Network Science

Differential Equations: Brief Review

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Summary

1. Basics
2. A function equal to its own derivative
3. Tricks and partial derivatives

Basics

Equations involving derivatives

$$\frac{d}{dx} f(x) = g(x)$$

$$\frac{d}{dx} f(x) = f(x)$$

$$f(x) + x \frac{d}{dx} f(x) = h(x)$$

Need to solve for $f(x)$

Alternative notation

$$f'(x) = \frac{d}{dx} f(x)$$

The equations become

$$f' = g$$

$$f' = f$$

$$f + xf' = h$$

The easiest case

$$\frac{d}{dx} f(x) = g(x)$$

or

$$f' = g$$

This is easy. Just integrate:

$$f = \int g$$

A function equal to its own derivative

Example of differential equation

Do you know of any function that is its own derivative?

$$\frac{d}{dx} f(x) = f(x)$$

The equation becomes

$$f'(x) = f(x)$$

or

$$\frac{f'}{f} = 1$$

Chain rule

$$\frac{f'}{f} = 1$$

If one remembers the chain rule, this becomes:

$$(\ln f)' = 1$$

Integrating,

$$\ln f = x + C$$

or

$$f(x) = e^{x+C} = e^C e^x = ce^x$$

Tricks and partial derivatives

Last example

$$f + xf' = h$$

Product rule:

$$\begin{aligned} (xf)' &= h \\ xf &= \int h \\ f(x) &= \frac{\int h}{x} \end{aligned}$$

Partial derivatives

$$\frac{\partial}{\partial x} f(x, y) \Rightarrow \text{treat } y \text{ as a constant}$$

Example:

$$\begin{aligned} 2p(k, m) &= -p(k, m) - k\frac{\partial p(k, m)}{\partial k} \\ 2p(k, m) &= -\frac{\partial}{\partial k}\left[kp(k, m)\right] \end{aligned}$$

Guess what?

$$2p(k,m) = -\frac{\partial}{\partial k}\left[kp(k,m)\right]$$

Is there a solution of the form:

$$p(k,m) = k^{\alpha} f(m)?$$

Substitute:

$$\begin{aligned} 2k^{\alpha} f(m) &= -\frac{\partial}{\partial k}\left[k^{\alpha+1} f(m)\right] \\ 2k^{\alpha} f(m) &= -(\alpha+1)k^{\alpha} f(m) \\ 2 &= -(\alpha+1) \\ -3 &= \alpha \end{aligned}$$