Network Science

Class 8: Network Robustness

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with

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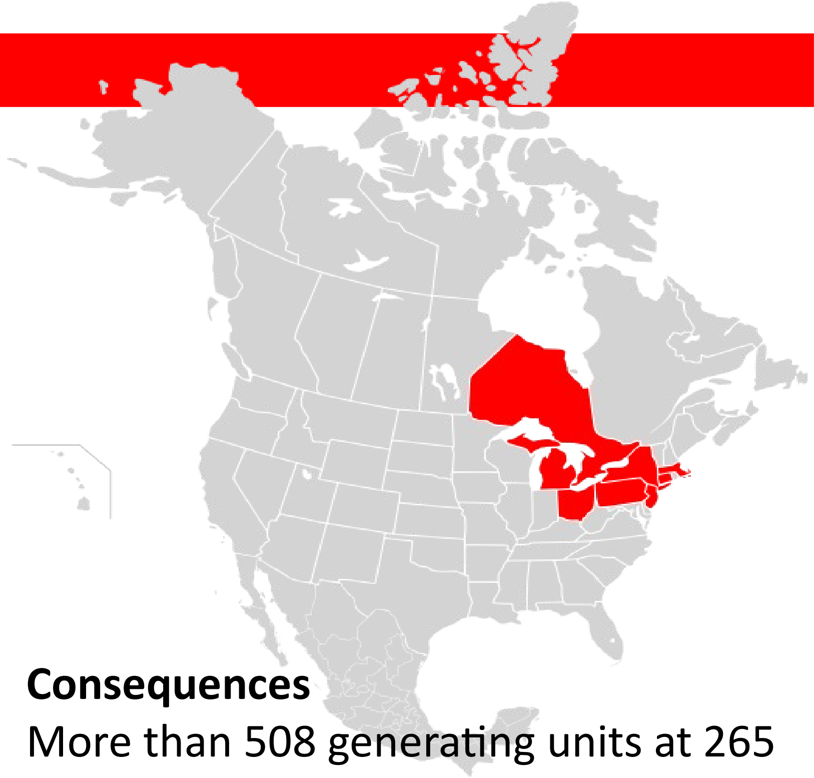
Cascading failures: Empirical Results

Cascades: The Domino Effect

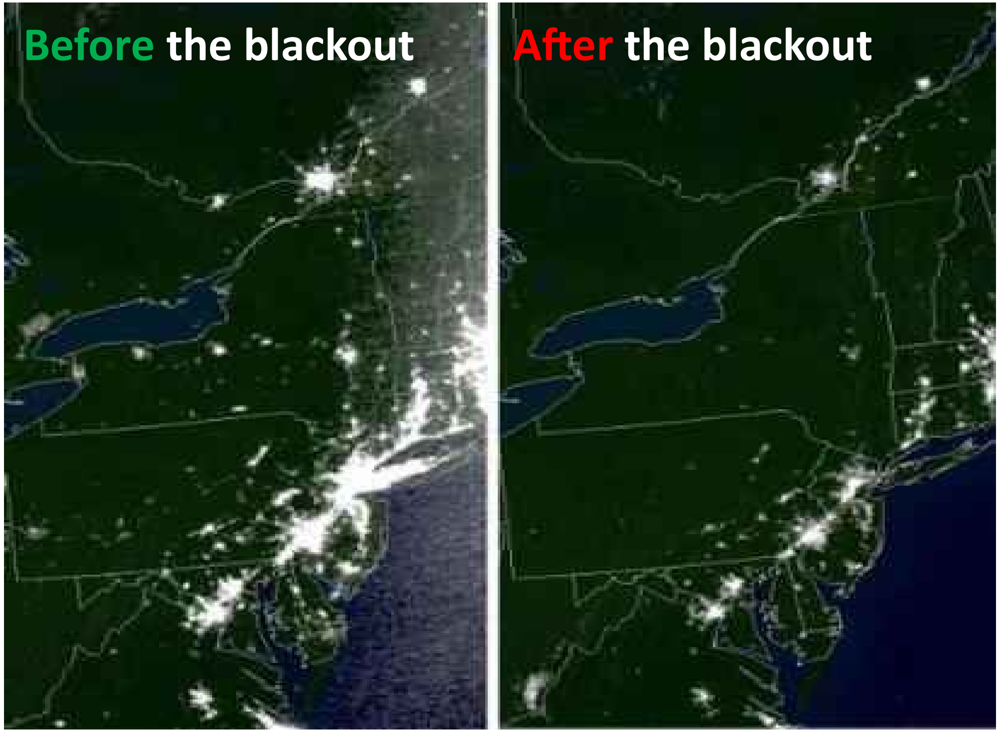
Large events triggered by small initial shocks

Northeast Blackout of 2003



Origin

A 3,500 MW power surge (towards Ontario) affected the transmission grid at 4:10:39 p.m. EDT. (Aug-14-2003)



Consequences

More than 508 generating units at 265 power plants shut down during the outage. In the minutes before the event, the NYISO-managed power system was carrying 28,700 MW of load. At the height of the outage, the load had dropped to 5,716 MW, a loss of 80%.

Section 8.5

- **Denial of Service Attacks (Internet)**

  If a router fails to transmit the packets received by it, the Internet protocols will alert the neighboring routers to avoid the troubled equipment by re-routing the packets using alternative routes. Consequently a failed router increases traffic on other routers, potentially inducing a series of denial of service attacks throughout the Internet [13].

- **Financial Crises**

  Cascading failures are common in economic systems. For example, the drop in the house prices in 2008 in the U.S. has spread along the links of the financial network, inducing a cascade of failed banks, companies and even nations [14, 15, 16]. It eventually caused the worst global financial meltdown since the 1930s Great Depression.

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Suggested retail price
$1.00
$1.50 outside of
Metro Boston
*

The Boston Globe

MONDAY, MARCH 14, 2011

A NEW WEAK

TODAY: *Partly sunny and colder. H*
37-42. Low 27-32.
TOMORROW: *Mostly sunny, milde*
High 42-47. Low 32-37.

HIGH TIDE: *6:42 a.m., 7:25 p.m.*
SUNRISE: *6:59* SUNSET: *6:49*

FULL REPORT: PAGE B13

Cascading disaster in Japan

Blast shakes a second reacto death toll soar

By Martin Fackler and Mark McDonald
NEW YORK TIMES

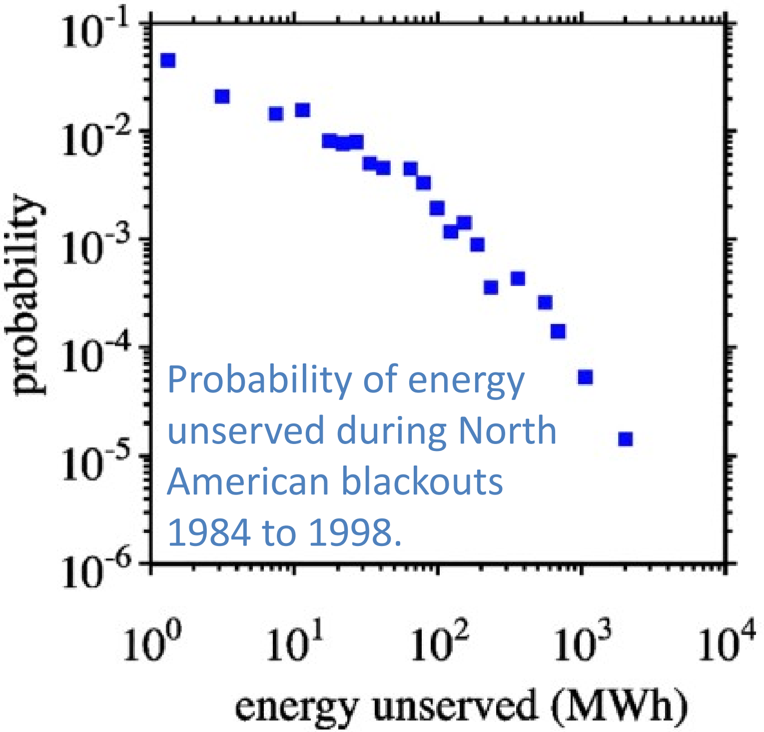
SENDAI, Japan — Japan reel from a rapidly unfolding disaster epic scale yesterday, pummeled by death toll, destruction, and homele ness caused by the earthquake a tsunami and new hazards from da aged nuclear reactors. The prime m ister called it Japan's worst crisis si World War II.

Japan's $5 trillion economy, t world's third largest, was threaten with severe disruptions and partial ralysis as many industries shut do temporarily. The armed forces and v unteers mobilized for the far more gent crisis of finding survivors, eva ating residents near the strick power plants and caring for the v tims of the record 8.9 magnitu quake that struck on Friday.

The disaster has left more th 10,000 dead, many thousands hom less, and millions without water, po er, heat, or transportation.

Cascades Size Distribution of Blackouts

Unserved energy/power magnitude (*S*) distribution



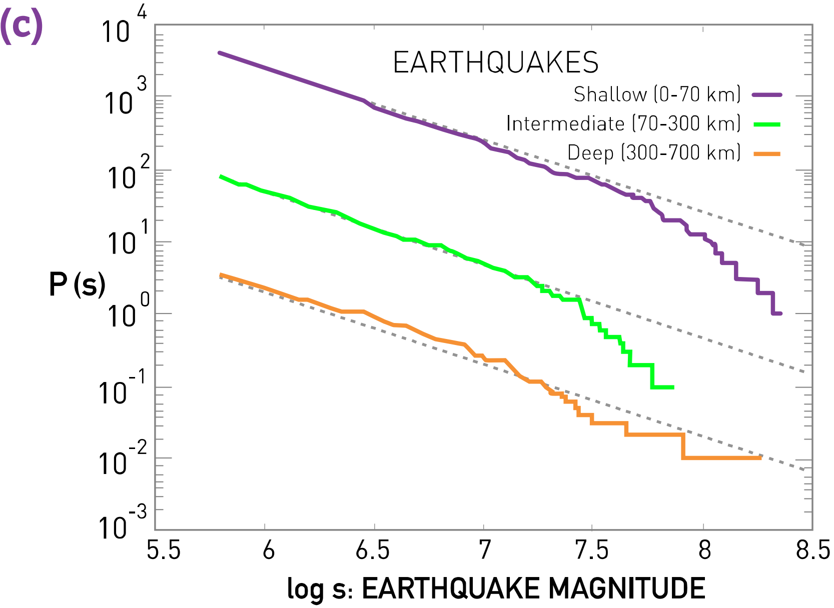
Probability of energy unserved during North American blackouts 1984 to 1998.

$$P(S) \sim S^{-\alpha},\ 1 < \alpha < 2$$

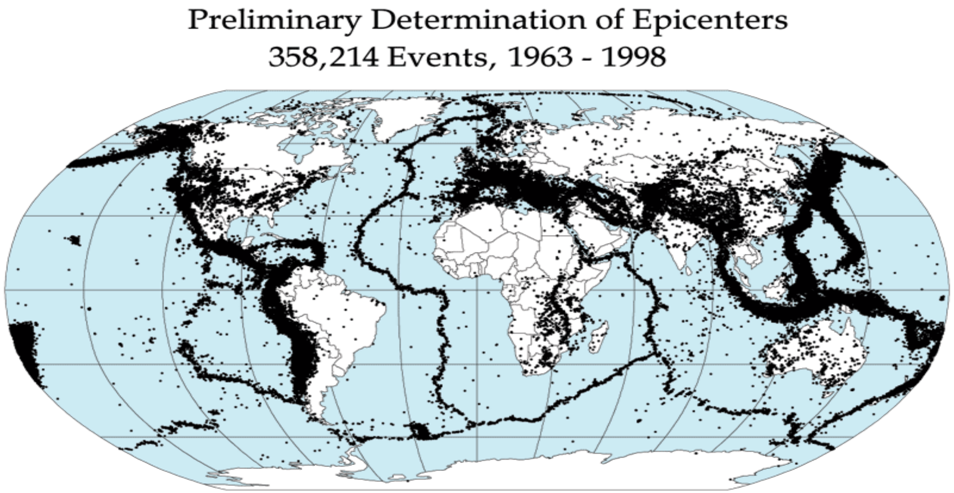
Source	Exponent	Quantity
North America	2.0	Power
Sweden	1.6	Energy
Norway	1.7	Power
New Zealand	1.6	Energy
China	1.8	Energy

I. Dobson, B. A. Carreras, V. E. Lynch, D. E. Newman, *CHAOS* 17, 026103 (2007)

Cascades Size Distribution of Earthquakes



Earthquakes during 1977–2000.

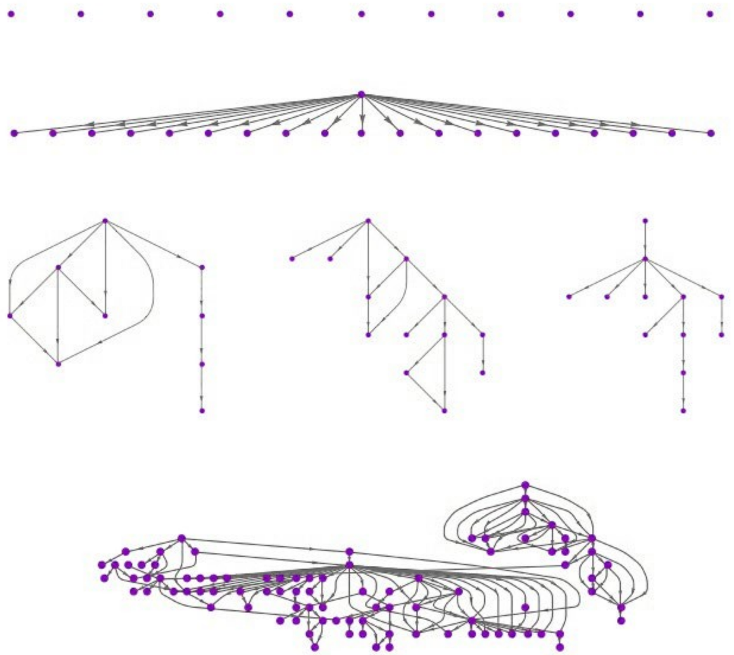


Earthquake size *S* distribution

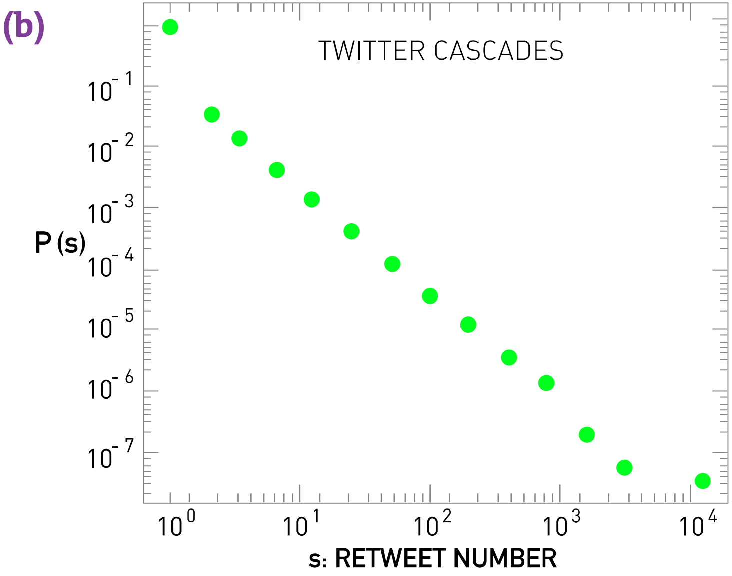
$$P(S) \sim S^{-\alpha}, \alpha \approx 1.67$$

Y. Y. Kagan, *Phys. Earth Planet. Inter.* 135 (2–3), 173–209 (2003)

Information Cascades



(b)



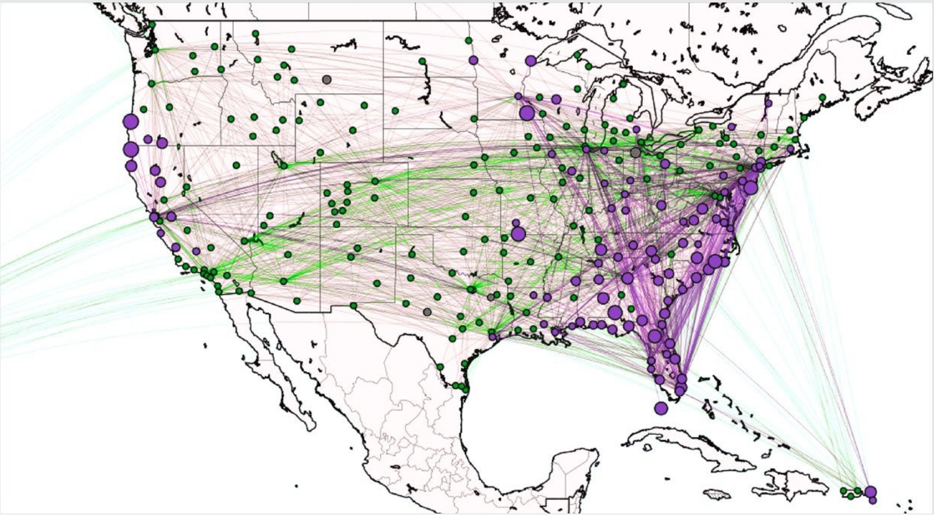
$$p(s) \sim s^{-\alpha}$$

$$\alpha \simeq 1.75$$

Empirical Results

Cascading failures are documented in many other environments:

- The consequences of bad weather or mechanical failures can cascade through airline schedules, delaying multiple flights and stranding thousands of passengers (BOX 8.3) [22].
- The disappearance of a species can cascade through the food web of an ecosystem, inducing the extinction of numerous species and altering the habitat of others [23, 24, 25, 26].
- The shortage of a particular component can cripple supply chains. For example, the 2011 floods in Thailand have resulted in a chronic shortage of car components that disrupted the production chain of more than 1,000 automotive factories worldwide. Therefore the damage was not limited to the flooded factories, but resulted in worldwide insurance claims reaching $20 billion [27].



U.S. aviation map showing congested airports as purple nodes, while those with normal traffic as green nodes. The lines correspond to the direct flights between them on March 12, 2010. The clustering of the congested airports indicate that the dealys are not independent of each other, but cascade through the airport network. After [22].

Empirical Results: Summary

SOURCE	EXPONENT	CASCADE
Power grid (North America)	2.0	Power
Power grid (Sweden)	1.6	Energy
Power grid (Norway)	1.7	Power
Power grid (New Zealand)	1.6	Energy
Power grid (China)	1.8	Energy
Twitter Cascades	1.75	Retweets
Earthquakes	1.67	Seismic Wave

Section 8.6

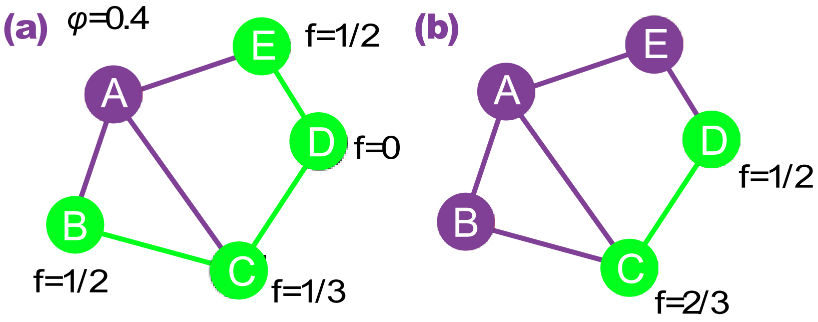
Modeling Cascading failures

(i) The system is characterized by some flow over a network, like the flow of electric current in the power grid or the flow of information in communication systems.

(ii) Each component has a local breakdown rule that determines when it contributes to a cascade, either by failing (power grid, earthquakes) or by choosing to pass on a piece of information (Twitter).

(iii) Each system has a mechanism to redistribute the traffic to other nodes upon the failure or the activation of a component.

Section 8.6 Failure Propagation Model



(a,b) The development of a cascade in a small network in which each node has the same breakdown threshold $\varphi = 0.4$. Initially all nodes are in state 0, shown as green circles. After node A changes its state to 1 (purple), its neighbors B and E will have a fraction $f = 1/2 > 0.4$ of their neighbors in state 1. Consequently they also fail, changing their state to 1, as shown in (b). In the next time step C and D will also fail, as both have $f > 0.4$. Consequently the cascade sweeps the whole network, reaching a size $s = 5$. One can check that if we initially flip node B, it will not induce an avalanche.

Initial Setup

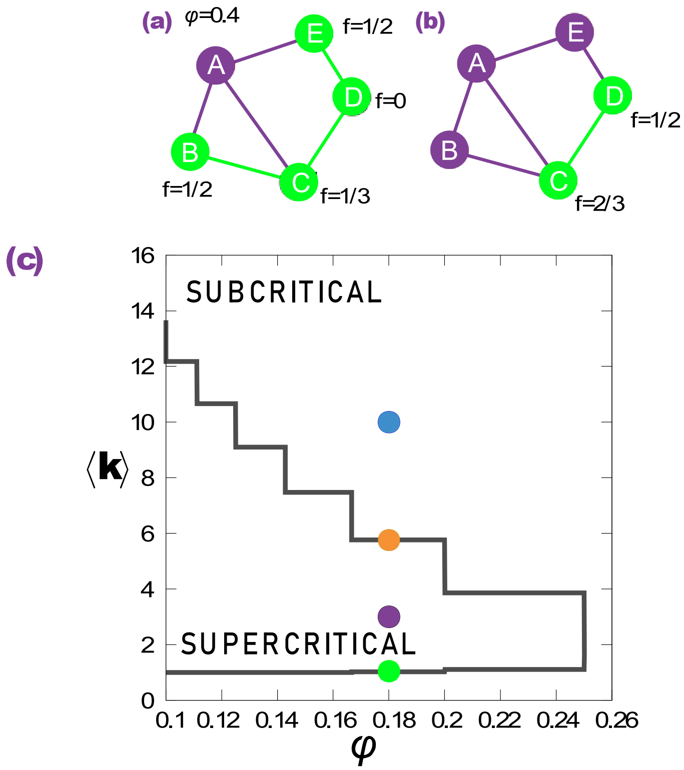
- Random graph with *N* nodes
- Initially each node is functional.

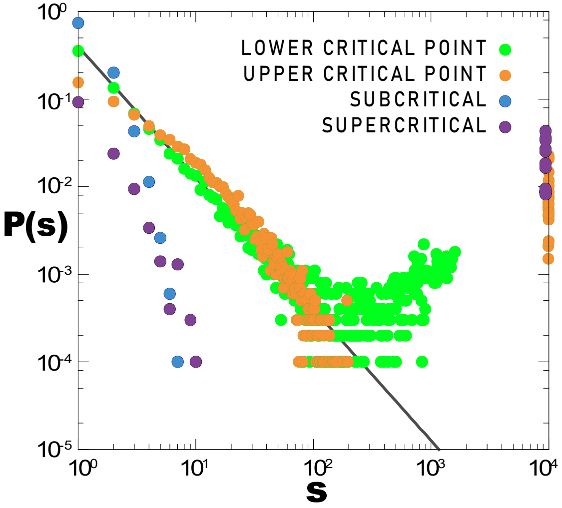
Cascade

- Initiated by the failure of one node.
- f_i : fraction of failed neighbors of node *i*. Node *i* fails if f_i is greater than a global threshold ϕ.

D. Watts, *PNAS* 99, 5766-5771 (2002)

Section 8.6 Failure Propagation Model





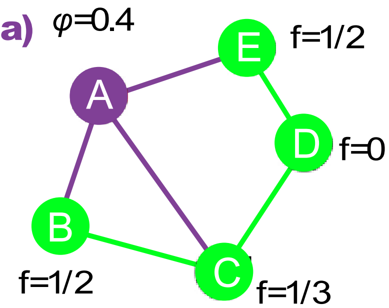
Erdos-Renyi network

$$P(S) \sim S^{-3/2}$$

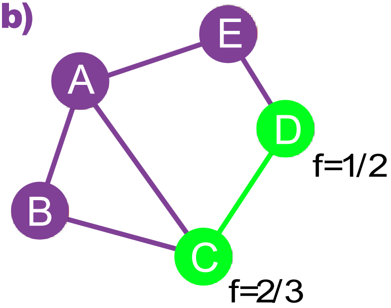
D. Watts, *PNAS* 99, 5766-5771 (2002)

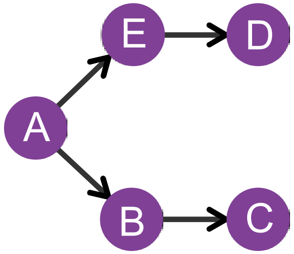
Section 8.6 Branching Model

(a)

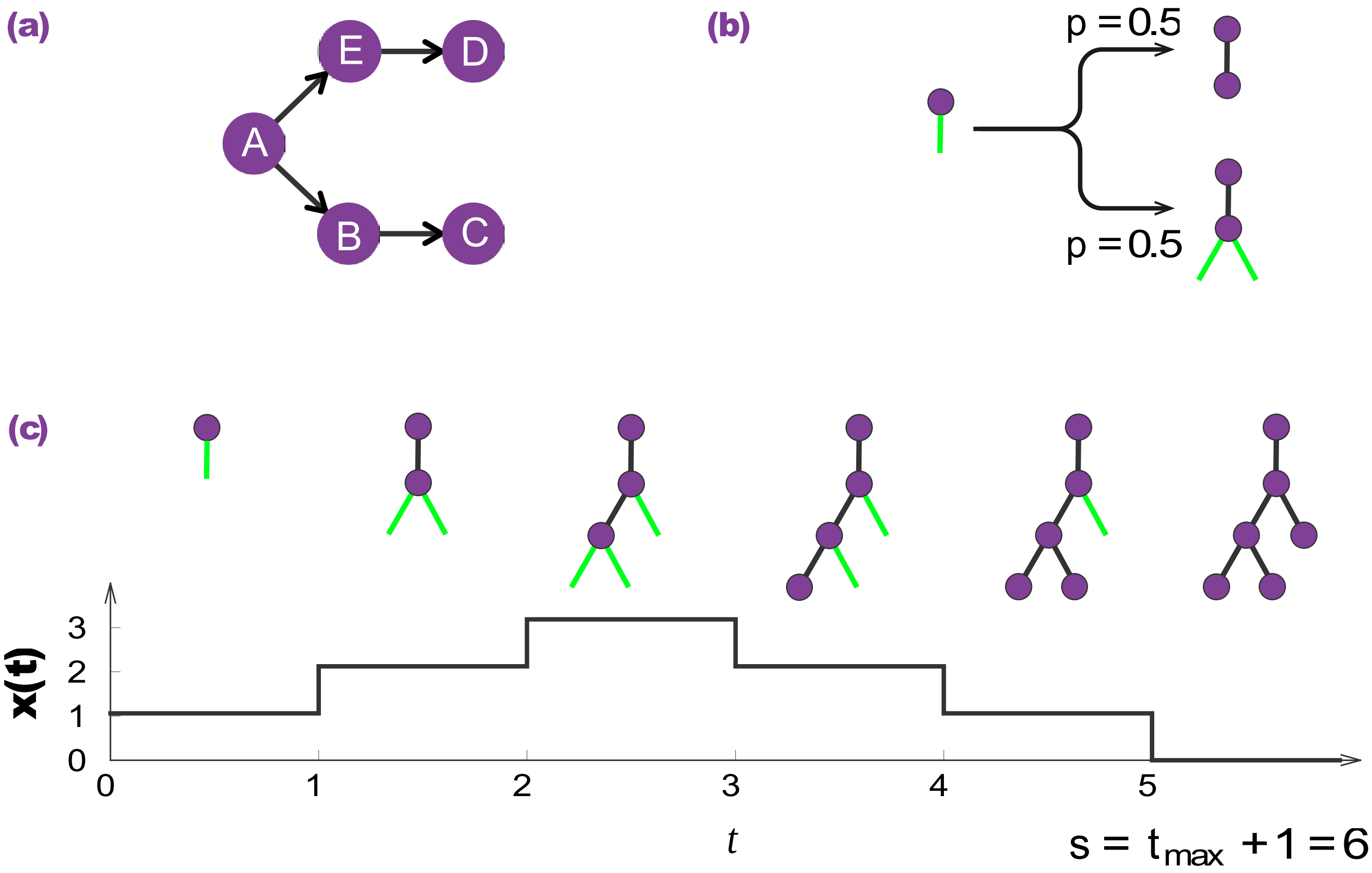


(b)





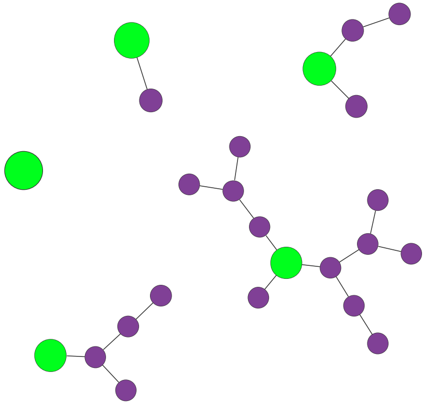
Section 8.6 Branching Model



Section 8.6 Branching Model

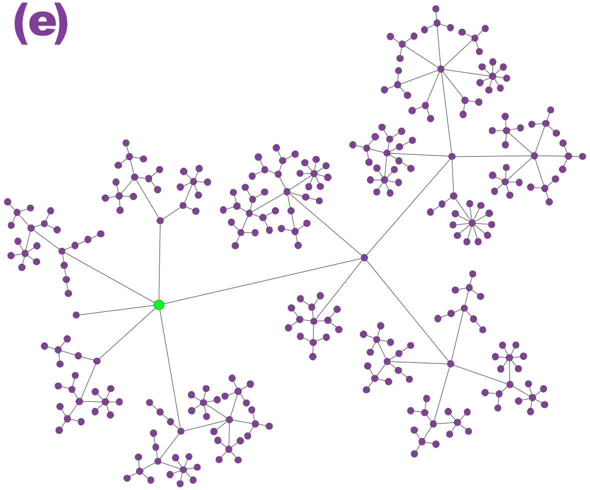
SUBCRITICAL

(d)



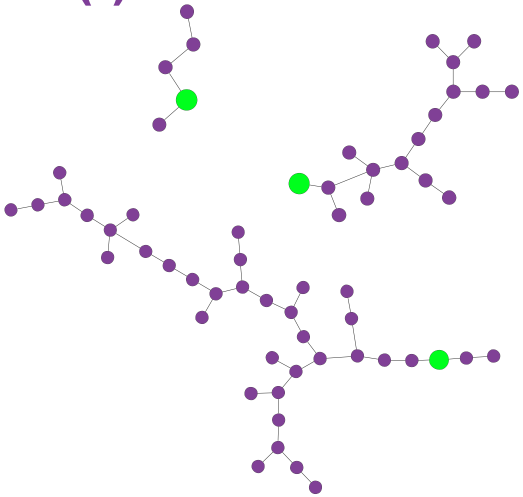
SUPERCRITICAL

(e)



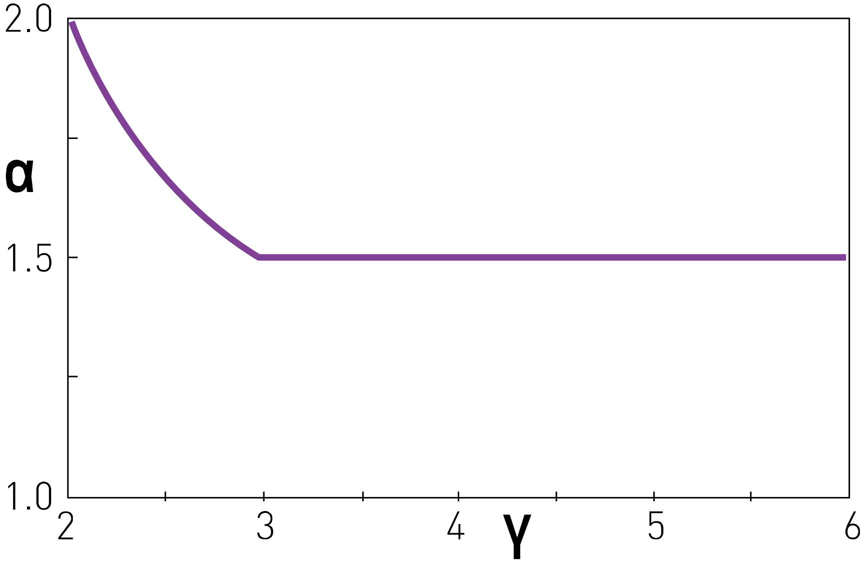
CRITICAL

(f)



Section 8.6 Branching Model

$$\alpha = \begin{cases} 3/2, & \gamma \geq 3 \\ \gamma/(\gamma-1), & 2 < \gamma < 3 \end{cases} .$$



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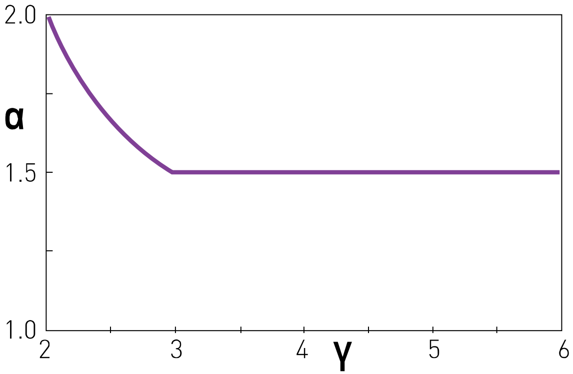

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Section 8.7

Building Robustness

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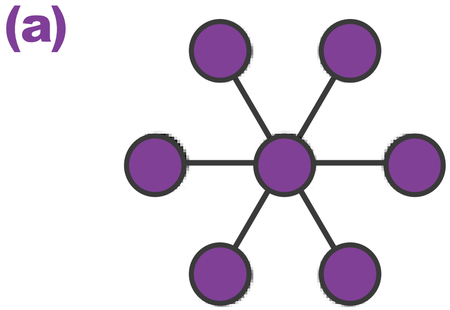
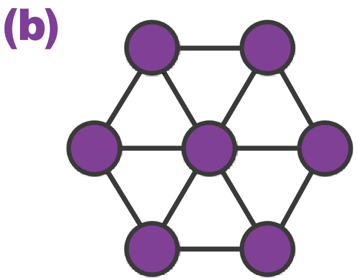


$\langle k \rangle = 12/7$



$\langle k \rangle = 24/7$

Can we maximize the robustness of a network to both random failures and targeted attacks without changing the cost?

Building Robustness

A network's robustness against random failures is captured by its percolation threshold f_c, which is the fraction of the nodes we must remove for the network to fall apart. To enhance a network's robustness we must increase f_c. According to (8.7) f_c depends only on $\langle k\rangle$ and $\langle k^2\rangle$. Consequently the degree distribution which maximizes f_c needs to maximize $\langle k^2\rangle$ if we wish to keep the cost $\langle k\rangle$ fixed. This is achieved by a bimodal distribution, corresponding to a network with only two kinds of nodes, with degrees k_{min} and k_{max} (Figure 8.23a,b).

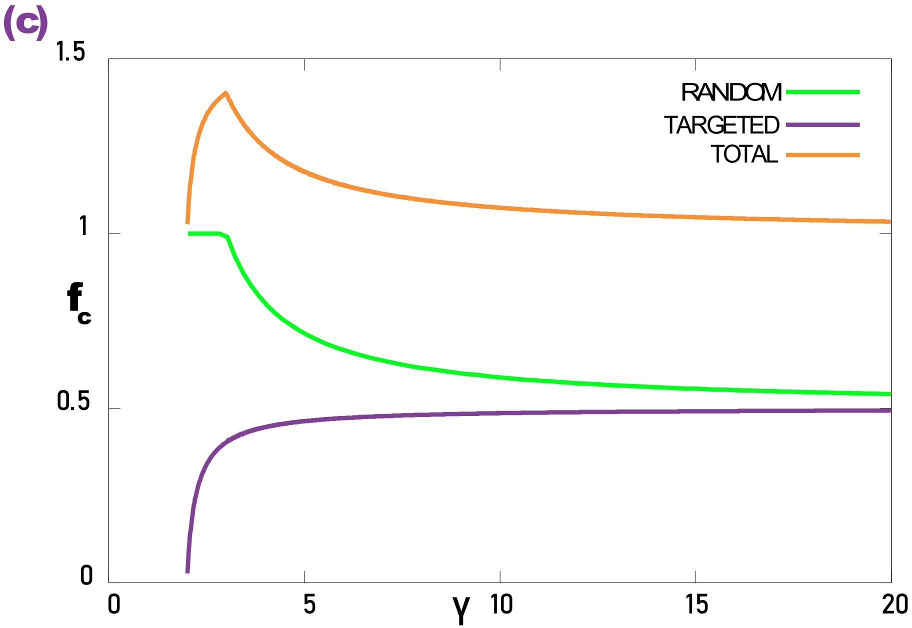
$$f_c^{tot} = f_c^{rand} + f_c^{targ}$$

Building Robustness

$$f_c^{\,tot} = f_c^{\,rand} + f_c^{\,targ}$$

$$p_k \equiv (1-r)\delta(k-k_{\min}) + r\delta(k-k_{\max}),$$

$$k_{\max} = AN^{2/3}.$$



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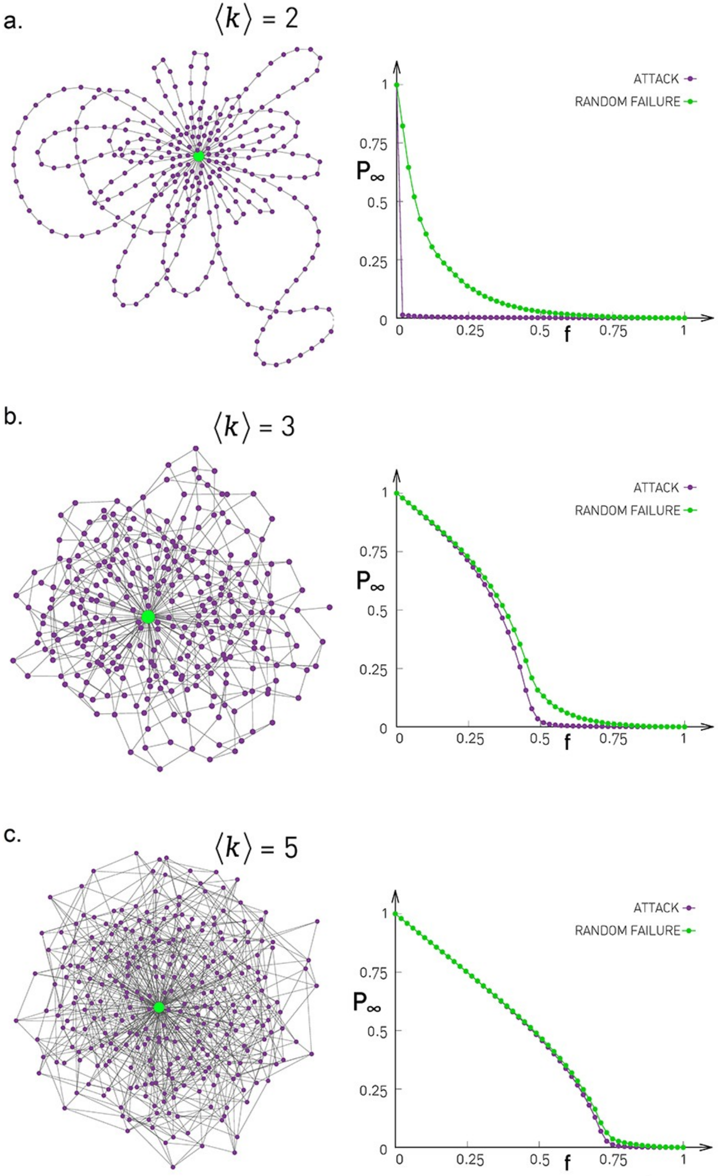
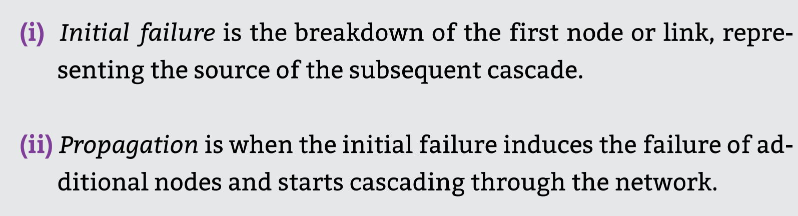


Section 8.7

Halting Cascading Failures

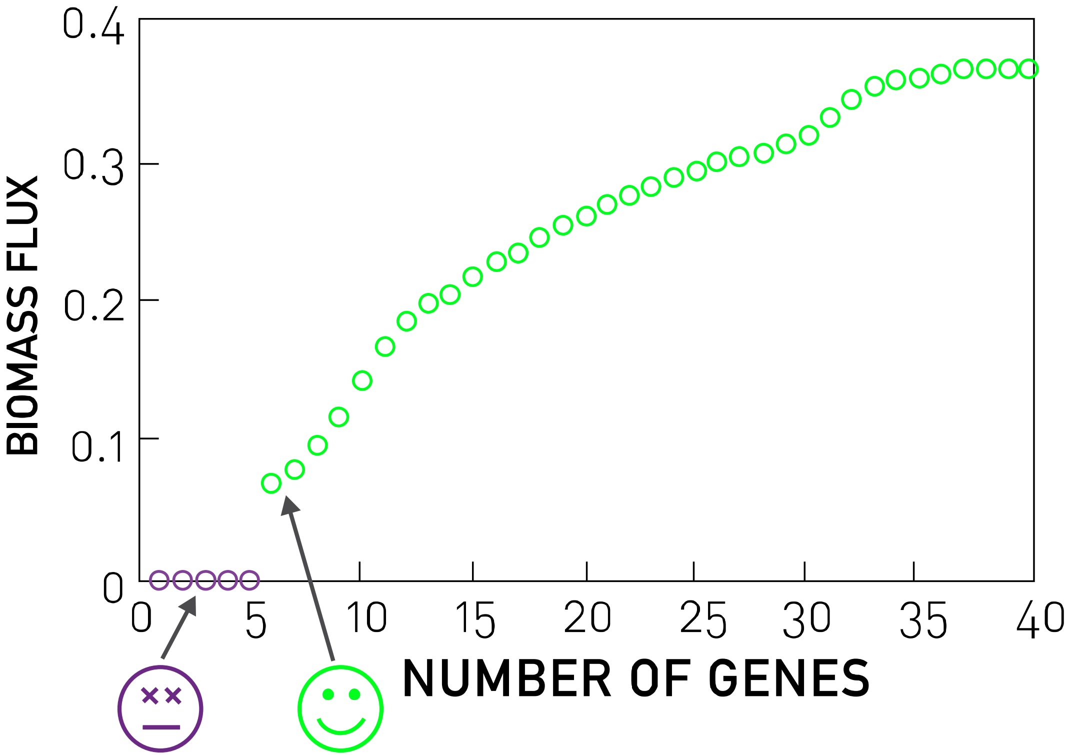


(i) *Initial failure* is the breakdown of the first node or link, representing the source of the subsequent cascade.

(ii) *Propagation* is when the initial failure induces the failure of additional nodes and starts cascading through the network.

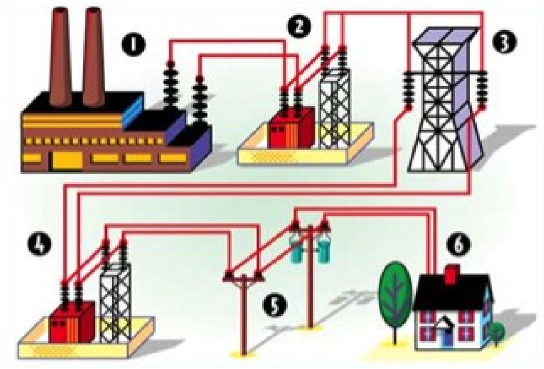
Simulations indicate that to limit the size of the cascades we must remove nodes with small loads and links with large excess load in the vicinity of the initial failure. The mechanism is similar to the method used by firefighters, who set a controlled fire in the fire- line to consume the fuel in the path of a wildfire.

Lazarus Effect



Section 8.7 Case Study: Power Grid

(a)



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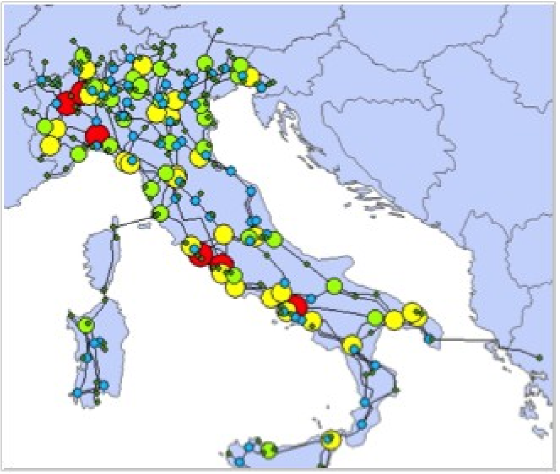
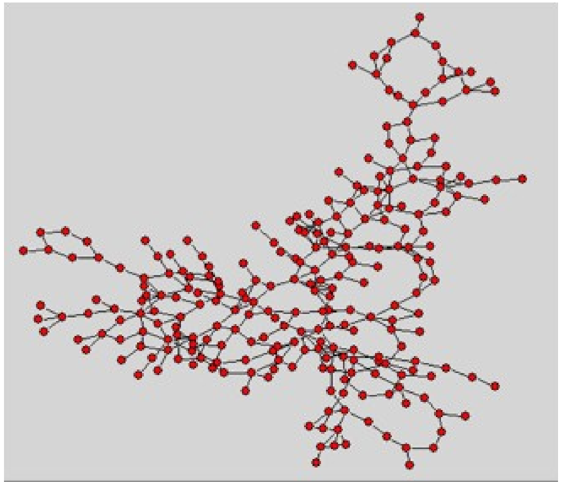
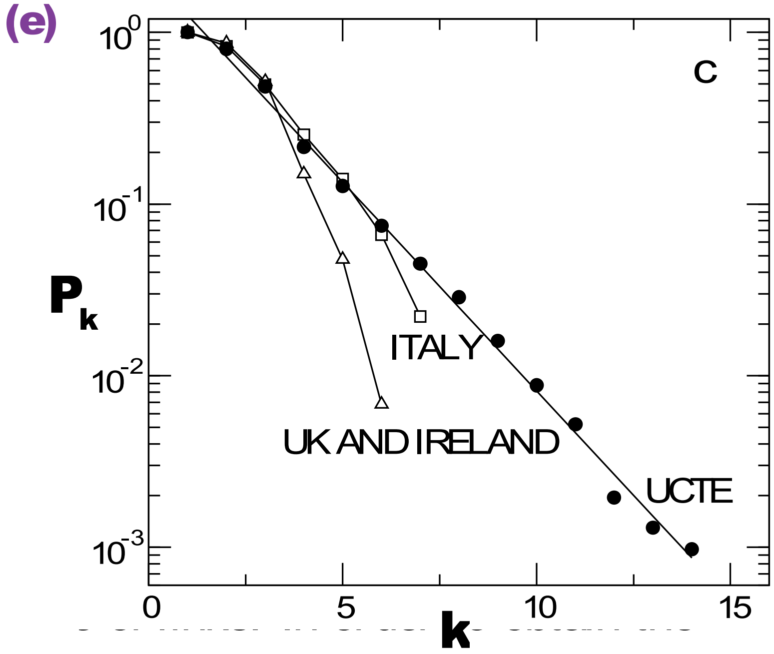


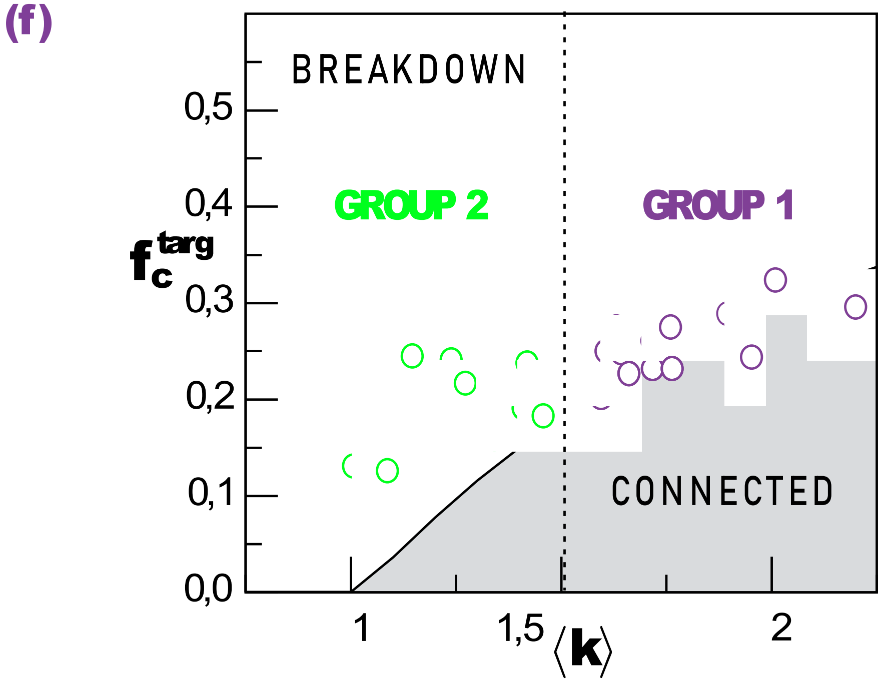
(c)

(d)



Section 8.7 Case Study: Power Grid

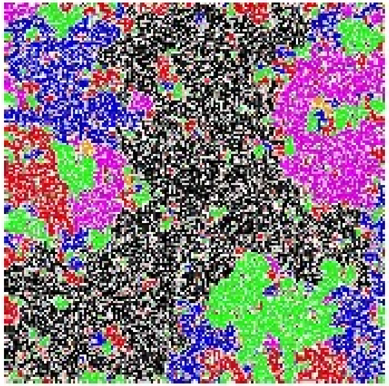




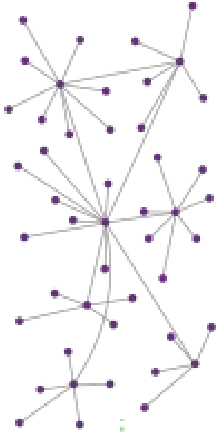
$$p_k = \frac{e^{-k/\langle k\rangle}}{\langle k\rangle}$$

Group 2: these networks are more robust to attacks than expected based on their degree distribution.

Section 8.8 Summary



PERCOLATION



ACHILLES' HEEL

SHLOMO HAVLIN

Mathematicians **Simon Broadbent** and **John Hammersey** introduce percolation and formalize many of its mathematical concepts [5]. The theory rose to prominence in the 1960s and 70s, finding applications from oil exploration to superconductivity.

Paul Baran explores the vulnerability of communication networks to Soviet nuclear attacks, concluding that they are too centralized to be viable under attack. Proposes instead a mesh-like network architecture [BOX 8.2].

Albert, Jeong and **Barabási** study the error and attack tolerance of complex networks, discovering their joint robustness to failures and fragility to attacks.

Shlomo Havlin and his collaborators establish a formal link between network robustness and percolation theory, showing that the percolation threshold of a scale-free network is determined by the first two moments of the degree distribution.

Section 8.8

Summary

AT A GLANCE: NETWORK ROBUSTNESS

Malloy-Reed criteria:

A giant component exists if

$$\frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

Random failures:

$$f_c = 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

Random Network: $f_c^{ER} = 1 - \frac{1}{\langle k \rangle}$

Enhanced robustness: $f_c > f_c^{ER}$

Attacks:

$$f_c^{\frac{2-\gamma}{1-\gamma}} = 2 + \frac{2-\gamma}{3-\gamma} k_{\min} \left(f_c^{\frac{3-\gamma}{1-\gamma}} - 1 \right)$$

Cascading failures:

$$p(s) \sim s^{-\alpha}$$

$$\alpha = \begin{cases} 3/2 & \gamma > 3 \\ \frac{\gamma}{\gamma - 1} & 2 < \gamma < 3 \end{cases}$$

Robustness

A system is robust if it can maintain its basic functions in the presence of internal and external errors. In a network context robustness refers to the system's ability to carry out its basic functions even when some of its nodes and links may be missing.

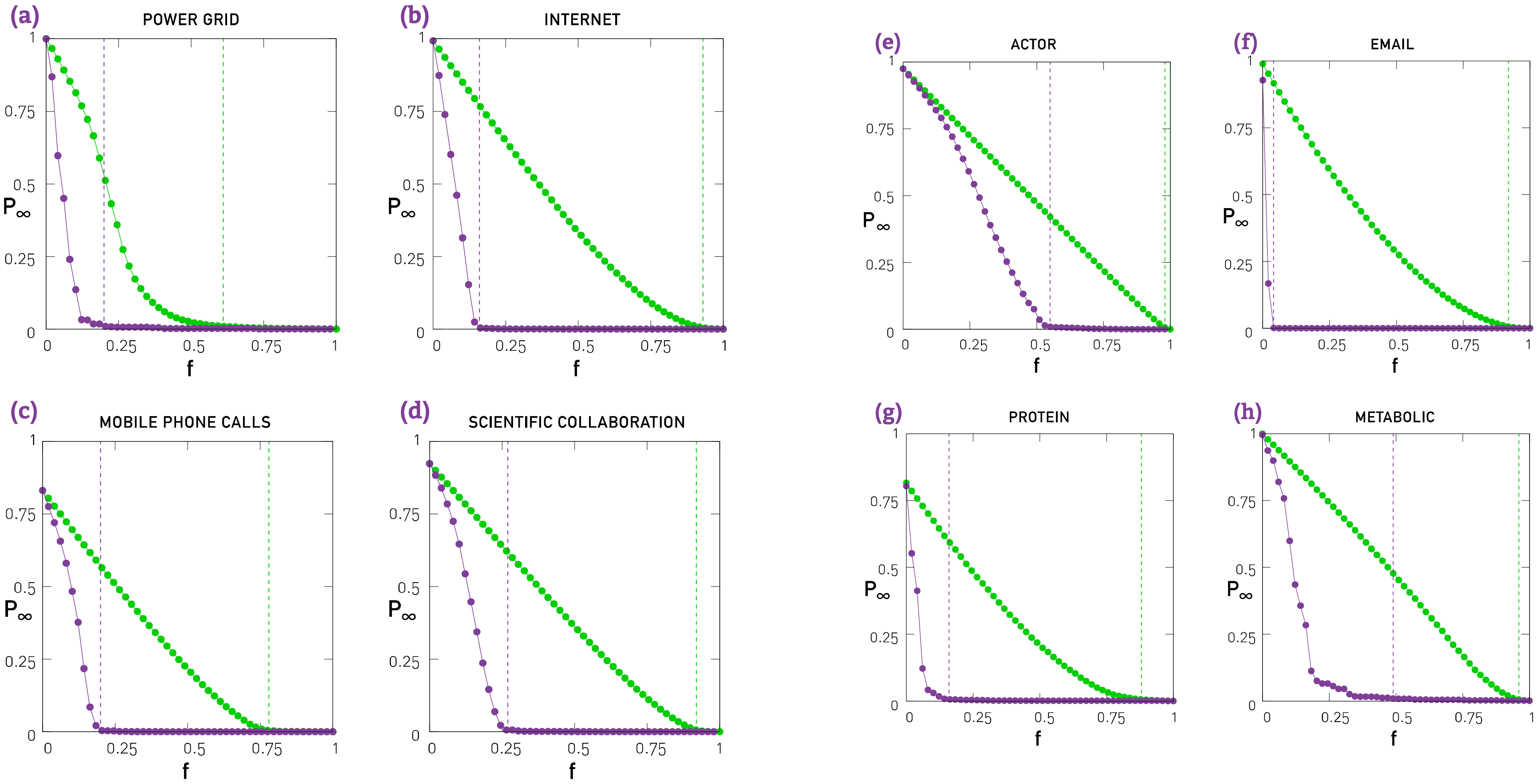
Resilience

A system is resilient if it can adapt to internal and external errors by changing its mode of operation, without losing its ability to function. Hence resilience is a dynamical property that requires a shift in the system's core activities.

Redundancy

Redundancy implies the presence of parallel components and functions that, if needed, can replace a missing component or funciton. Networks show considerable redundancy in their ability to navigate information between two nodes, thanks to the multiple independent paths between most node pairs.

Section 8.8 Achilles’ Heel



The end