Network Science

Class 3: Random Networks
(Chapter 3 in textbook)

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with

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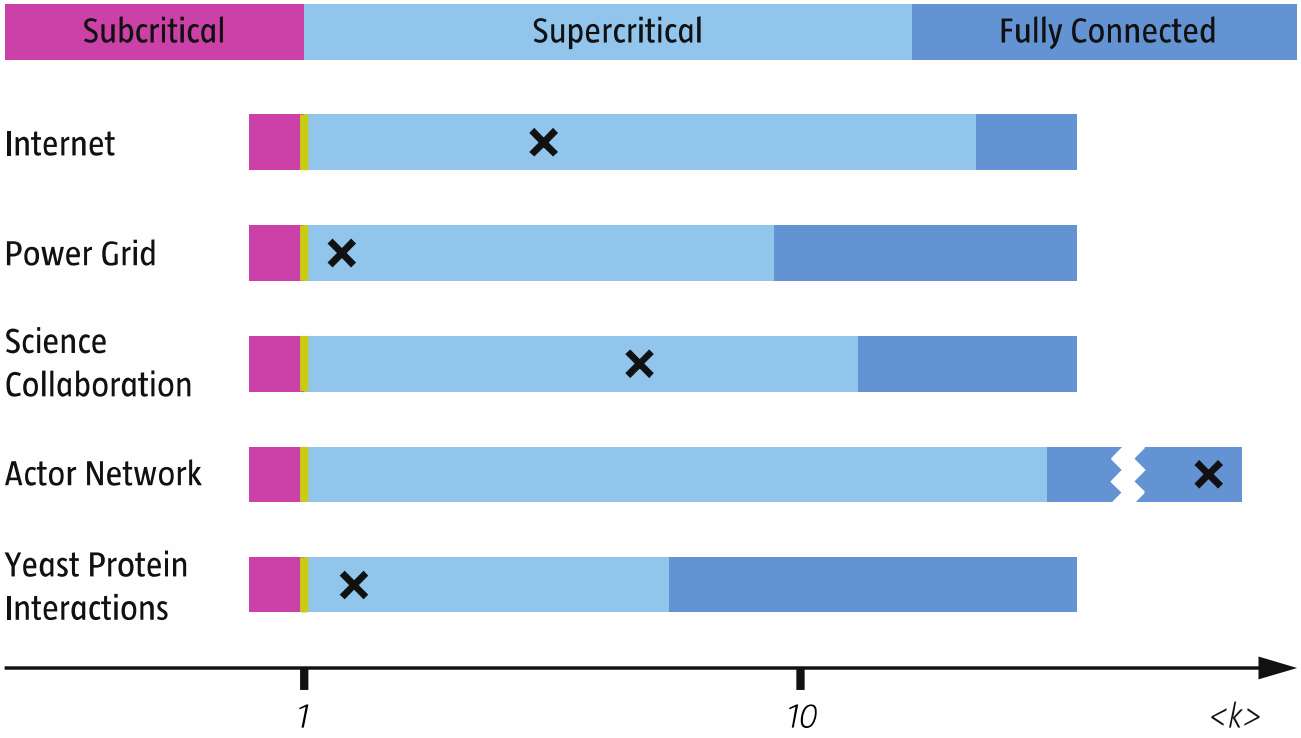
Real networks are supercritical

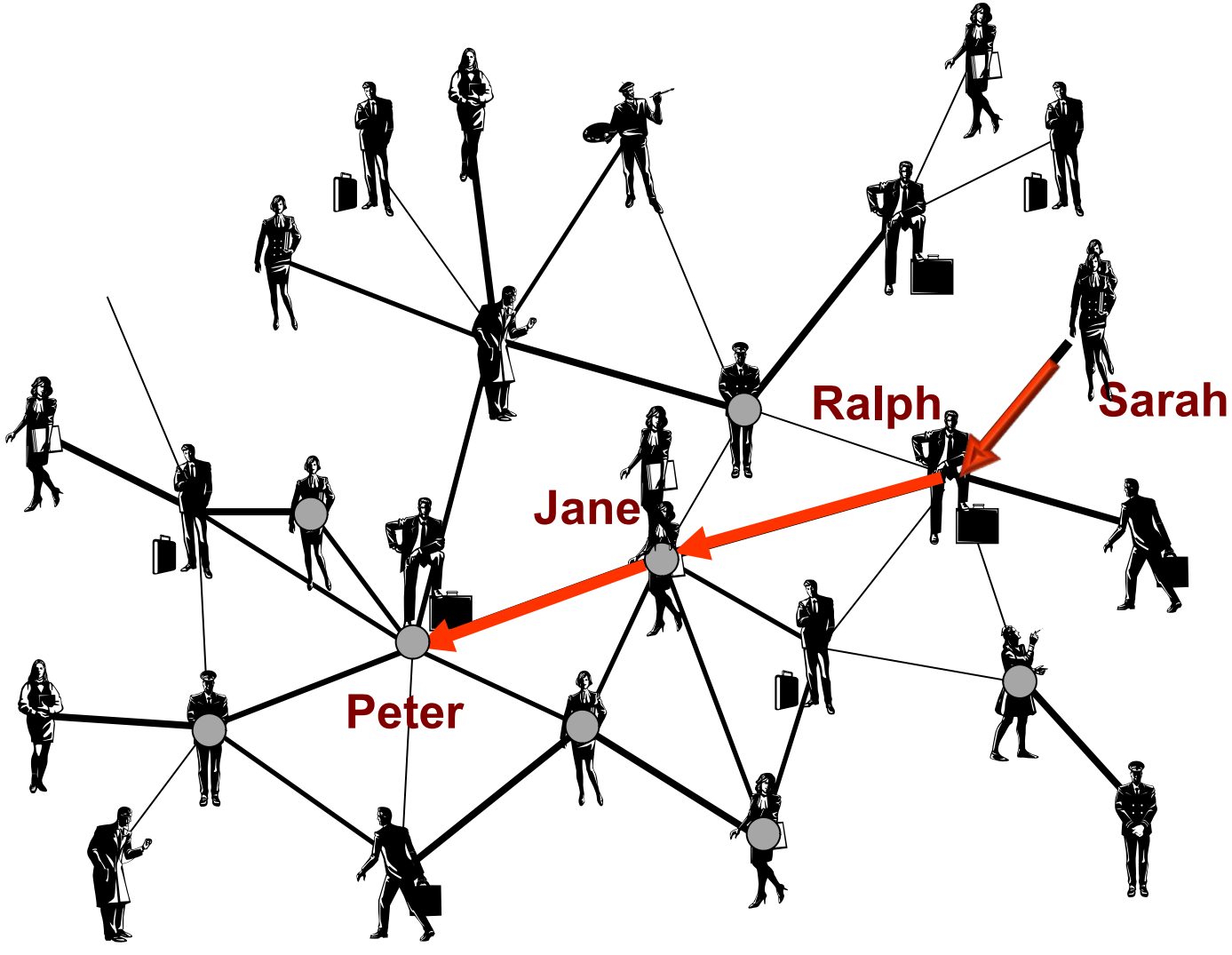
Section 7



Network	*N*	*L*	<*k*>	ln *N*
Internet	192,244	609,066	6.34	12.17
Power Grid	4,941	6,594	2.67	8.51
Science Collaboration	23,133	186,936	8.08	10.04
Actor Network	212,250	3,054,278	28.78	12.27
Yeast Protein Interactions	2,018	2,930	2.90	7.61

Small worlds

SIX DEGREES | small worlds



Frigyes Karinthy, 1929
Stanley Milgram, 1967

SIX DEGREES | 1929: Frigyes Kartinthy

Frigyes Karinthy (1887-1938)
Hungarian Writer

1929: *Minden másképpen van* (Everything is Different)
Láncszemek (Chains)

"Look, Selma Lagerlöf just won the Nobel Prize for Literature, thus she is bound to know King Gustav of Sweden, after all he is the one who handed her the Prize, as required by tradition. King Gustav, to be sure, is a passionate tennis player, who always participates in international tournaments. He is known to have played Mr. Kehrling, whom he must therefore know for sure, and as it happens I myself know Mr. Kehrling quite well."

"The worker knows the manager in the shop, who knows Ford; Ford is on friendly terms with the general director of Hearst Publications, who last year became good friends with Arpad Pasztor, someone I not only know, but to the best of my knowledge a good friend of mine. So I could easily ask him to send a telegram via the general director telling Ford that he should talk to the manager and have the worker in the shop quickly hammer together a car for me, as I happen to need one."

SIX DEGREES | 1967: Stanley Milgram

HOW TO TAKE PART IN THIS STUDY

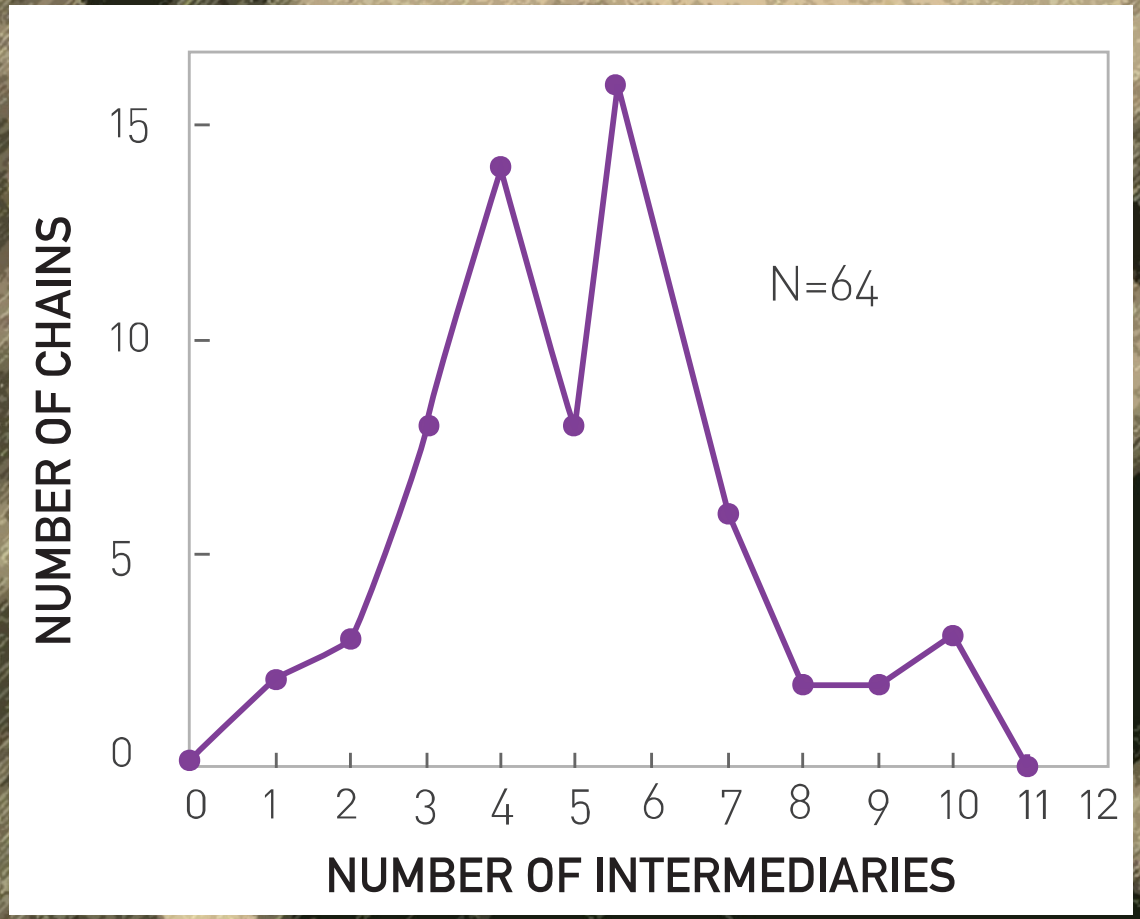
1. ADD YOUR NAME TO THE ROSTER AT THE BOTTOM OF THIS SHEET, so that the next person who receives this letter will know who it came from.

2. DETACH ONE POSTCARD. FILL IT AND RETURN IT TO HARVARD UNIVERSITY. No stamp is needed. The postcard is very important. It allows us to keep track of the progress of the folder as it moves toward the target person.

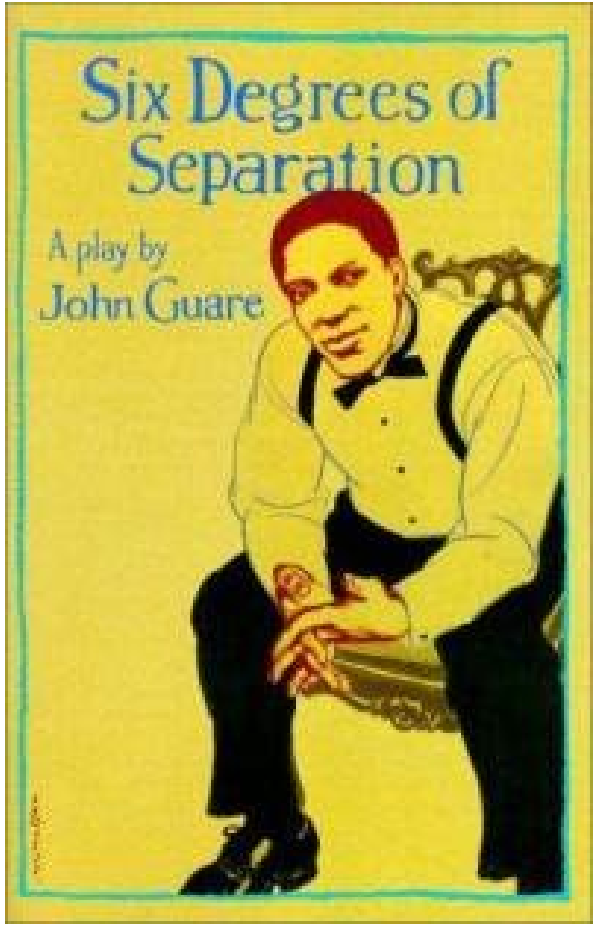
3. IF YOU KNOW THE TARGET PERSON ON A PERSONAL BASIS, MAIL THIS FOLDER DIRECTLY TO HIM (HER). Do this only if you have previously met the target person and know each other on a first name basis.

4. IF YOU DO NOT KNOW THE TARGET PERSON ON A PERSONAL BASIS, DO NOT TRY TO CONTACT HIM DIRECTLY. INSTEAD, MAIL THIS FOLDER (POST CARDS AND ALL) TO A PERSONAL ACQUAINTANCE WHO IS MORE LIKELY THAN YOU TO KNOW THE TARGET PERSON. You may send the folder to a friend, relative or acquaintance, but it must be someone you know on a first name basis.

SIX DEGREES | 1967: Stanley Milgram



SIX DEGREES | 1991: John Guare



"Everybody on this planet is separated by only six other people. Six degrees of separation. Between us and everybody else on this planet. The president of the United States. A gondolier in Venice…. It's not just the big names. It's anyone. A native in a rain forest. A Tierra del Fuegan. An Eskimo. I am bound to everyone on this planet by a trail of six people. It's a profound thought. How every person is a new door, opening up into other worlds."

WWW: 19 DEGREES OF SEPARATION

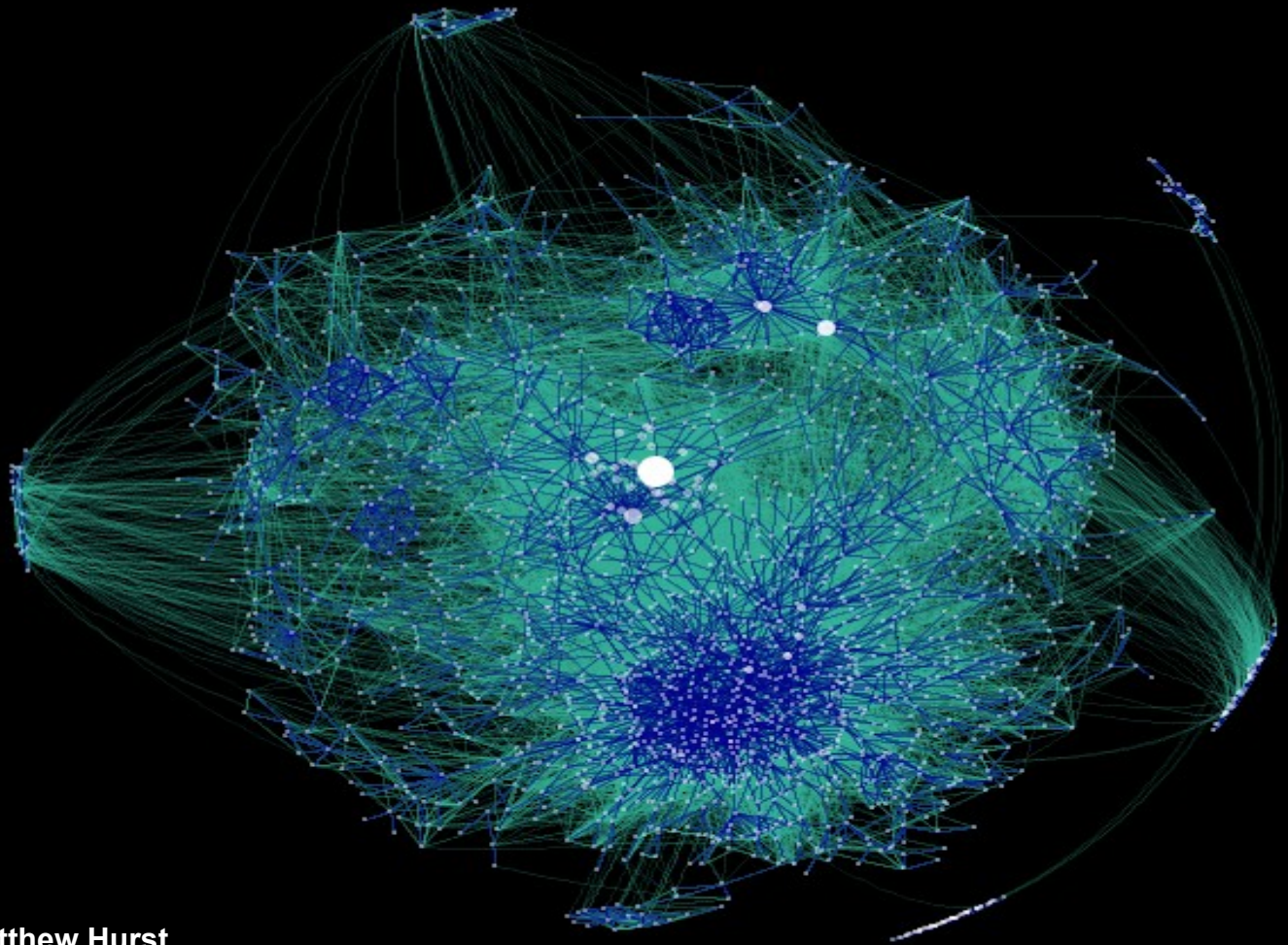
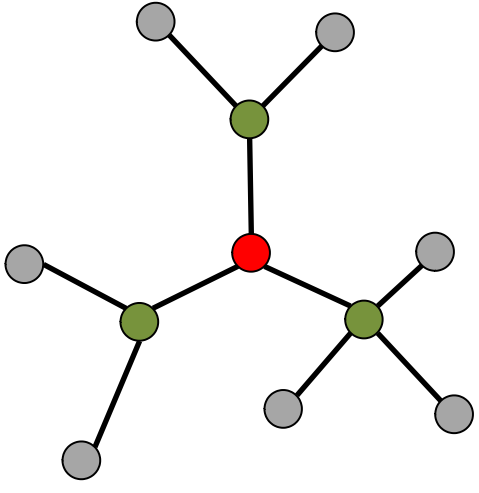


Image by **Matthew Hurst**
Blogosphere

DISTANCES IN RANDOM GRAPHS

Random graphs tend to have a tree-like topology with almost constant node degrees.



<k> nodes at distance one (d=1).

<k>^{2} nodes at distance two (d=2).

<k>^{3} nodes at distance three (d =3).

...

<k>^{d} nodes at distance d.

$$N = 1 + \langle k \rangle + \langle k \rangle^2 + \ldots + \langle k \rangle^{d_{\max}} = \frac{\langle k \rangle^{d_{\max}+1} - 1}{\langle k \rangle - 1} \approx \langle k \rangle^{d_{\max}} \quad \Rightarrow \quad d_{\max} = \frac{\log N}{\log \langle k \rangle}$$

DISTANCES IN RANDOM GRAPHS

$$d_{\max} = \frac{\log N}{\log \langle k \rangle}$$

In most networks this offers a better approximation to the average distance between two randomly chosen nodes, $\langle d \rangle$, than to d_{max}.

$$< d > = \frac{\log N}{\log \langle k \rangle}$$

We will call the *small world phenomena* the property that the average path length or the diameter depends logarithmically on the system size. Hence, ”small” means that $\langle d \rangle$ is proportional to log N, rather than N.

The $1/\log\langle k \rangle$ term implies that denser the network, the smaller will be the distance between the nodes.

DISTANCES IN RANDOM GRAPHS

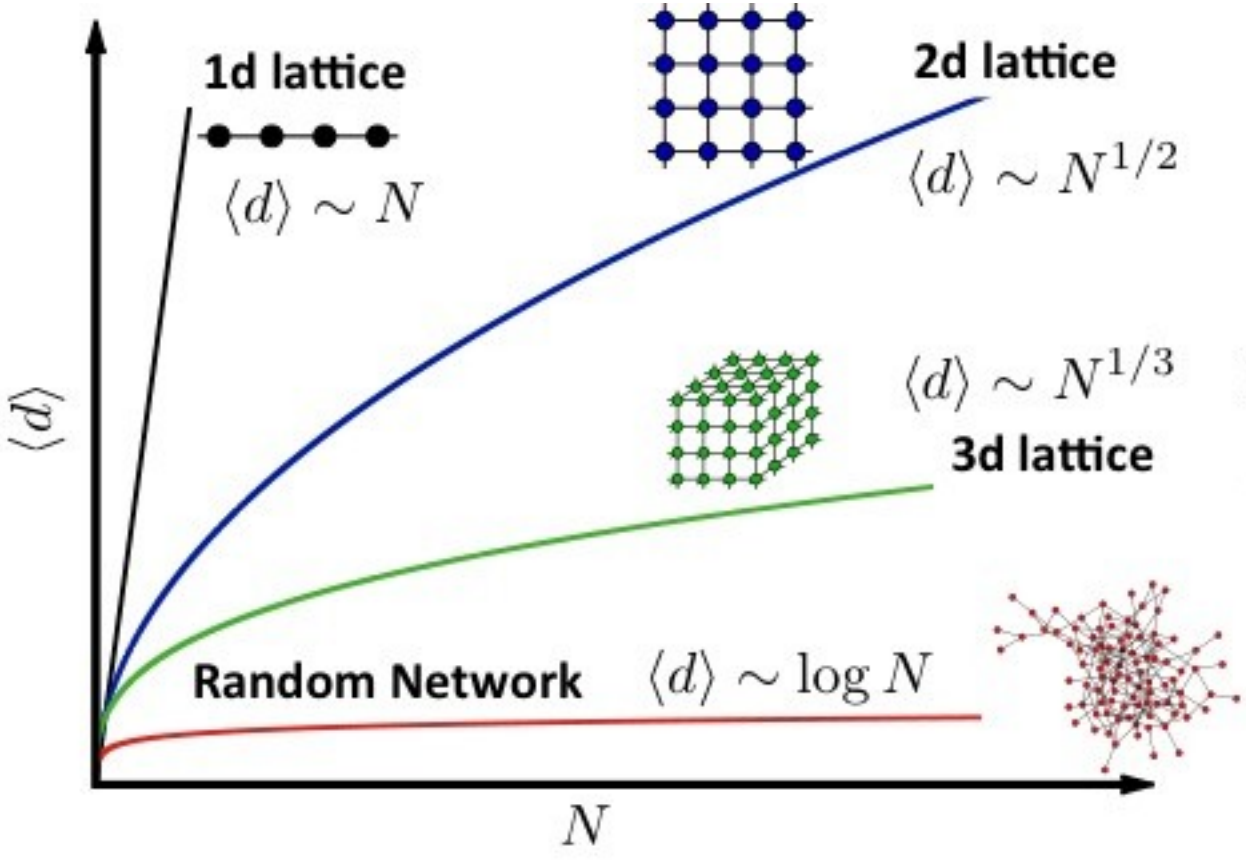
compare with real data

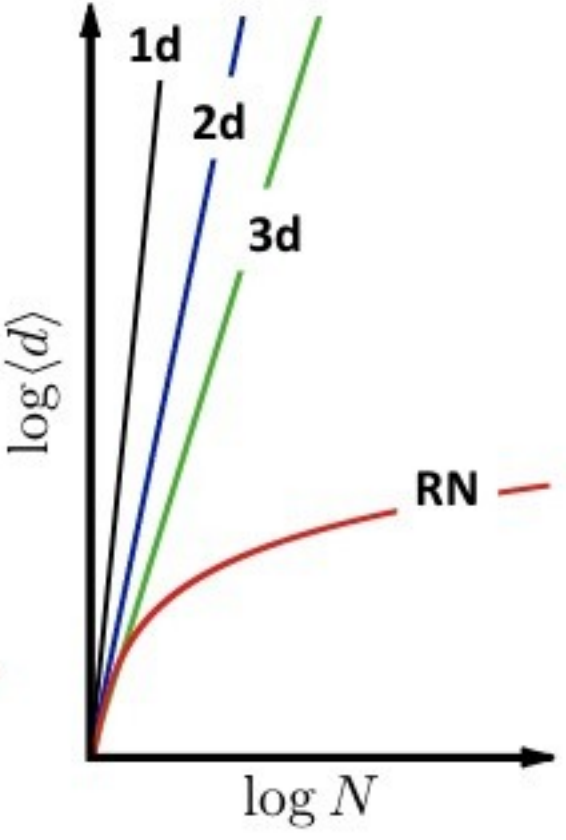
Network	N	L	‹k›	‹d›	dmax	lnN/ln‹k›
Internet	192244	609066	6.34	6.98	26	6.58
WWW	325729	1497134	4.6	11.27	93	8.31
Power Grid	4941	6594	2.67	18.99	46	8.66
Mobile-Phone Calls	36595	91826	2.51	11.72	39	11.42
Email	57194	103731	1.81	5.88	18	18.4
Science Collaboration	23133	93437	8.08	5.35	15	4.81
Actor Network	702388	29397908	83.71	3.91	14	3.04
Citation Network	449673	4707958	10.43	11.21	42	5.55
E. Coli Metabolism	1039	5802	5.58	2.98	8	4.04
Protein Interactions	2018	2930	2.9	5.61	14	7.14

Given the huge differences in scope, size, and average degree, the agreement is excellent.

Why are small worlds surprising?

Suprising compared to what?





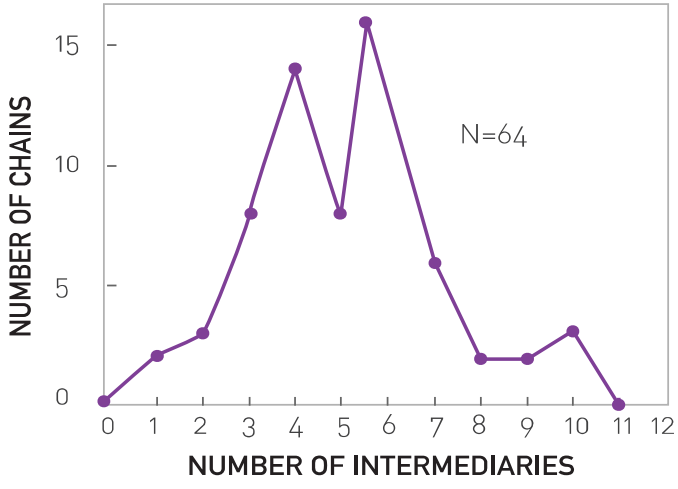
Three, Four or Six Degrees?

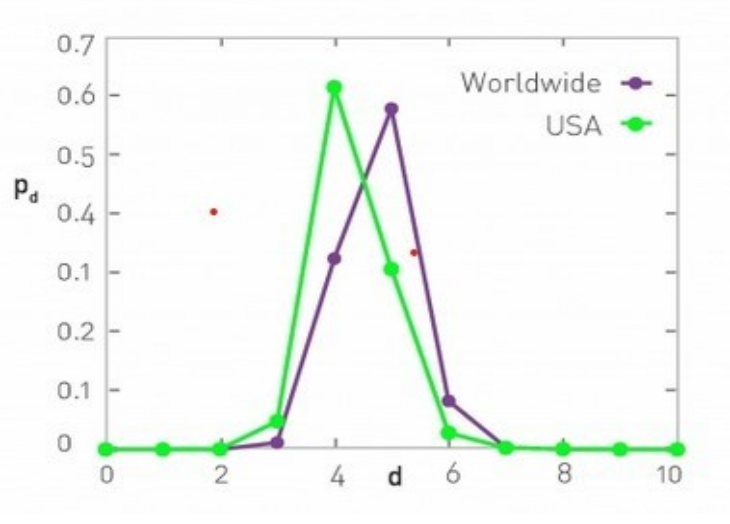
For the globe's social networks:

$\langle k \rangle \simeq 10^3$

$N \simeq 7 \times 10^9$ for the world's population.

$$d >= \frac{\ln(N)}{\ln\langle k \rangle} = 3.28$$





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Karinthy, 1929

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Manfred Kochen

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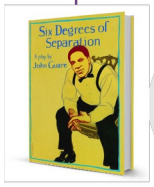

John Guare

Duncan J. Watts

Steven Strogatz

4-DEGREE OF SEPARATION

6-DEGREE OF SEPARATION

DISCOVERY

PUBLISHED 20 YEARS LATER

MILESTONES

WWII

XXI

PUBLICATION DATE

1929 1935 1940 1945 1950 1958 1960 1967 1970 1978 1980 1985 1991 1998 2000 2005 2011

Frigyes Karinthy (1887-1938)
Hungarian writer, journalist and playwright, the first to describe the small world property. In his short story entitled 'Láncszemek' (Chains) he links a worker in Ford's factory to himself [23, 24].

Manfred Kochen (1928-1989), **Ithiel de Sola Pool** (1917-1984)
Scientific interest in small worlds started with a paper by political scientist Ithiel de Sola Pool and mathematician Manfred Kochen. Written in 1958 and published in 1978, their work addressed in mathematical detail the small world effect, predicting that most individuals can be connected via two to threee acquaintances. Their paper inspired the experiments of Stanley Milgram.

Stanley Milgram (1933-1984)
American social psychologist who carried out the first experiment testing the small-world phenomena. (**BOX 3.6**).

John Guare (1938)
The phrase 'six degrees of separation' was introduced by the playwright John Guare, who used it as the title of his Broadway play.

Duncan J. Watts (1971), **Steven Strogatz** (1959)
A new wave of interest in small worlds followed the study of Watts and Strogatz, finding that the small world property applies to natural and technological networks as well.

The **Facebook Data Team** measures the average distance between its users, finding "4 degrees" (**BOX 3.6**).