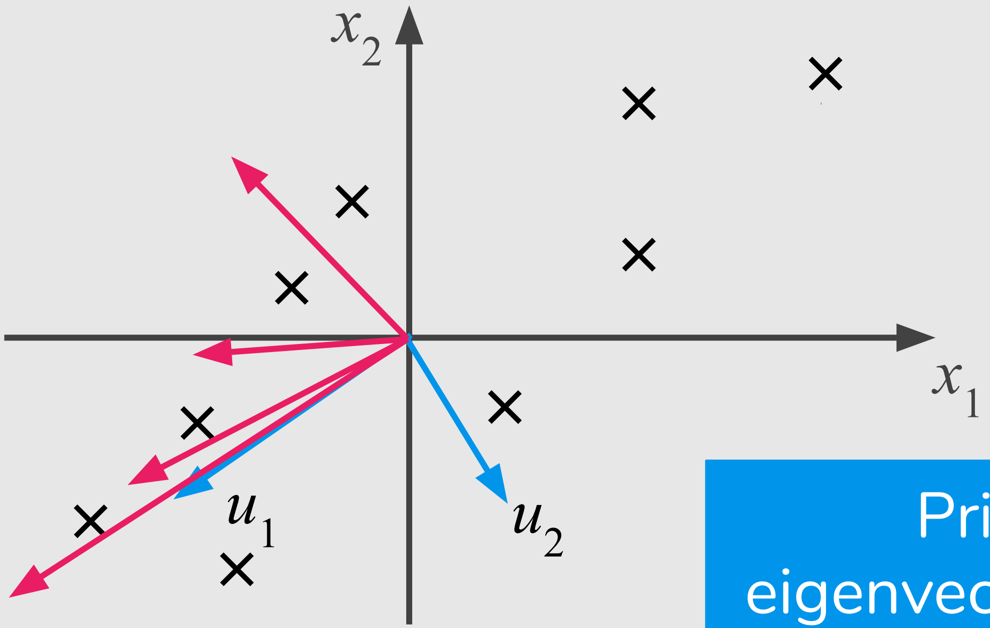


Want vectors u which aren't turned: $\boldsymbol{\Sigma} u = \lambda u$

u = eigenvectors of $\boldsymbol{\Sigma}$

λ = eigenvalues

PCA Algorithm



Want vectors u which aren't turned: $\mathbf{\Sigma} u = \lambda u$

u = eigenvectors of $\mathbf{\Sigma}$

λ = eigenvalues

Principal components = eigenvectors w. largest eigenvalues

PCA in a Nutshell (Eigen Decomposition)

1. Center the data (and normalize)
2. Compute covariance matrix $\mathbf{\Sigma}$
3. **Find eigenvectors $\boldsymbol{u}$ and eigenvalues $\boldsymbol{\lambda}$**
4. Sort eigenvalues and pick first k eigenvectors
5. Project data to k eigenvectors

Finding Principal Components

1. Find eigenvalues by solving: $\det(\mathbf{\Sigma} - \lambda \mathrm{I}) = 0$

$$\det\begin{bmatrix} 2.0-\lambda & 0.8 \\ 0.8 & 0.6-\lambda \end{bmatrix} =$$

Finding Principal Components

1. Find eigenvalues by solving: $\det(\mathbf{\Sigma} - \lambda \mathrm{I}) = 0$

$$\det\begin{bmatrix} 2.0-\lambda & 0.8 \\ 0.8 & 0.6-\lambda \end{bmatrix} = (2.0-\lambda)(0.6-\lambda)-(0.8)(0.8)$$

Finding Principal Components

1. Find eigenvalues by solving: $\det(\boldsymbol{\Sigma} - \lambda \mathrm{I}) = 0$

$$\det\begin{bmatrix} 2.0-\lambda & 0.8 \\ 0.8 & 0.6-\lambda \end{bmatrix} = (2.0-\lambda)(0.6-\lambda)-(0.8)(0.8) = \lambda^2-2.6\lambda+0.56 = 0$$

$$\{\lambda_1,\lambda_2\} = \{2.36,0.23\}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\mathbf{\Sigma} u_i = \lambda_i u_i$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\boldsymbol{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\boldsymbol{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} \Rightarrow \begin{aligned} 2.0u_{11} + 0.8u_{12} &= 2.36u_{11} \\ 0.8u_{11} + 0.6u_{12} &= 2.36u_{12} \end{aligned}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\mathbf{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} \Rightarrow \begin{matrix} 2.0u_{11} + 0.8u_{12} = 2.36u_{11} \\ 0.8u_{11} + 0.6u_{12} = 2.36u_{12} \end{matrix} \Rightarrow u_{11} = 2.2u_{12}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\boldsymbol{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} \Rightarrow \begin{matrix} 2.0u_{11} + 0.8u_{12} = 2.36u_{11} \\ 0.8u_{11} + 0.6u_{12} = 2.36u_{12} \end{matrix} \Rightarrow u_{11} = 2.2u_{12}$$

$$\Downarrow$$

$$\mathrm{u}_1 \sim \begin{bmatrix} 2.2 \\ 1 \end{bmatrix}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\mathbf{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix}$$

$$2.0u_{11} + 0.8u_{12} = 2.36u_{11}$$
$$0.8u_{11} + 0.6u_{12} = 2.36u_{12}$$

$$u_{11} = 2.2u_{12}$$

$$\mathrm{u}_1 \sim \begin{bmatrix} 2.2 \\ 1 \end{bmatrix}$$

Want $\|u_1\|=1$

$$\begin{bmatrix} 0.91 \\ 0.41 \end{bmatrix}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\boldsymbol{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}\begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36\begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} \Rightarrow \begin{matrix} 2.0u_{11} + 0.8u_{12} = 2.36u_{11} \\ 0.8u_{11} + 0.6u_{12} = 2.36u_{12} \end{matrix} \Rightarrow u_{11} = 2.2u_{12}$$

$$\Downarrow$$

$$\mathrm{u}_1 \sim \begin{bmatrix} 2.2 \\ 1 \end{bmatrix}$$

$\Downarrow$ Want $\|u_1\| = 1$

$$\begin{bmatrix} 0.91 \\ 0.41 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix}\begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0.23\begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} \Rightarrow u_2 = \begin{bmatrix} -0.41 \\ 0.91 \end{bmatrix}$$

Finding Principal Components

2. Find i^{th} eigenvector by solving: $\mathbf{\Sigma} u_i = \lambda_i u_i$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = 2.36 \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} \Rightarrow \begin{matrix} 2.0u_{11} + 0.8u_{12} = 2.36u_{11} \\ 0.8u_{11} + 0.6u_{12} = 2.36u_{12} \end{matrix} \Rightarrow u_{11} = 2.2u_{12}$$

$$\Downarrow$$

$$u_1 \sim \begin{bmatrix} 2.2 \\ 1 \end{bmatrix}$$

$\Downarrow$ Want $\|u_1\|=1$

$$\begin{bmatrix} 0.91 \\ 0.41 \end{bmatrix}$$

$$\begin{bmatrix} 2.0 & 0.8 \\ 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = 0.23 \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} \Rightarrow u_2 = \begin{bmatrix} -0.41 \\ 0.91 \end{bmatrix}$$

3. 1^{st} PC: $\begin{bmatrix} 0.91 \\ 0.41 \end{bmatrix}$ and 2^{nd} PC: $\begin{bmatrix} -0.41 \\ 0.91 \end{bmatrix}$

PCA in a Nutshell (Eigen Decomposition)

1. Center the data (and normalize)
2. Compute covariance matrix $\mathbf{\Sigma}$
3. Find eigenvectors u and eigenvalues λ
4. **Sort eigenvalues and pick first k eigenvectors**
5. Project data to k eigenvectors

How many PCs?

- Have eigenvectors $u_1, u_2, \ldots, u_n$, want $k < n$
- eigenvalue λ_i = variance along u_i

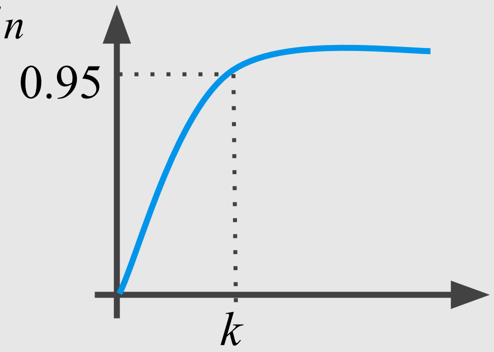
How many PCs?

- Have eigenvectors $u_1, u_2, ..., u_n$, want $k < n$
- eigenvalue λ_i = variance along u_i
- Pick u_i that explain the most variance:
  - Sort eigenvectors s.t. $\lambda_1 > \lambda_2 > \lambda_3 > \ldots > \lambda_n$
  - Pick first k eigenvectors which explain 95% of total variance

How many PCs?

- Have eigenvectors $u_1, u_2, \ldots, u_n$, want $k < n$
- eigenvalue λ_i = variance along u_i
- Pick u_i that explain the most variance:
    - Sort eigenvectors s.t. $\lambda_1 > \lambda_2 > \lambda_3 > \ldots > \lambda_n$
    - Pick first k eigenvectors which explain 95% of total variance

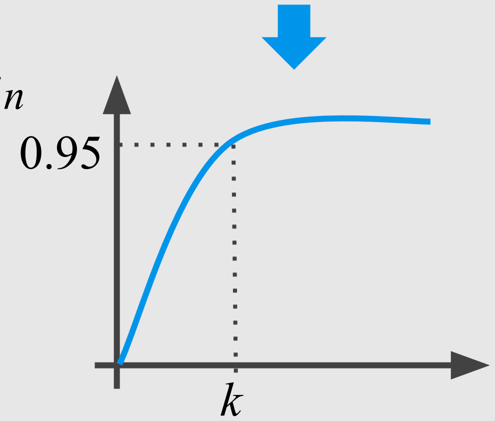
$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{n} \lambda_i} \leq 1$$



How many PCs?

- Have eigenvectors $u_1, u_2, \ldots, u_n$, want $k < n$
- eigenvalue λ_i = variance along u_i
- Pick u_i that explain the most variance:
  - Sort eigenvectors s.t. $\lambda_1 > \lambda_2 > \lambda_3 > \ldots > \lambda_n$
  - Pick first k eigenvectors which explain 95% of total variance
    - Typical threshold: 90%, 95%, 99%

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{n} \lambda_i} \leq 1$$



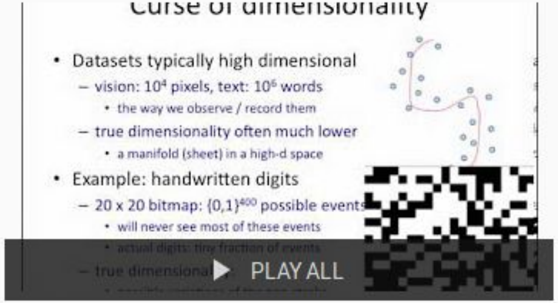
PCA in a Nutshell (Eigen Decomposition)

1. Center the data (and normalize)
2. Compute covariance matrix $\mathbf{\Sigma}$
3. Find eigenvectors u and eigenvalues λ
4. Sort eigenvalues and pick first k eigenvectors
5. **Project data to k eigenvectors**

Principal Component Analysis (12 videos, 3-15min)

https://www.youtube.com/playlist?list=PLBv09BD7ez_5_yapAg86Od6JeeypkS4YM

Search



Principal Component Analysis

12 videos • 119,895 views • Last updated on May 21, 2014

Victor Lavrenko

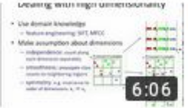
SUBSCRIBE 19K

Lectures 18 and 19 in the Introductory Applied Machine Learning (IAML) course by Victor Lavrenko at the

1 

PCA 1: curse of dimensionality

Victor Lavrenko

2 

PCA 2: dimensionality reduction

Victor Lavrenko

3 

PCA 3: direction of greatest variance

Victor Lavrenko

4 

PCA 4: principal components = eigenvectors

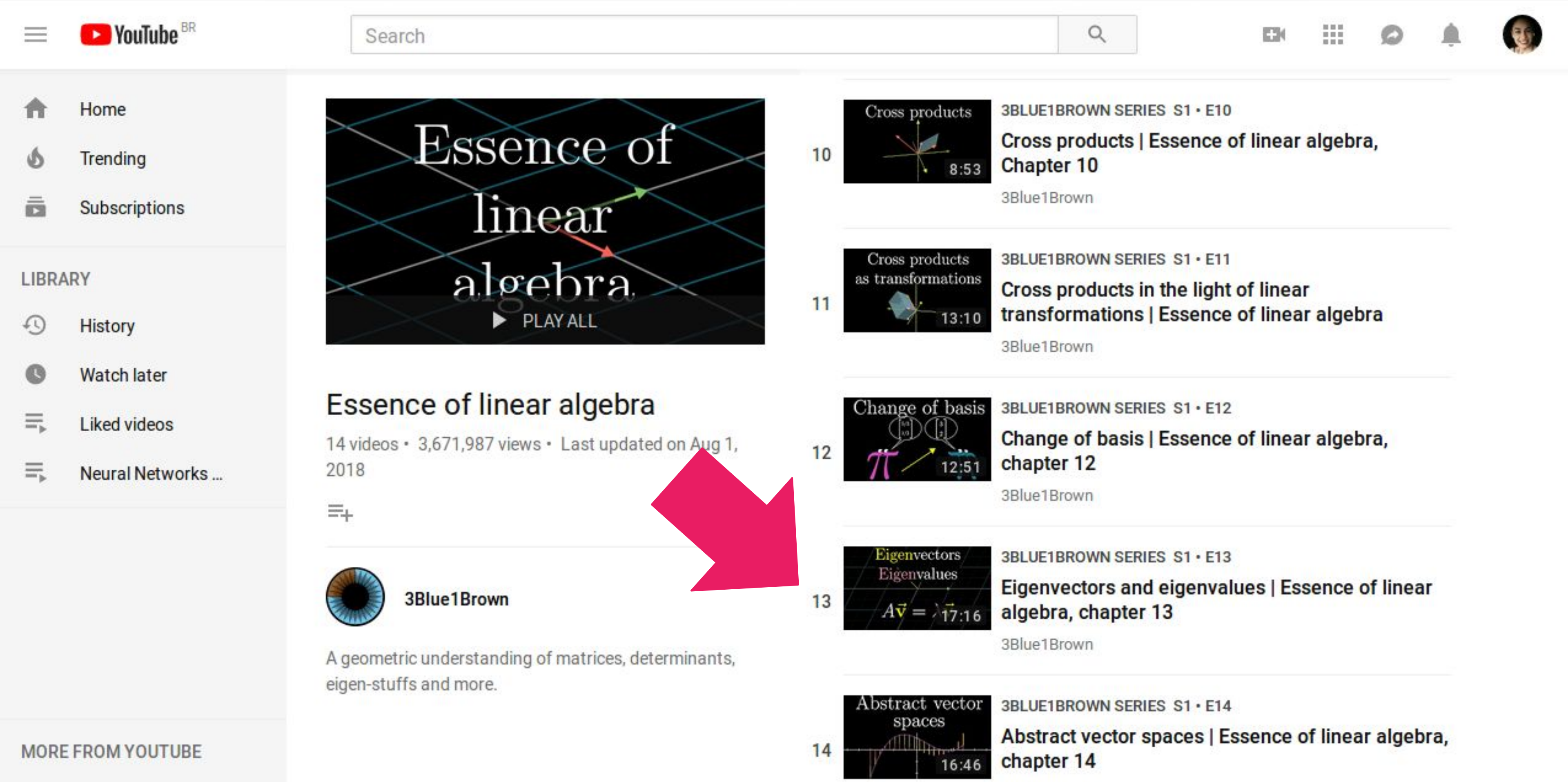
Victor Lavrenko

5 

PCA 5: finding eigenvalues and eigenvectors

Victor Lavrenko

https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab



References

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 8 “Dimensionality Reduction”
- Pattern Recognition and Machine Learning, Chap. 12 “Continuous Latent Variables”
- Pattern Classification, Chap. 10 “Unsupervised Learning and Clustering”

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 8