Network Science

Calculus: Brief Review

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Summary

Definitions

Definition of derivative

$$\left(\frac{d}{dx}f(x)\right)(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Example:

$$\begin{aligned}\left(\frac{d}{dx}x\right)(x_0) &= \lim_{\Delta x \to 0} \frac{x_0 + \Delta x - x_0}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} 1 \\ &= 1.\end{aligned}$$

Approximation

Definition of derivative:

$$\left(\frac{d}{dx}f(x)\right)(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

We can approximate:

$$\left(\frac{d}{dx}f(x)\right)(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

if Δx is very small

Examples

Powers of x

$$
\begin{aligned}
\frac{d}{dx}x &= 1 \\
\frac{d}{dx}x^2 &= 2x \\
\frac{d}{dx}x^3 &= 3x^2 \\
\frac{d}{dx}x^4 &= 4x^3 \\
&\vdots \\
\frac{d}{dx}x^n &= nx^{n-1}
\end{aligned}
$$

Fractional, negative powers

$$\frac{d}{dx}x^n \quad = \quad nx^{n-1}$$

For which values of n?

$n = 1, 2, 3, 4, \ldots$? ok

Perhaps $n = 0.5$? YES!

Perhaps $n = -1$? YES!!

Perhaps $n = -5.6$? YES!!

Perhaps $n = 0$? YES!!

Log

$$\frac{d}{dx}x^{-3} = -3x^{-4} \qquad \frac{d}{dx}x^{1} = 1$$

$$\frac{d}{dx}x^{-2} = -2x^{-3} \qquad \frac{d}{dx}x^{2} = 2x$$

$$\frac{d}{dx}x^{-1} = -1x^{-2} \qquad \frac{d}{dx}x^{3} = 3x^{2}$$

$$\frac{d}{dx}x^{0} = 0 \qquad \frac{d}{dx}x^{4} = 4x^{3}$$

We do not get x^{-1} as a derivative in this table

$$\frac{d}{dx}\ln x = x^{-1}$$

Here is the missing link!

Integrals

$$
\begin{aligned}
\int x &= \frac{1}{2}x^2 + C \\
\int x^2 &= \frac{1}{3}x^3 + C \\
\int x^3 &= \frac{1}{4}x^4 + C \\
&\vdots \\
\int x^n &= \frac{1}{n+1}x^{n+1} + C
\end{aligned}
$$

Works for fractionary, negative n; except for $n = -1$

$$\int x^{-1} = \ln x + C$$

Properties

Product by constant

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$$

Example:

$$\frac{d}{dx}\left(3x^2\right) = 3\frac{d}{dx}x^2 = 3 \cdot 2x = 6x$$

Product of two functions

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right]$$

Example:

$$\begin{aligned}\frac{d}{dx}(x \cdot x) &= \left[\frac{d}{dx}x\right]x + x\left[\frac{d}{dx}x\right] \\ &= 1 \cdot x + x \cdot 1 \\ &= 2x\end{aligned}$$