Technology-Related Examples

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Chapter 3: Technology-Related Modeling Examples

- Routing, Flow, and Capacity Design in Communication & Computer Networks by Michal Pioro and Deepankar Medhi

- Permission to reproduce the figures in this file was given by Deepankar Medhi

IP Networks: Intra-Domain Traffic Engineering

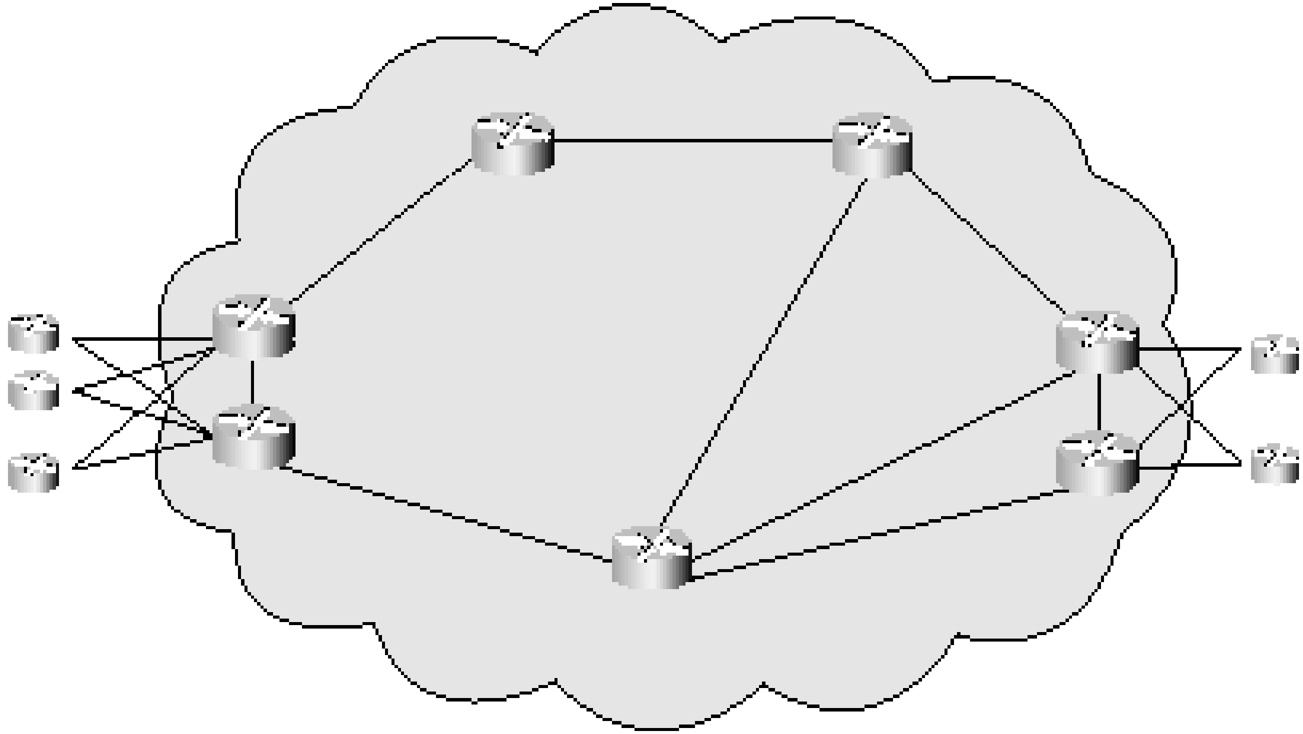

FIGURE 3.1 Intra-Domain IP Network

RFaCD, dm, figs/tables

IP Networks: Intra-Domain Traffic Engineering

- *How to minimize the delay for packets transversing through the networks?*
- *How to determine (costs) of links so that the shortest paths are determined in a way minimizing the delay as an overall network goal?*
- *How to formulate the problem of minimizing the maximum link utilization given under routing policies imposed by OSPF and IS-IS?*

Intra-Domain Traffic Engineering (3.1.1)

- **indices**

  $d = 1, 2, ..., D$ demands

  $p = 1, 2, ..., P_d$ candidate paths for flows realizing demand d

  $e = 1, 2, ..., E$ links

- **constants**

  c_e capacity of link e

  δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise

  h_d volume of demand d

- **variables**

  w_e metric of link e, $\boldsymbol{w} = (w_1, w_2, ..., w_E)$ (primary)

  $x_{dp}(\boldsymbol{w})$ flow allocated to path p of demand d determined by the link system w

  $\underline{y}_e(\boldsymbol{w})$ load of link e determined by the link system w

  r maximum link utilization variable, r = $\max_{e=1,\ldots,E} \left\{ \underline{y}_e(\boldsymbol{w})/c_e \right\}$

- The problem of minimizing the maximum link utilization can be formulated as

minimize$_{\boldsymbol{w},r}$ $\boldsymbol{F} = r$

subject to $\sum_p x_{dp}(\boldsymbol{w}) = h_d \quad d = 1, 2, ..., D$

$\sum_d \sum_p \delta_{edp} x_{dp}(\boldsymbol{w}) \le c_e r \quad e = 1, 2, ..., E$

r continuous

w_e non-negative integers.

MPLS Networks: Tunneling Optimization

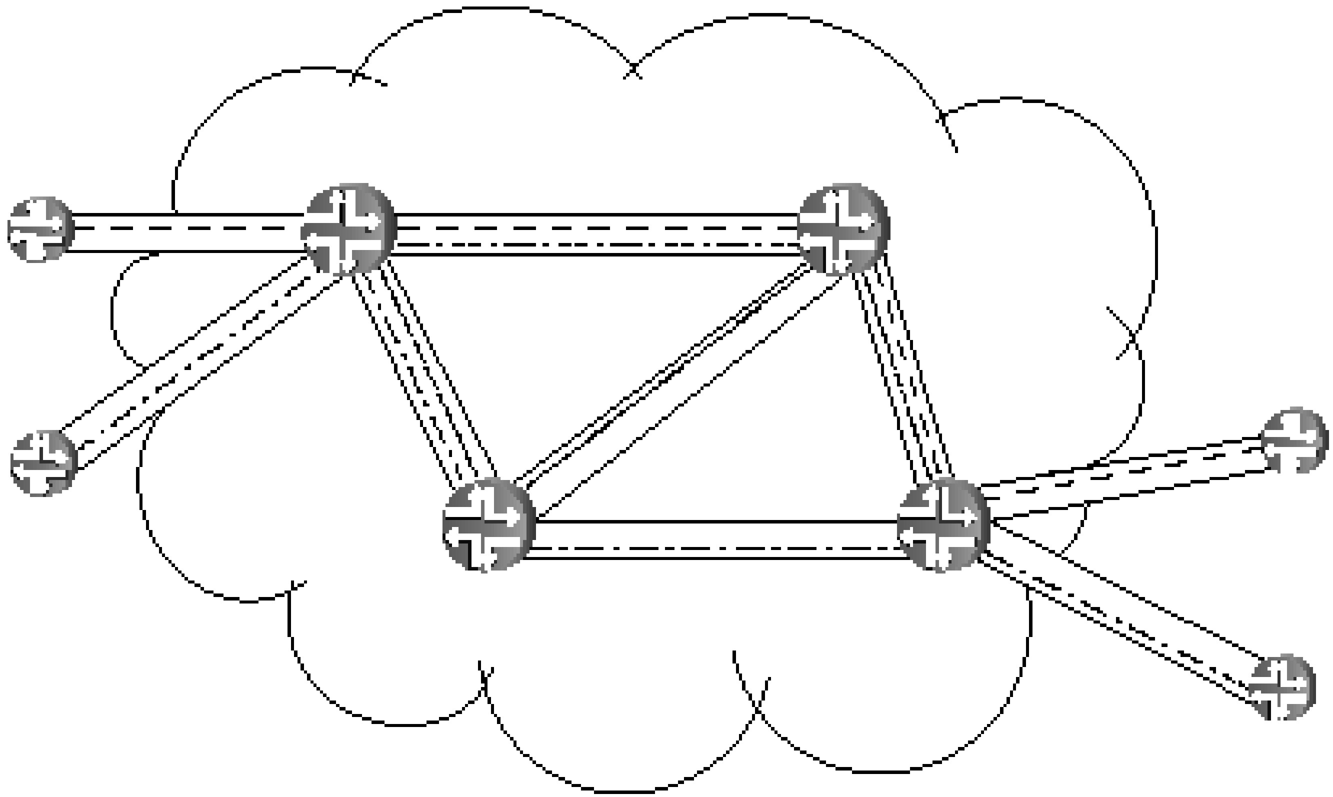

FIGURE 3.2 MPLS Network

RFaCD, dm, figs/tables

MPLS Networks: Tunneling Optimization

- End-to-end virtual paths of predefined capacities to different streams associated to different service classes
- ***How to carry different traffic classes in an MPLS network through the creation of tunnels in such a way that the number of tunnels on each MPLS router/link is minimized and balanced?***

Tunneling Optimization (3.2.1)

- **indices**

  $d = 1, 2, ..., D$ demands

  $p = 1, 2, ..., P_d$ number of possible tunnels for demand d

  $e = 1, 2, ..., E$ links

- **constants**

  c_e capacity of link e

  δ_{edp} = 1, if link e belongs to tunnel p realizing demand d; 0, otherwise

  h_d volume of demand d

- **variables**

  x_{dp} fraction of the demand volume d carried over tunnel p

  ε lower bound on fraction of flow on a tunnel (path)

  u_{dp} = 1, to denote selection of a tunnel if the lower bound is satisfied; 0, otherwise

  r maximum number of tunnels over all links.

(Contd.)

Tunneling Optimization (3.2.1)

(Contd.)

- The problem of minimizing the number of tunnels on each MPLS router/link and load balancing in an MPLS network can be formulated as

$$
\begin{array}{lll}
\textbf{minimize}_{\boldsymbol{x},\boldsymbol{u},r} & \boldsymbol{F} = r & \\
\textbf{subject to} & \sum_p x_{dp} = 1 & d = 1, 2, ..., D \\
& \sum_d h_d \sum_p \delta_{edp} x_{dp} \le c_e & e = 1, 2, ..., E \\
& \varepsilon u_{dp} \le h_d x_{dp} & d = 1, 2, ..., D \quad p = 1, 2, ..., P_d \\
& x_{dp} \le u_{dp} & d = 1, 2, ..., D \quad p = 1, 2, ..., P_d \\
& \sum_d \sum_p \delta_{edp} u_{dp} \le r & e = 1, 2, ..., E \\
& x_{dp} \ \text{continuous and non-negative} & \\
& u_{dp} \ \text{binary}, \ r \ \text{integer.} &
\end{array}
$$

ATM Networks: Virtual Path Design

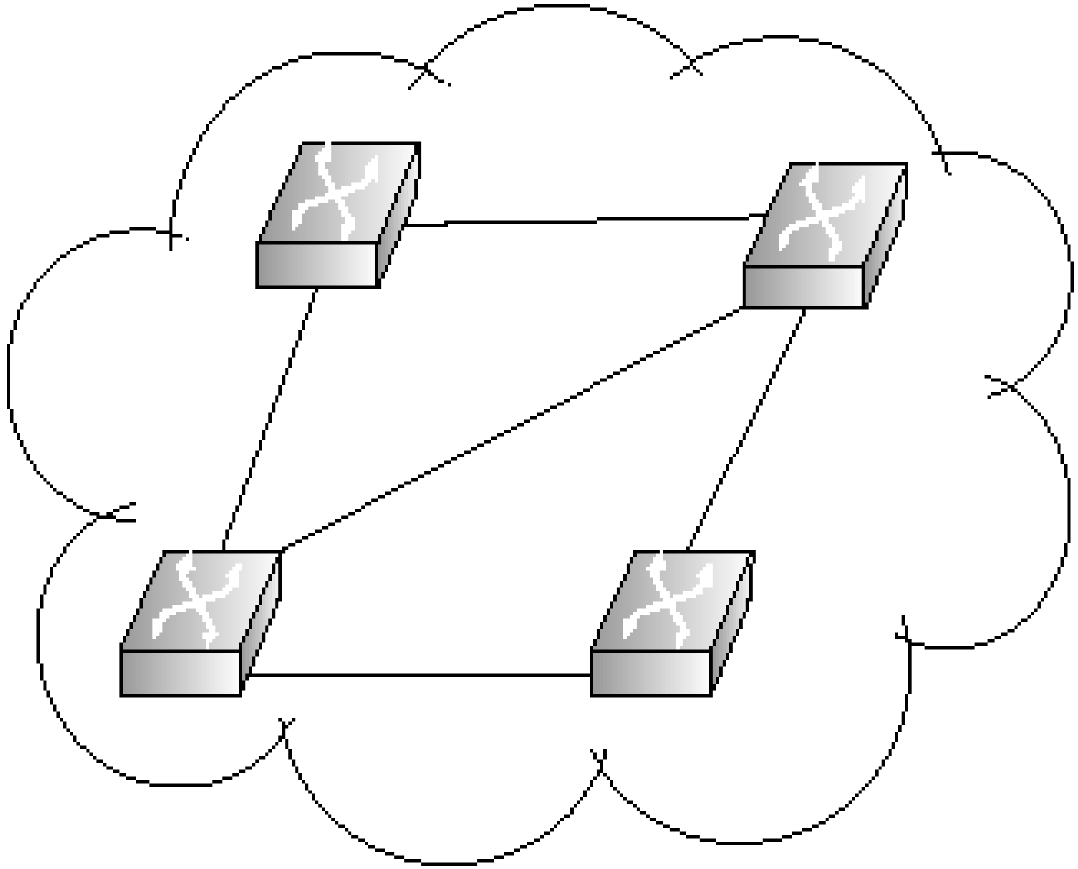

FIGURE 3.3 ATM Network

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ATM Networks: Virtual Path Design

- Permanent and semi-permanent virtual paths with guaranteed data rates
- *How to determine link capacity so that the total link cost is minimized given that the ATM virtual path demand requirement and so that link capacacity can be in modular units such as 155 Mbps?*

ATM Networks

Virtual Path Design (3.3.2)

- **indices**

  $d = 1, 2, ..., D$ demands

  $p = 1, 2, ..., P_d$ number of possible paths for a virtual path (VP) for demand d

  $e = 1, 2, ..., E$ links

- **constants**

  δ_{edp} = 1, if VP path p for demand d uses link e; 0, otherwise

  h_d volume of demand d (Mbps)

  ξ_e unit cost of a 155 Mbps link(LCU) on link e

  M capacity unit of an ATM link (in terms of the the number of modules)

- **variables**

  u_{dp} = 1, if path p for demand d is selected link e; 0, otherwise

  y_e capacity of link e (expressed in 155Mbps modules)

- The problem of determining the link capacity so that the total cost is minimized given that the ATM virtual path demand requirement and link capacity is in modular units (155 Mbps)

minimize$_{\boldsymbol{u},\boldsymbol{y}}$ $\boldsymbol{F} = \sum_e \xi_e y_e$

subject to $\sum_p u_{dp} = 1, \quad d = 1, 2, ..., D$

$\sum_d h_d \sum_p \delta_{edp} u_{dp} \leq M y_e, \quad e = 1, 2, ..., E$

u_{dp} binary, y_e integers.

Telephone Networks
Single-busy and Multi-busy hours
Network Dimensioning

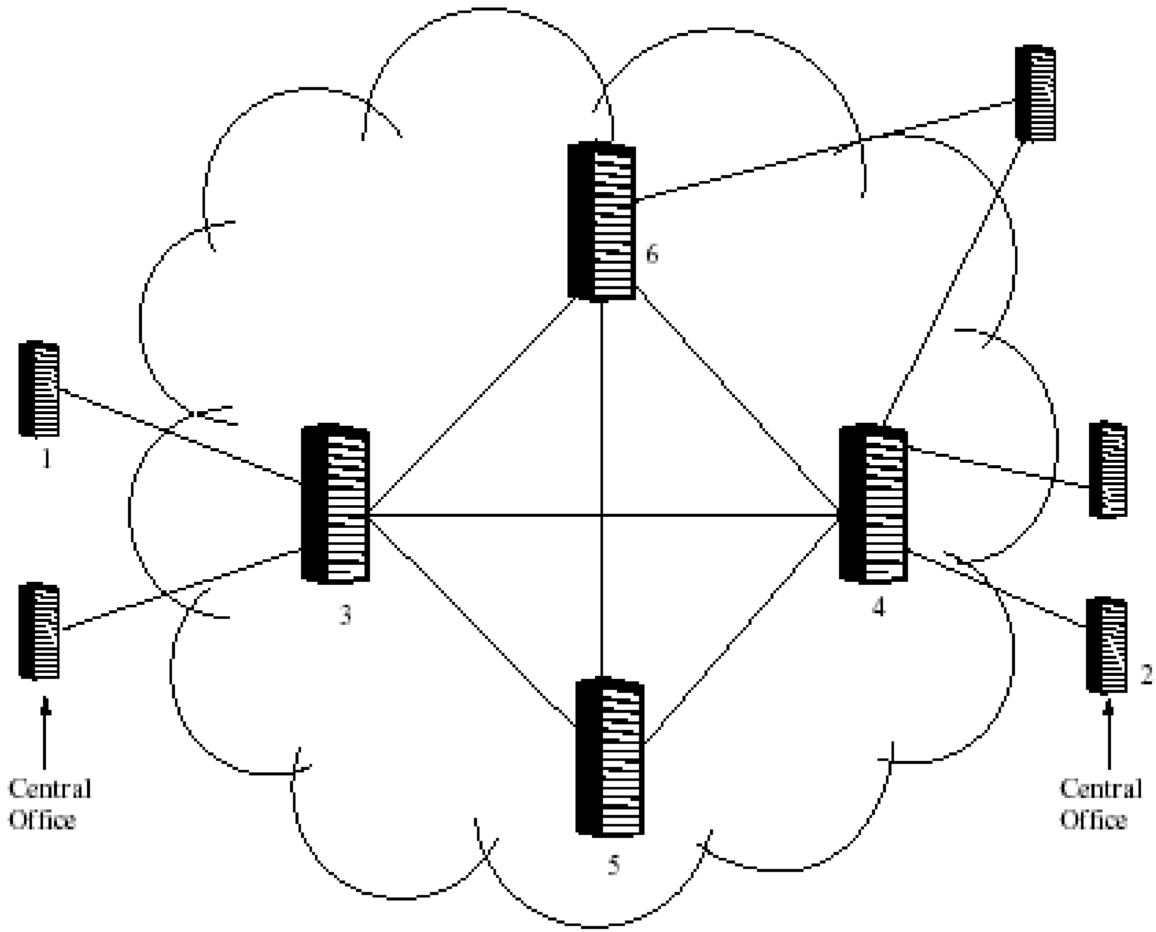

FIGURE 3.4 Circuit-Switched Network

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Telephone Networks
Single-busy and Multi-busy hours
Network Dimensioning

- Access nodes (end nodes) and digital exchanges (switches)
- Deman expressed in Erlangs
- Trunk-groups of 24 (T1) or 30 (E1) voice channels of 64 kbps
- Single-busy hour – peak offered traffic estimated over the entire day
- Multi-busy hours – different offered traffic during the day

Telephone Networks
Single-busy and Multi-busy hours
Network Dimensioning

- Originally fixed order of trying different routes, in the 80's changed with the introduction of dynamic non-hierarchical rputing (DNHR), dynamically controled routing (DHR) etc
- ***How to do modular capacity design given that traffic volume is different for different hours of a day, and by taking into account functional characteristics of routing scheme?***

Digital Circuit-Switched Telephone Networks

Single Busy-Hour Network Dimensioning (3.4.2)

- **indices**

  $d = 1, 2, ..., D$ demands

  $e = 1, 2, ..., E$ links

  $p = 1, 2, ..., P_d$ number of available routes for demand d
- **constants**

  δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise

  h_d volume of demand d in Erlangs(Erl)

  ξ_e unit modular capacity cost of link e

  b_e call blocking probability for link e to maintain a certain grade of service

  M modular capacity (e.g., in T1 – 24 voice circuits, or, E1 – 30 voice circuits)
- **variables**

  x_{dp} flow allocated to path p of demand d

  y_e capacity of link e expressed as number of modules M.
- The problem of determining the modular capacity needed in a network so that offered traffic is carried with an acceptable grade -of-service can be formulated as

  minimize$_{\boldsymbol{x},\boldsymbol{y}}$ $\quad \boldsymbol{F} = \sum_e \xi_e y_e$

  subject to $\quad \sum_p x_{dp} = h_d, \qquad d = 1, 2, ..., D$

  $F_e(\sum_d \sum_p \delta_{edp} x_{dp}) \leq M y_e, \qquad e = 1, 2, ..., E$

  x_{dp} continuous, non-negative

  $y_e$integers

  where $F_e(a) = \mathcal{C}(a; b_e)$. The function $\mathcal{C}(a; b)$ is the inverse of the Erlang blocking formula for offered load a and blocking b.

Multi Busy-Hour Network Dimensioning (3.4.3)

- **indices**

  $d = 1, 2, ..., D$ demands
  $e = 1, 2, ..., E$ links
  $p = 1, 2, ..., P_d$ number of available routes for demand d
  $t = 1, 2, ..., T$ number of traffic hour partitions

- **constants**

  h_{dt} volume of demand d in Erlangs(Erl) for time partition (hour) t
  δ_{edpt} = 1, if link e belongs to path p demand d for time partition t;
  = 0, otherwise
  b_{et} call blocking probability for link e for time partition t
  ξ_e unit modular capacity cost of link e

- **variables**

  x_{dpt} flow allocated to path p of demand d for time partition t
  y_e capacity of link e expressed as number of modules M.

- The problem of determining the modular capacity given that traffic volume is different for different hours of a day

minimize$_{\boldsymbol{x},\boldsymbol{y}}$ $\quad \boldsymbol{F} = \sum_e \xi_e y_e$

subject to

$$\sum_p x_{dpt} = h_{dt}, \qquad d = 1, 2, ..., D \quad t = 1, 2, ..., T$$

$$F_{et}\left(\sum_d \sum_p \delta_{edpt} x_{dpt}\right) \le M y_e, \qquad e = 1, 2, ..., E \quad t = 1, 2, ..., T$$

x_{dpt} continuous, non-negative $\qquad y_e$ integers,

where $F_{et}(a) = \mathcal{C}(a; b_{et})$. The function $\mathcal{C}(a; b)$ is the inverse of the Erlang blocking formula for offered load a and blocking b.

SONET/SDH
Capacity and Protection Design

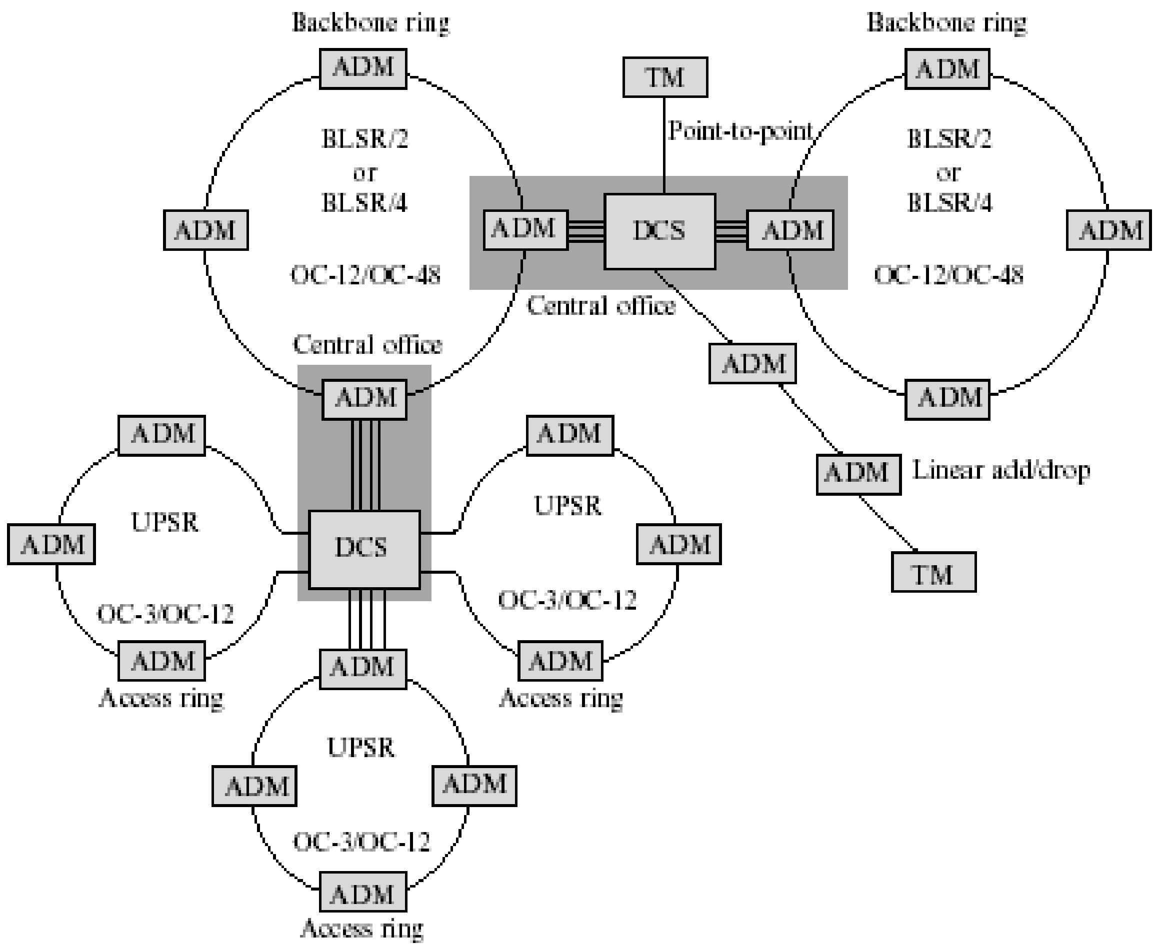

FIGURE 3.5 Elements of a SONET Infrastructure

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SONET/SDH
Capacity and Protection Design

TABLE 3.1 Transmission Rates for SONET/SDH.

SONET Signal	SDH Signal	Bit Rate (Mbps)
STS-1 (OC-1)	–	51.84
STS-3 (OC-3)	STM-1	155.52
STS-12 (OC-12)	STM-4	622.08
STS-48 (OC-48)	STM-16	2,488.32
STS-192 (OC-192)	STM-64	9,953.28

TABLE 3.2 Rates for VT (Subrates for STM) and VC (Subrates for STS).

VC Type	Bit Rate (Mbps)
VC-11	1.728
VC-12	2.304
VC-3	48.960
VC-4	150.336

VT Type	Bit Rate (Mbps)
VT-1.5	1.728
VT-2	2.304
VT-3	3.456
VT-6	6.912

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SONET/SDH
Capacity and Protection Design

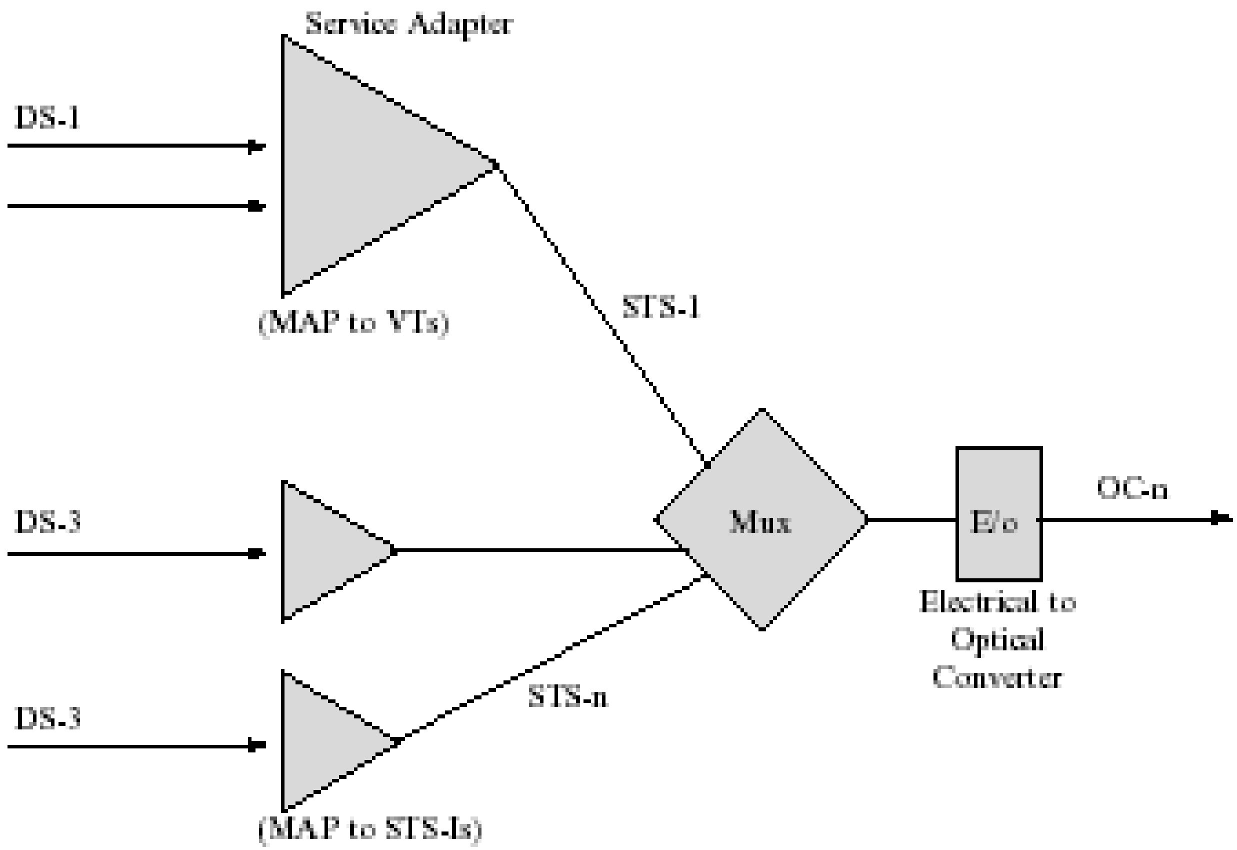

FIGURE 3.6 SONET Multiplexing Interfacing

SONET/SDH Transport Networks

Capacity and Protection Design (3.5.1)

- **indices**
  - $d = 1, 2, ..., D$ demands
  - $e = 1, 2, ..., E$ links
  - $p = 1, 2, ..., P_d$ number of available routes for demand d
- **constants**
  - δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise
  - h_d volume of demand d in term,s of VC-12s
  - ξ_e cost of one LCU(STM-1 system) on link e
  - M= 63 (Each STM-1 module can carry 63 VC-12 containers)
- **variables**
  - x_{dp} flow allocated to path p of demand d
  - y_e capacity of link e (expressed in STM-1 modules)
- The SDH transport network capacity design problem can be formulated as follows :

minimize$_{\boldsymbol{x},\boldsymbol{y}}$ $\boldsymbol{F} = \sum_e \xi_e y_e$

subject to $\sum_p x_{dp} = h_d, \quad d = 1, 2, ..., D$

$\sum_d \sum_p \delta_{edp} x_{dp} \leq M y_e, \quad e = 1, 2, ..., E$

x_{dp}, y_e non-negative integers

Capacity and Protection Design (3.5.2)

- **indices**

  $d = 1, 2, ..., D$ demands

  $e = 1, 2, ..., E$ links

  $p = 1, 2, ..., P_d$ number of available routes for demand d
- **constants**

  δ_{edp} = 1, if link e belongs to path p realizing demand d; 0, otherwise

  h_d volume of demand d in term,s of VC-12s

  ξ_{en} cost of one transmission system STM-n realized on link e

  M_n = 63 n, the modularity of STM-n system
- **variables**

  x_{dp} flow allocated to path p of demand d

  y_{en} number of STM-n systems realized on link e
- The SDH transport network capacity design problem that differentiates costs of STM modules can be formulated as follows :

minimize$_{\boldsymbol{x},\boldsymbol{y}}$ $\quad \boldsymbol{F} = \sum_e \sum_n \xi_{en} y_{en}$

subject to

$$\sum_p x_{dp} = h_d, \qquad d = 1, 2, ..., D$$

$$\sum_d \sum_p \delta_{edp} x_{dp} \le \sum_n M_n y_{en}, \qquad e = 1, 2, ..., E$$

x_{dp}, y_{en} non-negative integers

Capacity and Protection Design (3.5.5)

- **indices**

  $d = 1, 2, ..., D$ demands

  $e = 1, 2, ..., E$ links

  $q = 1, 2, ..., Q_e$ list of restoration paths available for link e
- **constants**

  ξ_e unit cost of link e

  h_d volume of demand d in Erlangs(Erl)

  c_e capacity of link e

  β_{feq} =1, if link f belongs to path q restoring link; 0, otherwise
- **variables**

  y_e protection capacity of link e

  z_{fq} capacity restored by path q that restores link f

- The problem of minimizing the cost of the necessary link protection capacity can be formulated as follows :

minimize$_{\boldsymbol{z},\boldsymbol{y}}$ $\boldsymbol{F} = \sum_e \xi_e y_e$

subject to $\sum_q z_{eq} = c_e, \quad e = 1, 2, ..., E$

$\sum_e \sum_q \beta_{fek} z_{eq} = y_f, \quad f = 1, ..., E, \; e = 1, 2, ..., E, \quad f \neq e$

z_{eq}, y_e non-negative integers.

SONET/SDH RINGS: RING BANDWIDTH DESIGN

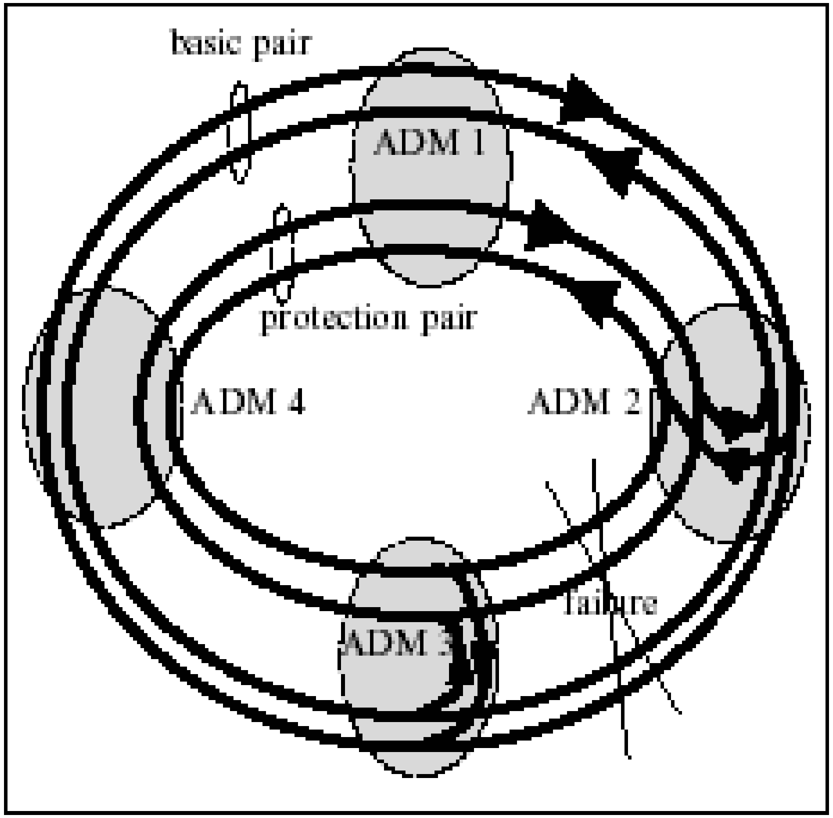

FIGURE 3.7 Bi-Directional Line-Switched Ring (BLSR)

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SONET/SDH RINGS: RING BANDWIDTH DESIGN

- Restoration intrinsic to network functionality: < 50 ms restoration capability from single-link failure
- ADM nodes capable of extracting containers
- ***Given the inherent routing nature of a SONET/SDH ring and the demand volume, how do we determine what is the minimal number and type of (parallel) rings needed?***

SONET/SDH RINGS: RING BANDWIDTH DESIGN

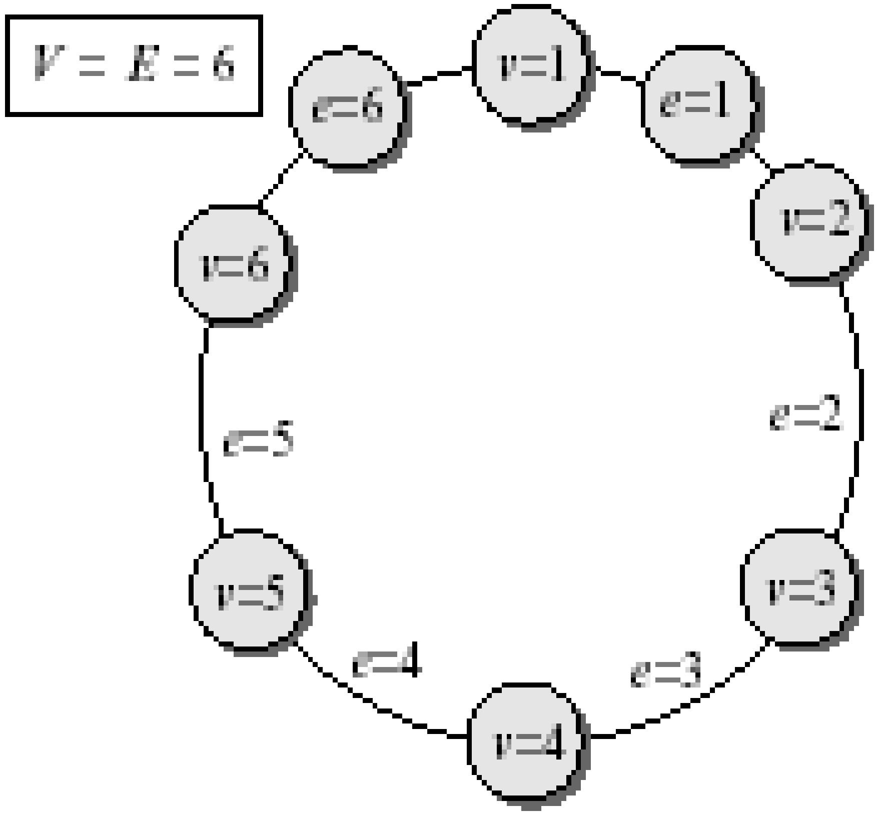

FIGURE 3.8 Node and Segment Labeling of BLSR

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SONET/SDH Rings

Ring Bandwidth Design (3.6.1)

- **indices**

  $v = 1, 2, ..., V$ nodes

  $e = 1, 2, ..., E$ segments

- **constants**

  h_{vw} demand volume between nodes v and w, with $v < w$

  M Modularity of the STM system

  δ_{evw} = 1, if $v \le e < w$; 0, otherwise

- **variables**

  u_{vw} flow on the clockwise path from w to v

  z_{vw} flow on the clockwise path from v to w

- The problem of determining the minimal number and type of (parallel) rings needed can be formulated as

$$\begin{array}{lll} \textbf{\textit{minimize}}_{\boldsymbol{u}, \boldsymbol{z}, r} & r & \\ \textbf{\textit{subject to}} & u_{vw} + z_{vw} = h_{vw}, & v, w = 1, 2, ..., V, v < w \\ & \delta_{evw} u_{vw} + (1 - \delta_{evw}) z_{vw} \le Mr, & e = 1, 2, ..., E \\ & u_{vw}, z_{vw}, r \text{ non-negative integers.} & \end{array}$$

WDM Networks: Restoration Design

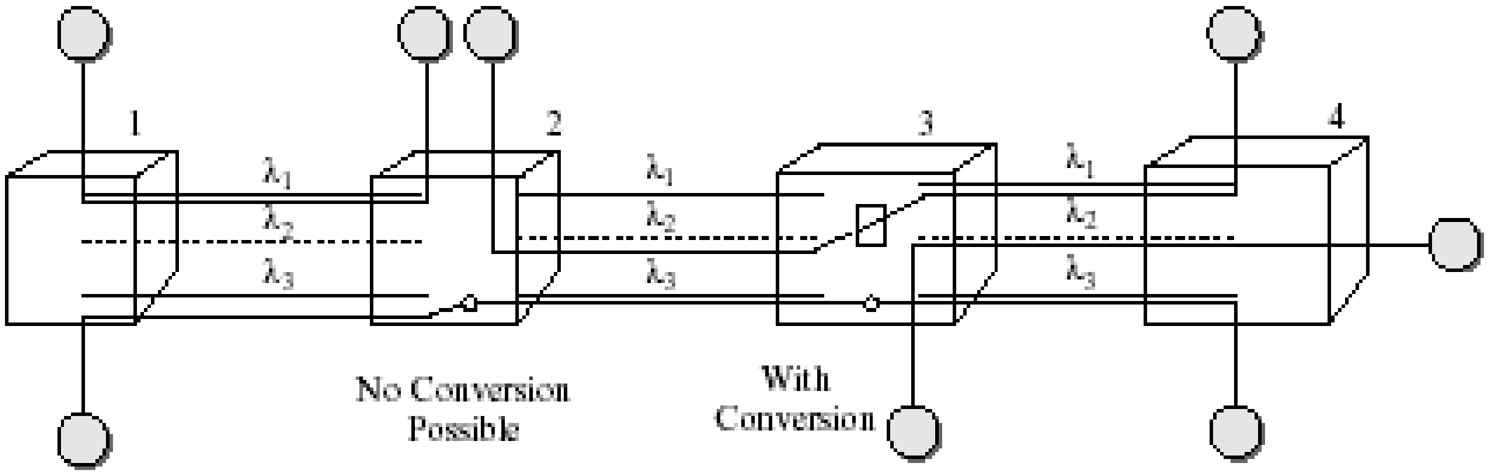


FIGURE 3.9 WDM Network

WDM Networks: Restoration Design

- Wavelengths typically carries 10 Gbps (1 DVU = 10 Gbps)
- Optical cross connect with and without wavelength conversion
- A light-path (route) can use different colours

Restoration Design with Optical Cross-Connects (3.7.1)

- **indices**

$c = 1, 2, ..., C$	colors
$e = 1, 2, ..., E$	links
$v = 1, 2, ..., V$	nodes
$s = 0, 1, ..., S$	failure situations

- **constants**

h_{ds} ($d = 1, 2, ..., D$)	volume of demand d to be realized in situation s,
ξ_e ($e = 1, 2, ..., E$)	cost of one LCU (i.e., optical fibre) on link e
α_{es}	= 0 if link e is failed in situation s ; 1, otherwise
δ_{edp}	= 1 if link e belongs to path p realizing demand d, ; 0, otherwise
θ_{dps}	=0 if path p of demand d is failed in situation s ; 1, otherwise

- **variables**

x_{dpc}	flow (number of light-paths) realizing demand d in color c on path p
z_{ce}	number of times the color c is used on link e
y_e	capacity of link e expressed in the number of fibers

- The optimization problem for the OXCs without wavelength conversion can be formulated as

minimize$_{\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{y}}$ $\quad \boldsymbol{F} = \sum_e \xi_e y_e$

subject to

$$\sum_p \theta_{dps} \sum_c x_{dpc} \geq h_{ds}, \quad d = 1, 2, ..., D \quad s = 0, 1, 2, ..., S$$

$$\sum_d \sum_p \delta_{edp} x_{dpc} = z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E$$

$$y_e \geq z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E.$$

x_{dpc}, z_{ce}, y_{es} non-negative integers

WDM Networks:

Restoration Design with Optical Cross-Connects (3.7.2)

- **indices**

  $c = 1, 2, ..., C$ colors

  $e = 1, 2, ..., E$ links

  $v = 1, 2, ..., V$ nodes

  $s = 0, 1, ..., S$ failure situations

- **constants**

  h_{ds} volume of demand d to be realized in situation s,

  ξ_e cost of one LCU (i.e., optical fibre) on link e

  κ_e link opening cost for the link e

  α_{es} = 0 if link e is failed in situation s ; 1, otherwise

  δ_{edp} = 1 if link e belongs to path p realizing demand d, ; 0, otherwise

  θ_{dps} =0 if path p of demand d is failed in situation s ; 1, otherwise

- **variables**

  x_{dpc} flow (number of light-paths) realizing demand d in color c on path p

  z_{ce} number of times the color c is used on link e

  y_e capacity of link e expressed in the number of fibers

  u_e =0 if the link e is installed; 1, otherwise

(Contd.)

Restoration Design with Optical Cross-Connects (3.7.2)

(Contd.)

- The optimization problem for the OXCs without wavelength conversion that takes in to account the link opening costs can be formulated as

minimize$_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{y},\boldsymbol{u}}$ $\quad \boldsymbol{F} = \sum_e (\xi_e y_e + \kappa_e u_e)$

subject to

$$\sum_p \theta_{dps} \sum_c x_{dpc} \geq h_{ds}, \quad d = 1, 2, ..., D \quad s = 0, 1, 2, ..., S$$

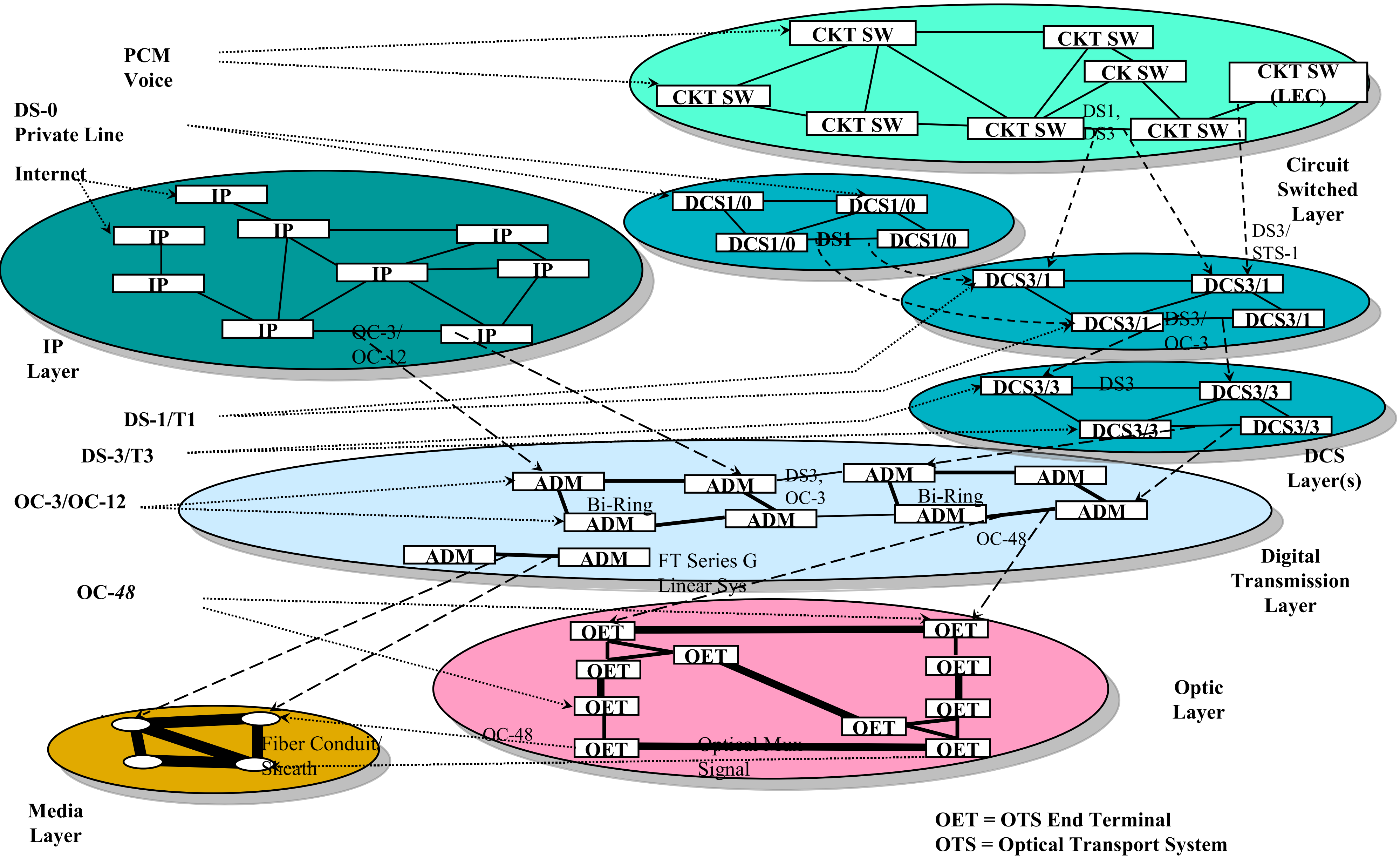
$$\sum_d \sum_p \delta_{edp} x_{dpc} = z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E$$

$$y_e \geq z_{ce}, \quad c = 1, ..., C, \quad e = 1, 2, ..., E$$

$$y_e \leq M u_e, \quad e = 1, 2, ..., E.$$

x_{dpc}, z_{ce}, y_{es} non-negative integers

M is a large number



PCM
Voice
DS-0
Private Line
Internet
CKT SW
CKT SW
CK SW
CKT SW
(LEC)
CKT SW
CKT SW
CKT SW
CKT SW
DS1,
DS3
Circuit
Switched
Layer
DS3/
STS-1
IP
IP
IP
IP
IP
IP
IP
IP
IP
OC-3/
OC-12
IP
Layer
DCS1/0
DCS1/0
DCS1/0
DS1
DCS1/0
DCS3/1
DCS3/1
DCS3/1
DS3/
OC-3
DCS3/1
DCS3/3
DS3
DCS3/3
DCS3/3
DCS3/3
DS-1/T1
DS-3/T3
DCS
Layer(s)
OC-3/OC-12
ADM
ADM
DS3,
OC-3
ADM
ADM
Bi-Ring
ADM
ADM
Bi-Ring
ADM
ADM
OC-48
ADM
ADM
FT Series G
Linear Sys
Digital
Transmission
Layer
OC-48
OET
OET
OET
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OET
OET
OET
OET
OET
OET
OC-48
Optical Mux
Signal
Optic
Layer
Fiber Conduit/
Sheath
Media
Layer
OET = OTS End Terminal
OTS = Optical Transport System

IP Over SONET

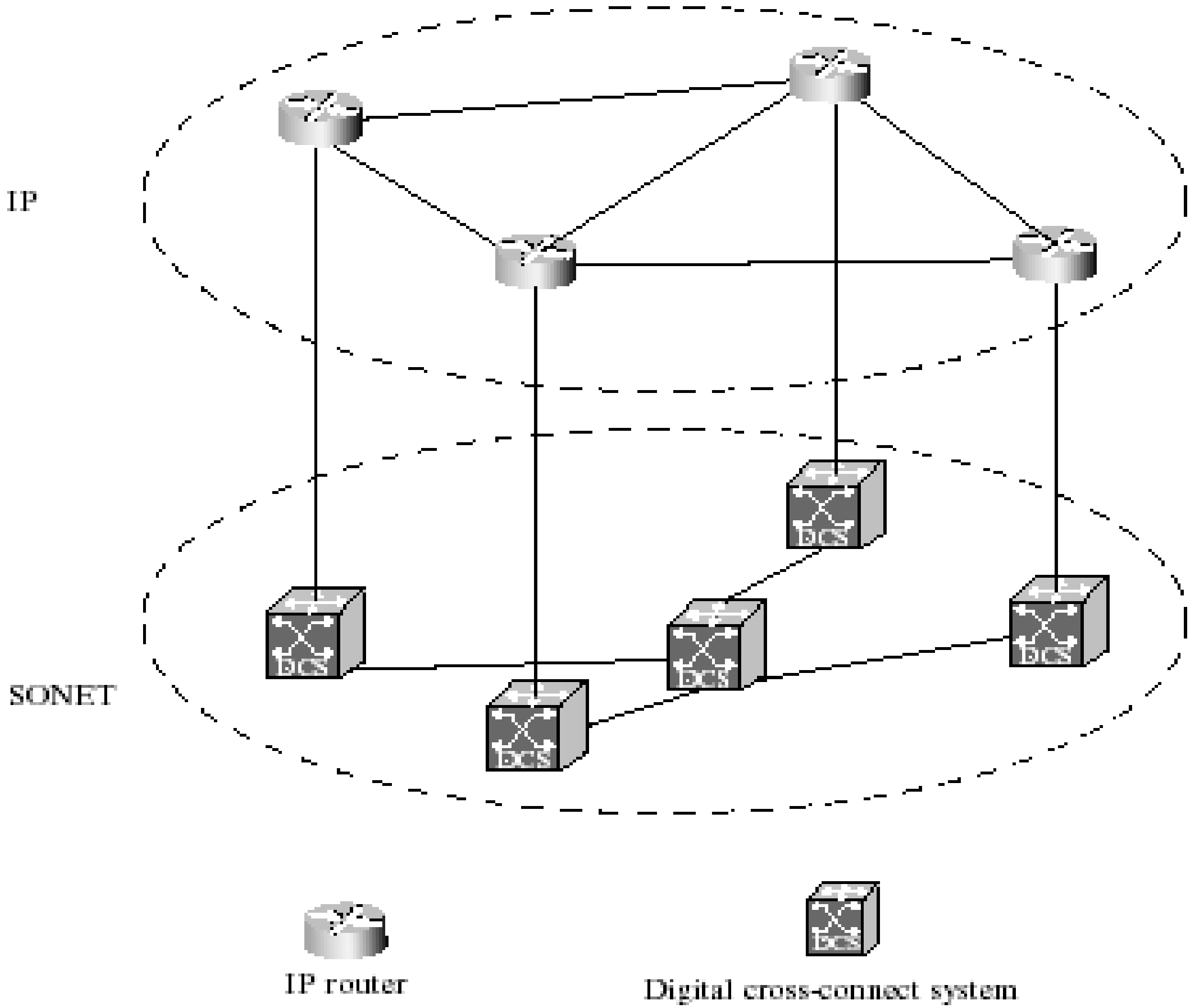

FIGURE 3.10 IP Over SONET: Two-Layer Architecture

IP over SONET

- ***Given an IP intra-domain network and that the IP links are realized as transmission paths over a capacitated SONET network, how do we determine capacity required for the IP links, and the routing of these links in the SONET network in an integrated manner to minimize the IP network cost?***

IP Over SONET: Combined Two-Layer Design (3.8.1)

- **indices**

$d = 1, 2, ..., D$	demands
$e = 1, 2, ..., E$	links
$q = 1, 2, ..., Q_e$	list of candidate paths for link e

- **constants**

c_g	capacity of link g in the SONET network expressed in OC-48 modules
δ_{edp}	= 1, if link e belongs to path p realizing demand d; 0, otherwise
h_d	volume of demand d
ξ_e	link termination cost by the cost of the OC-3 interfaces at the end routers of link e
ζ_{eq}	routing cost at the SONET layer
ρ	link utilization coefficient
M	=size of the link capacity in IP network(e.g., 155.52 Mbps)
N	=size of the link capacity in SONET network (e.g., 2,488.32 Mbps)

- **variables**

w_e	metric of link e, $\boldsymbol{w} = (w_1, w_2, ..., w_E)$
$x_{dp}(\boldsymbol{w})$	flow allocated to path p of demand d determined by the link system w
y_e	modular capacity of the IP layer link e
z_{eq}	flow allocated to path q realizing capacity link e
γ_{geq}	=1, if path q on the transport layer for demand e uses link g ; and 0, otherwise

(Contd.)

IP Over SONET: Combined Two-Layer Design (3.8.1)

(Contd.)

- The problem to determine the capacity required for the IP links, and the routing of these links in the SONET network in an integrated manner to minimize the IP network cost can be formulated as

$$
\begin{array}{lll}
\textbf{minimize}_{\boldsymbol{w},\boldsymbol{y},\boldsymbol{z}} & \sum_e \xi_e y_e + \sum_e \sum_q \zeta_{eq} z_{eq} & \\
\textbf{subject to} & \sum_p x_{dp}(\boldsymbol{w}) = h_d, & d = 1, 2, ..., D \\
 & \sum_d \sum_p \delta_{edp} x_{dp}(\boldsymbol{w}) \le \rho M y_e, & e = 1, 2, ..., E \\
 & \sum_q \sum_c z_{eq} = y_e, & e = 1, 2, ..., E \\
 & \sum_e M \sum_q \gamma_{geq} z_{eq} \le N c_g, & g = 1, 2, ..., G \\
 & w_e \text{ non-negative integer} & \\
 & y_e, z_{eq} \text{ non-negative integer .} &
\end{array}
$$