Algorithm 1: Computation of orthonormal bases for each subspace V_*.

Data: Three $n \times n$ permutation matrices A, B e C.
Result: Subspace bases S_1, S_2, S_3, S_4, S_5.

$S \leftarrow \emptyset$
$L \leftarrow \text{null}(\begin{bmatrix}(A-B)\\(A-C)\end{bmatrix})$
foreach $v \in L$ **do**
  $S \leftarrow$ `Add(`S,v`)`
$S_1 \leftarrow S$ `// Basis of` $V_*(.ABC.)$
$S \leftarrow S_1$
$L \leftarrow \text{null}(A-B)$
foreach $v \in L$ **do**
  $S \leftarrow$ `Add(`S,v`)`
$S_2 \leftarrow S - S_1$ `// Basis of` $V_*(.AB.C.)$
$S \leftarrow S_1$
$L \leftarrow \text{null}(B-C)$
foreach $v \in L$ **do**
  $S \leftarrow$ `Add(`S,v`)`
$S_3 \leftarrow S - S_1$ `// Basis of` $V_*(.BC.A.)$
$S \leftarrow S_1$
$L \leftarrow \text{null}(C-A)$
foreach $v \in L$ **do**
  $S \leftarrow$ `Add(`S,v`)`
$S_4 \leftarrow S - S_1$ `// Basis of` $V_*(.AC.B.)$
$L \leftarrow \text{null}(\begin{bmatrix}S_1 & S_2 & S_3 & S_4\end{bmatrix})$
$S_5 \leftarrow \emptyset$
`// Basis of` $V_*(.A.B.C.)$
foreach $v \in L$ **do**
  $S_5 \leftarrow$ `Add(`S,v`)`

As v_2 is in the image of S_2, it can be written as $S_2 u_2$, and therefore

$$\begin{aligned} P_2 v &= S_2(N_2^T S_2)^{-1} N_2^T S_2 u_2 \\ &= S_2 u_2 \\ &= v_2. \end{aligned}$$

Note that, in the computation of P_2, P_3, and P_4, it is not possible to simplify the right-hand expression $S_i(N_i^T S_i)^{-1} N_i^T$. In general, neither S_i nor N_i are square matrices, and therefore they are not invertible. Nevertheless, the product $N_i^T S_i$ is always invertible.

Knowing the projection matrices, we can finally compute the median candidates, in Algorithm 2. Previously, we saw how to compute M_A. It is also possible to define M_B and M_C in an analogous way. The matrix M_B follows B in $V_*(.A.B.C.)$ instead of A, and M_C follows C. The entire computation takes $O(n^3)$ arithmetic operations.