P-median and P-center applied to Color Quantization and Pixel Mapping

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Abstract

The use of digital images has becoming increasingly popular, mainly due to the drop in media equipament's prices, such as, cameras, scanners, video cameras, computers, among others. However, even for the current storage patterns [3,4,5], the digital images consume a large quantity of the resources. In this context, this paper presents a single and intuitive manner to reduce the size of digital images. Instead of sophisticated mathematic theories, such as Fourier transformation, Z theory, among others, our purpose is to reduce the number of colors, treating the problem through facility location optimization model approach, more specifically using p-median and p-center models [1]. The results, obtained by multi-spectrum images indexing like the GIF approach, is promising, in spite of its incipient phase.

1. Introduction

This work solves the color quantization problem for a palette with 16 millions of colors through two facility location model approaches:

1. It searches a central pixel called localization center area, which represents an area with radius R started a priori. This strategy is based on the set covering and p-center problems [1].
2. A localization area is found within a covering radius R previously started. The calculus of the center is made through the median among the pixels within this region. This strategy is based on the *p-median* problem[1].

The optimization model adopted in this work was formalized as follows:

$$\max \quad \sum_{i=1..m} Z_i h_i \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=1..m} x_j \leq p \qquad (2)$$

$$\sum_{j=1..n} a_{ij} x_j \geq Z_i, \quad i = 1, \ldots, m \qquad (3)$$

where:

- facilities j: colors of spectrum;
- localities i: colors of image;
- $Z_i \in \{0,1\}$: locality i covered when $Z_i=1$;
- h_i: demand of locality i;
- $x_j \in \{0,1\}$:facility j selected when $x_j = 1$;
- p: quantity of facilities;
- $a_{ij} \in \{0,1\}$: facility j can cover the locality I when $a_{ij}=1$.

2. Methodology

To select p colors at most (within a universe of 2^{24} colors), which better represent an image with MxN RGB pixels, we have adopted two approaches: *p-center* and *p-median* [1]. Suppose that we choose the color C, with RGB components equal to (C_r, C_g, C_b), to represent others colors. In *p-center* approach, any image color with RGB values between (C_r-r, C_g-r, C_b-r) and (C_r+r, C_g+r, C_b+r) is replaced by color C, where r is the **cover radius**. In *p-center* approach, we choose the color C traversing the image from left to right, and from top to down. For each pixel, if it is not covered yet, we select its color as the current cover color. Otherwise, we do not evaluate the next pixel. However, it can occur that the number of colors may be higher than the threshold *p*. If it happens, it is necessary to increase the radius *r* according to the following equation:

*newRadius = radius(1+(NumerOfRegions-p)/(10*p)),*

thereafter the process is repeated. Therefore, while the number of cover regions is higher than *p*, the image traversing process is repeated with a radius each time smaller than before.

The *p-median* approach is similar to the above approach, except that instead of using the color of a central region, it calculates the median on all colors within that region.

3. Results and Conclusions

To validate the two approaches we propose in this paper, we have implemented **wxOptimage:** a software for image quantization and pixel mapping using *p-meadian* and *p-center* approaches, which quantifies colors on multi-spectral images (c.f., Fig. 1).

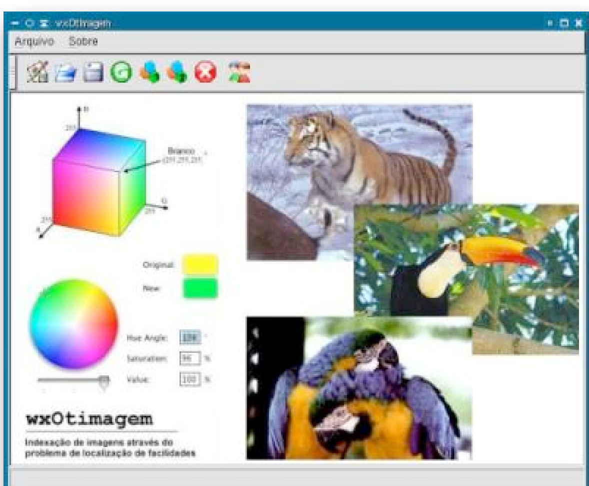

Figure 1. wxOptimage in execution.

We have developed the software using the C++ programming language [6] and wxWindows [7] graphic library.

The metric (denominated **gap**) used to compare an original image and its mapped image consists of the sum of the Euclidean distances between each RGB component contained in the pixels situated in the same position on each image. Therefore, the smaller the gap, the smaller the distortion between the original image and its mapped image, and the better the result.

Table 1 presents the gaps found in each image from test set used in this work with several distinct scenarios. This table also brings: the dimensions of the image, the initial radius R_i, the *p* value, and the number of regions R_c discovered for *p-center* and *p-median*.

Image	Dim.	R_i	p	center	R_c	media	R_c
water	270x204	5	25	9244880	6	9663194	6
amanda	214x454	5	256	2723449	248	2962529	213
canoe	288x193	10	100	1791959	95	1651840	88
flour	288x188	10	50	2272635	47	1831381	42
corcovado	288x204	5	100	1809694	99	1675418	94
Spectrum	229x326	5	120	1548612	112	1495188	117
flowers	300x369	5	256	2725000	243	3038608	184
imperials	277x187	10	200	1501058	157	1339592	173
jolie	300x295	10	256	1726486	188	1684615	213
sea	300x197	15	180	1237719	128	1426972	150
landscape	200x200	15	100	1477740	96	1206218	81
stone	288x177	5	80	2753456	74	1492937	70
train	330x237	5	256	2117088	249	2268443	198

Table 1. Comparison between *p-center* and *p-median.*

The Figure 2 shows an original image, while Figures 3 (b-c) present its mapping versions according to *p-center* and *p-median* approaches respectively.

Figure 2. Original image.

Analyzing the results, we conclude that, regarding performance, generally *p-center* is better than *p-median* approach.

(a)

(b)

Figure 3. (a) Image quantized by *p-center* with p = 100, (b) Image quantized by *p-median***.**

4. References

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