Network Science

Class 3: Random Networks
(Chapter 3 in textbook)

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with
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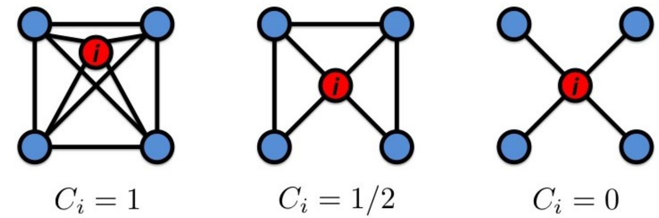
Clustering coefficient

CLUSTERING COEFFICIENT

$$C_i \equiv \frac{2 < L_i >}{k_i(k_i - 1)}$$

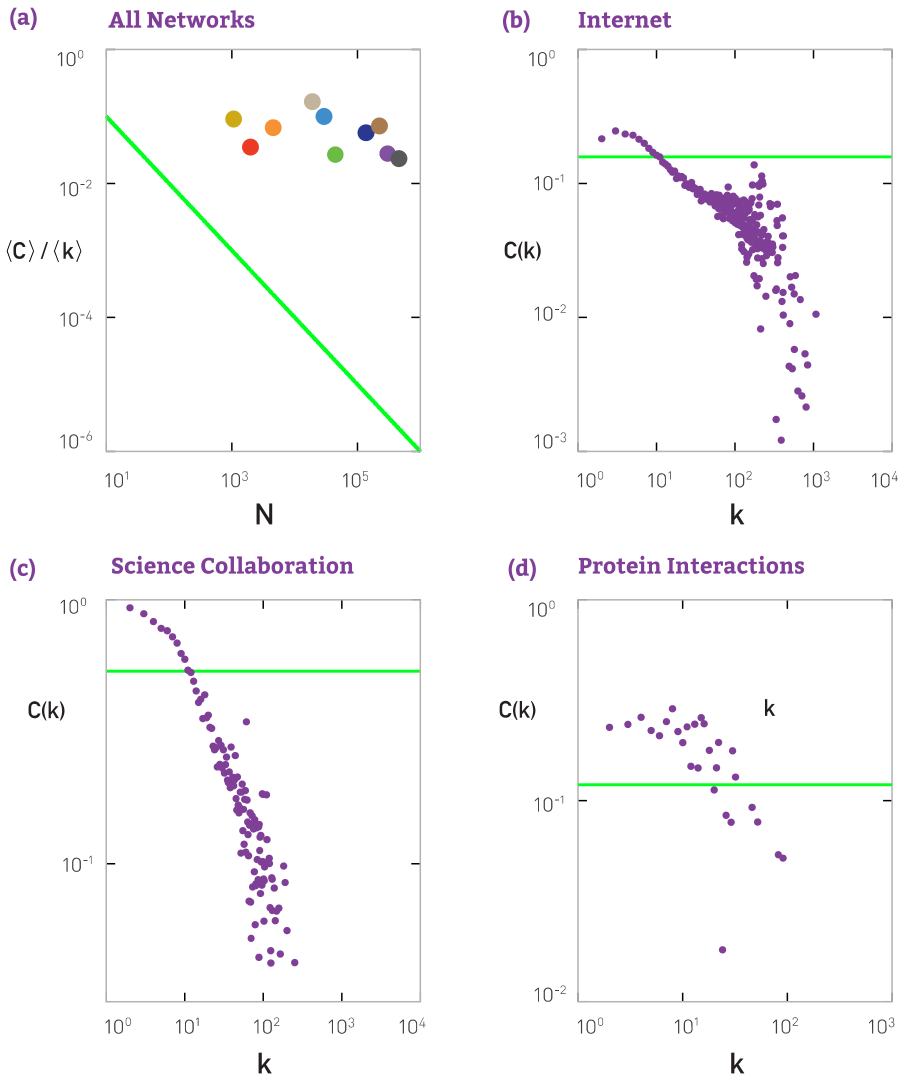


Since edges are independent and have the same probability *p*,

$$< L_i > \cong p \frac{k_i(k_i - 1)}{2} \quad \Rightarrow \quad C_i = \frac{2\langle L_i \rangle}{k_i(k_i - 1)} = p = \frac{\langle k \rangle}{N}.$$

- The clustering coefficient of random graphs is small.
- For fixed degree C decreases with the system size N.
- C is independent of a node's degree k.

CLUSTERING COEFFICIENT



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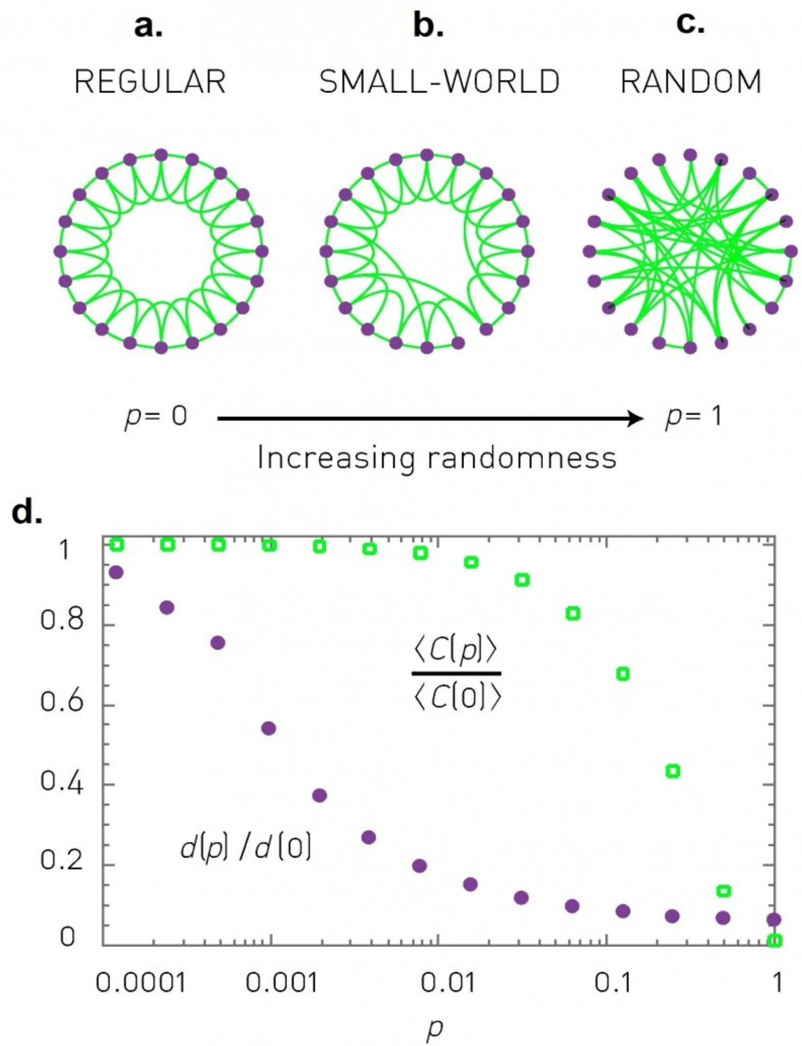
Watts-Strogatz Model



Real networks are not random

ARE REAL NETWORKS LIKE RANDOM GRAPHS?

As quantitative data about real networks became available, we can compare their topology with the predictions of random graph theory.

Note that once we have N and <k> for a random network, from it we can derive every measurable property. Indeed, we have:

Average path length:

$$< d > \sim \frac{\log N}{\log \langle k \rangle}$$

Clustering Coefficient:

$$C_i = \frac{2\langle L_i \rangle}{k_i(k_i - 1)} = p = \frac{\langle k \rangle}{N}.$$

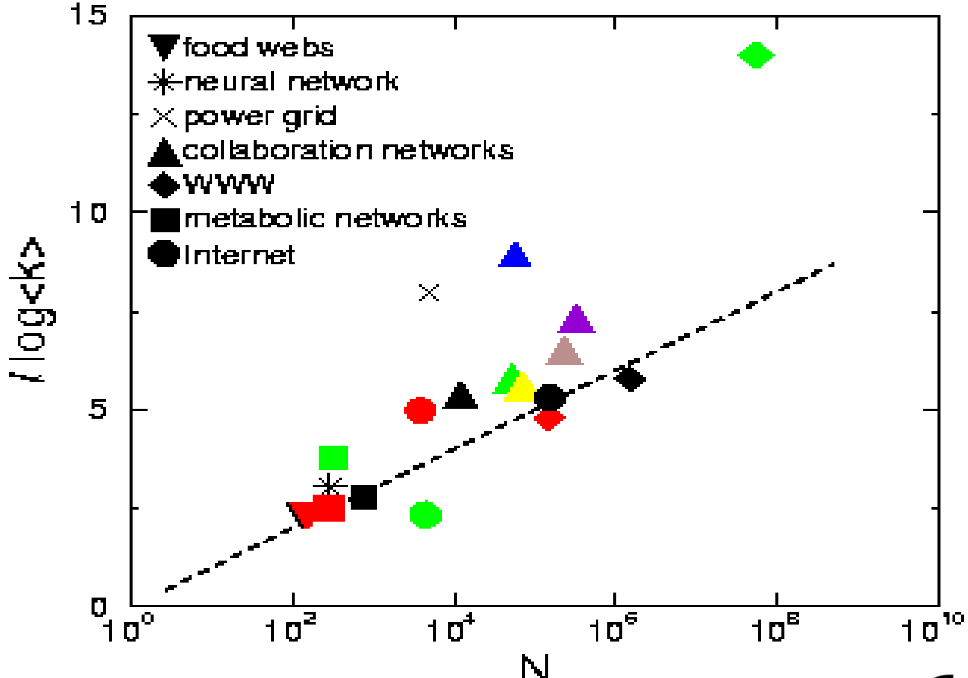
Degree Distribution:

$$P(k) = e^{-<k>} \frac{<k>^k}{k!}$$

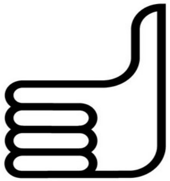
PATH LENGTHS IN REAL NETWORKS

Prediction:

$$<d>=\frac{\log N}{\log\langle k\rangle}$$



Real networks have short distances like random graphs.



CLUSTERING COEFFICIENT

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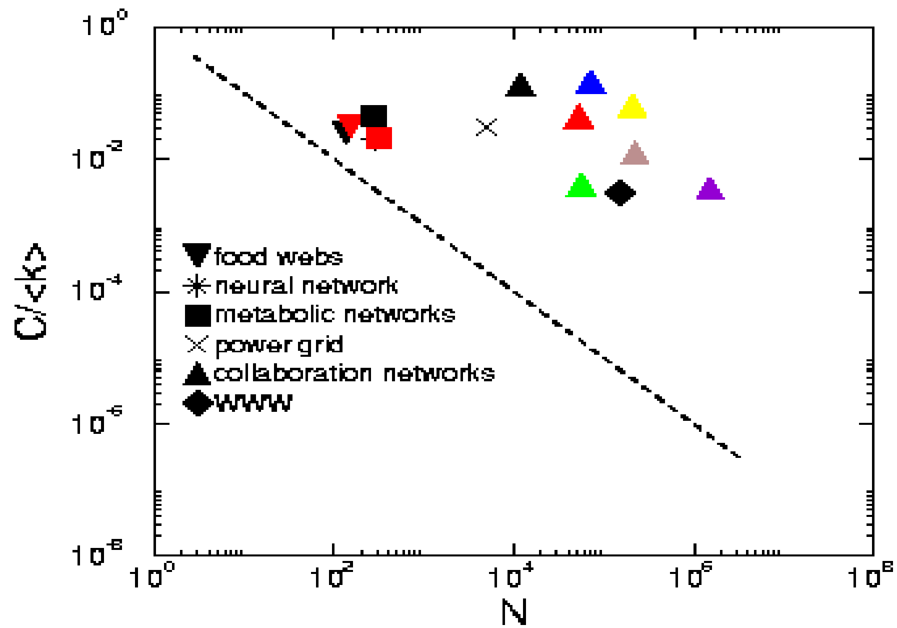


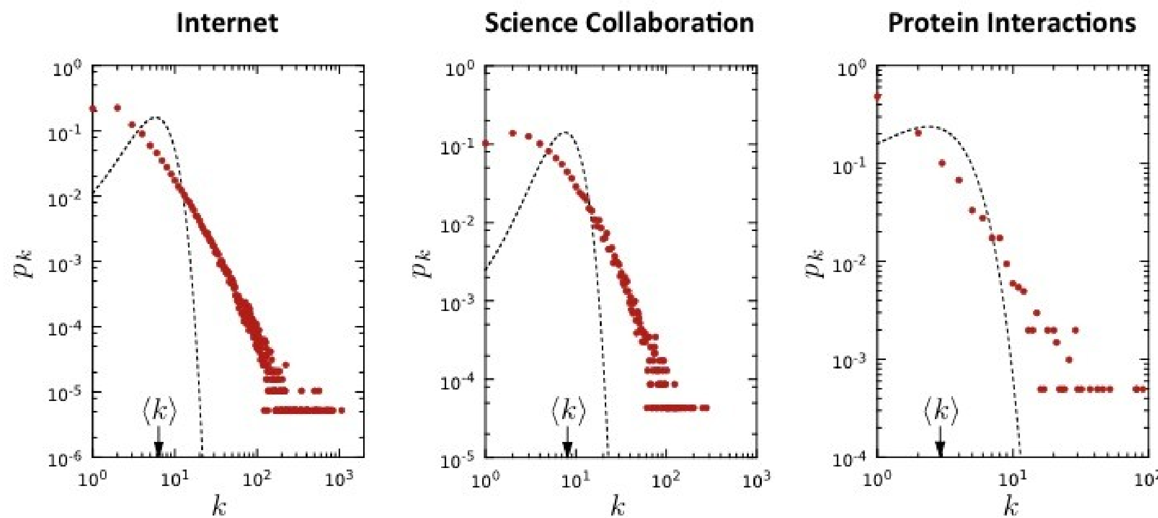
C_{rand} underestimates with orders of magnitudes the clustering coefficient of real networks.

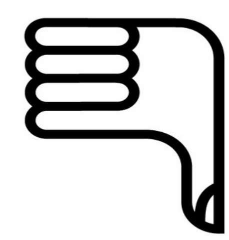
Prediction:

$$P(k) = e^{-<k>} \frac{<k>^k}{k!}$$

Data:

$P(k)$ » k^{-g}





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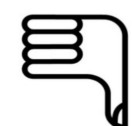


IS THE RANDOM GRAPH MODEL RELEVANT TO REAL SYSTEMS?

(B) Most important: we need to ask ourselves, are real networks random?

The answer is simply: NO

There is no network in nature that we know of that would be described by the random network model.

IF IT IS WRONG AND IRRELEVANT, WHY DID WE DEVOT TO IT A FULL CLASS?

It is the reference model for the rest of the class.

It will help us calculate many quantities, that can then be compared to the real data, understanding to what degree is a particular property the result of some random process.

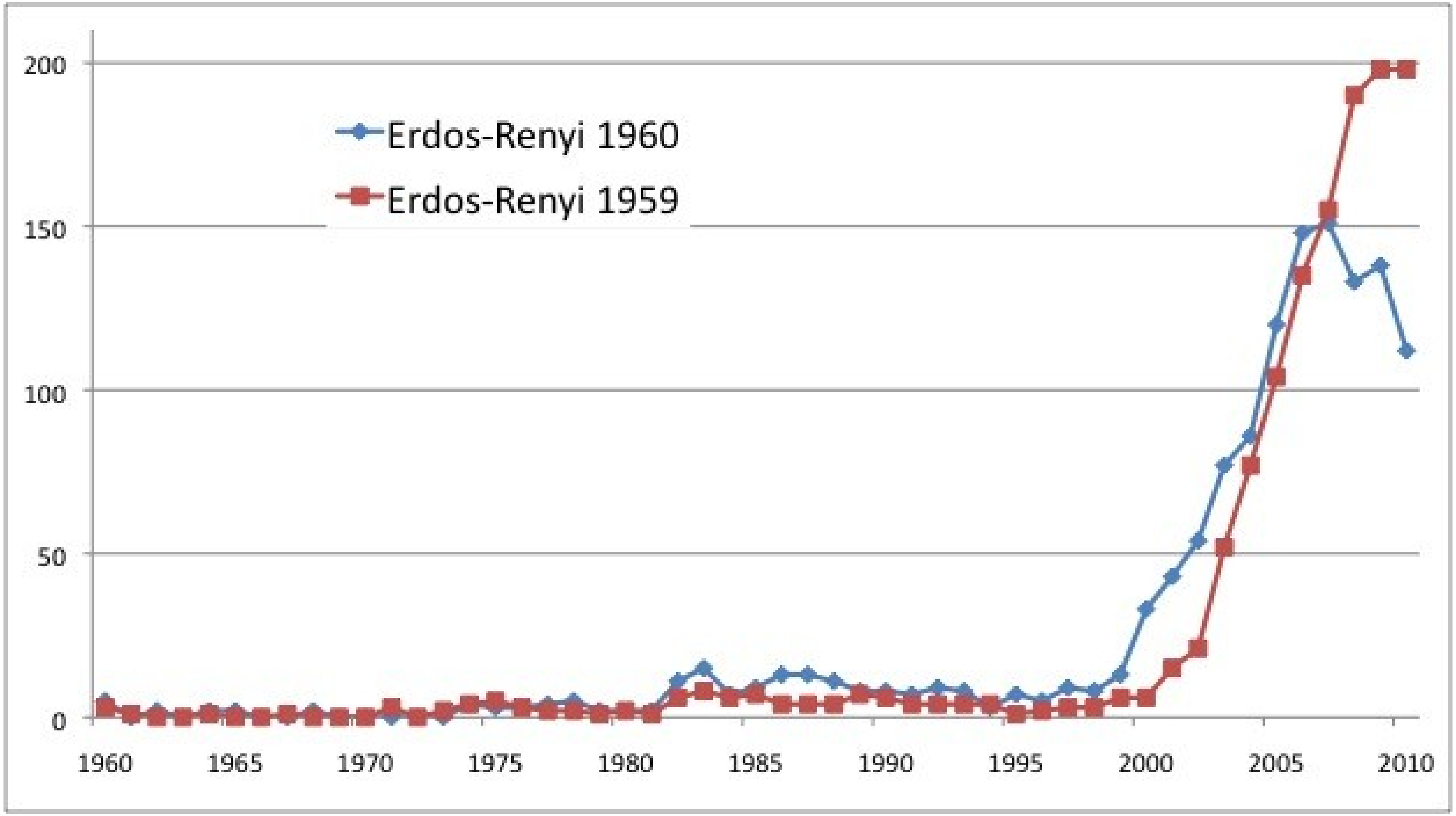
Patterns in real networks that are shared by a large number of real networks, yet which deviate from the predictions of the random network model.

In order to identify these, we need to understand how would a particular property look like if it is driven entirely by random processes.

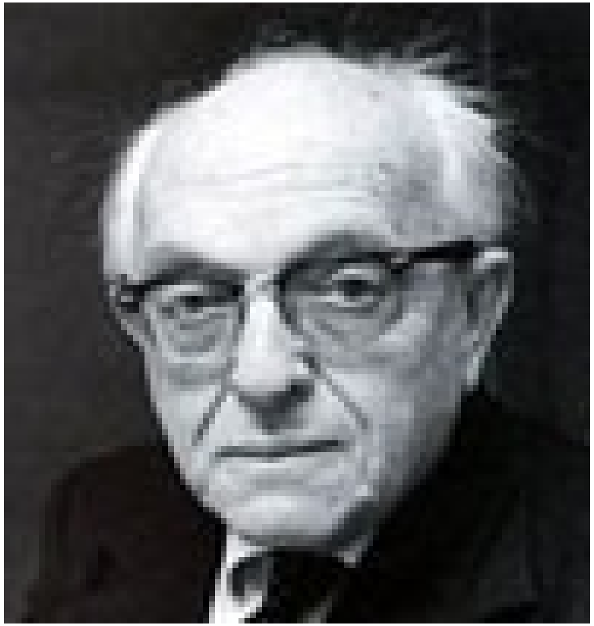
While WRONG and IRRELEVANT, it will turn out to be extremly USEFUL!

Summary

Erdös-Rényi MODEL (1960)



HISTORICAL NOTE



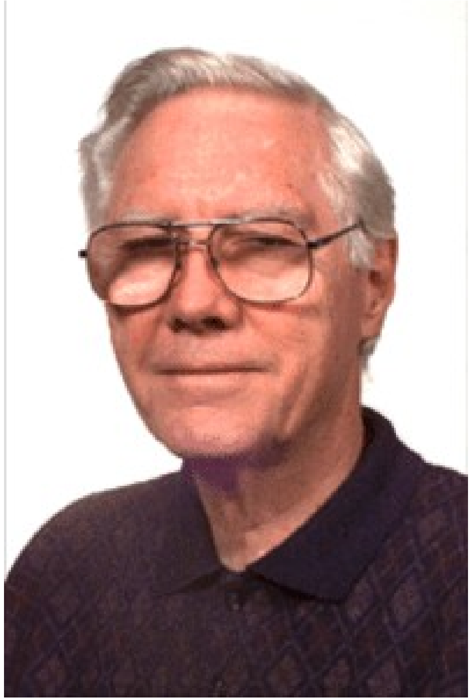
Anatol Rapoport
1911- 2007

1951, Rapoport and Solomonoff:

→ first systematic study of a random graph.
→demonstrates the phase transition.

→natural systems: neural networks; the social networks of physical contacts (epidemics); genetics.

1959: *G(N,p)*

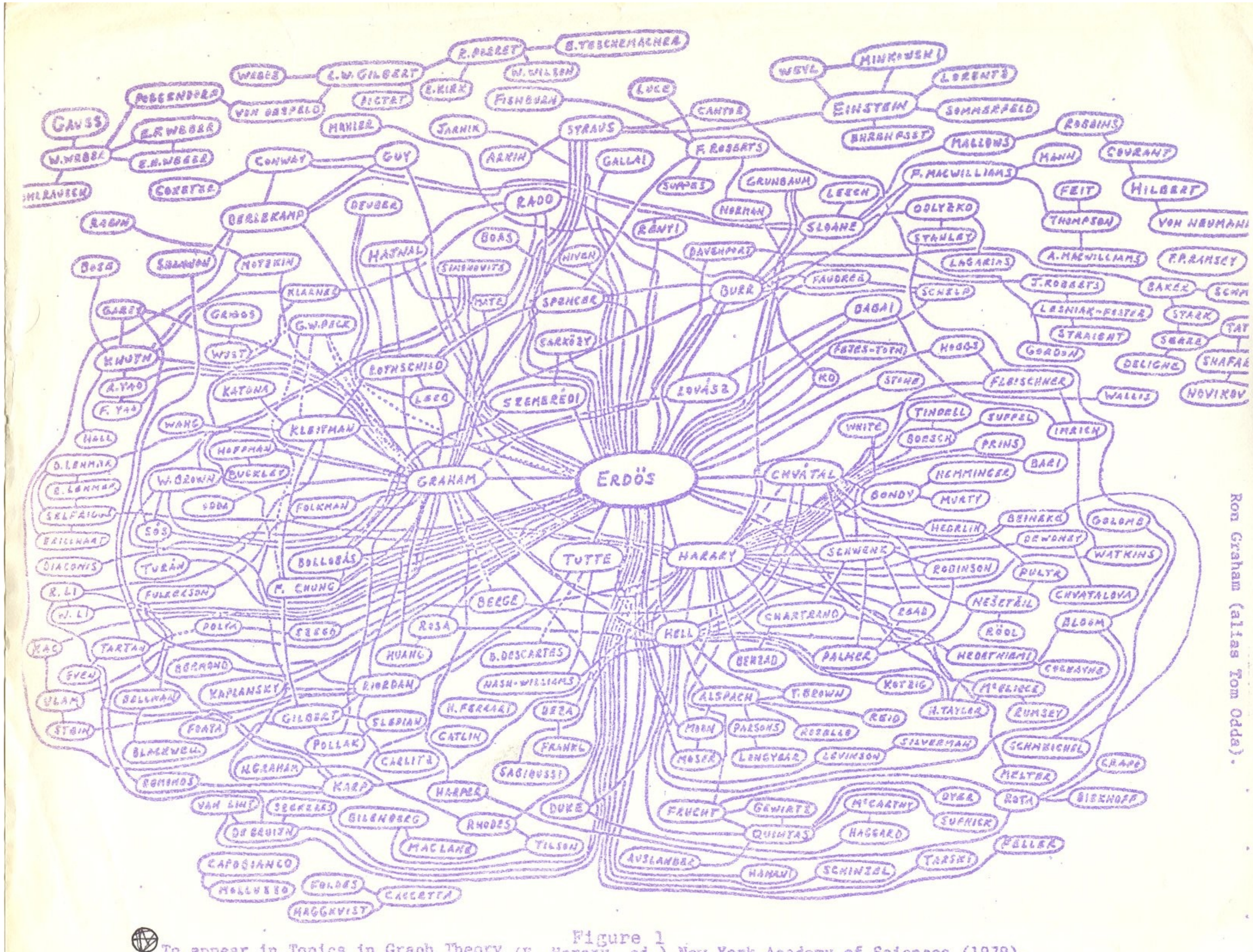


Edgar N. Gilbert
(**b**.1923)

Why do we call it the Erdos-Renyi random model?

HISTORICAL NOTE

NETWORK DATA: SCIENCE COLLABORATION NETWORKS



Erdos:
1,400 papers
507 coauthors

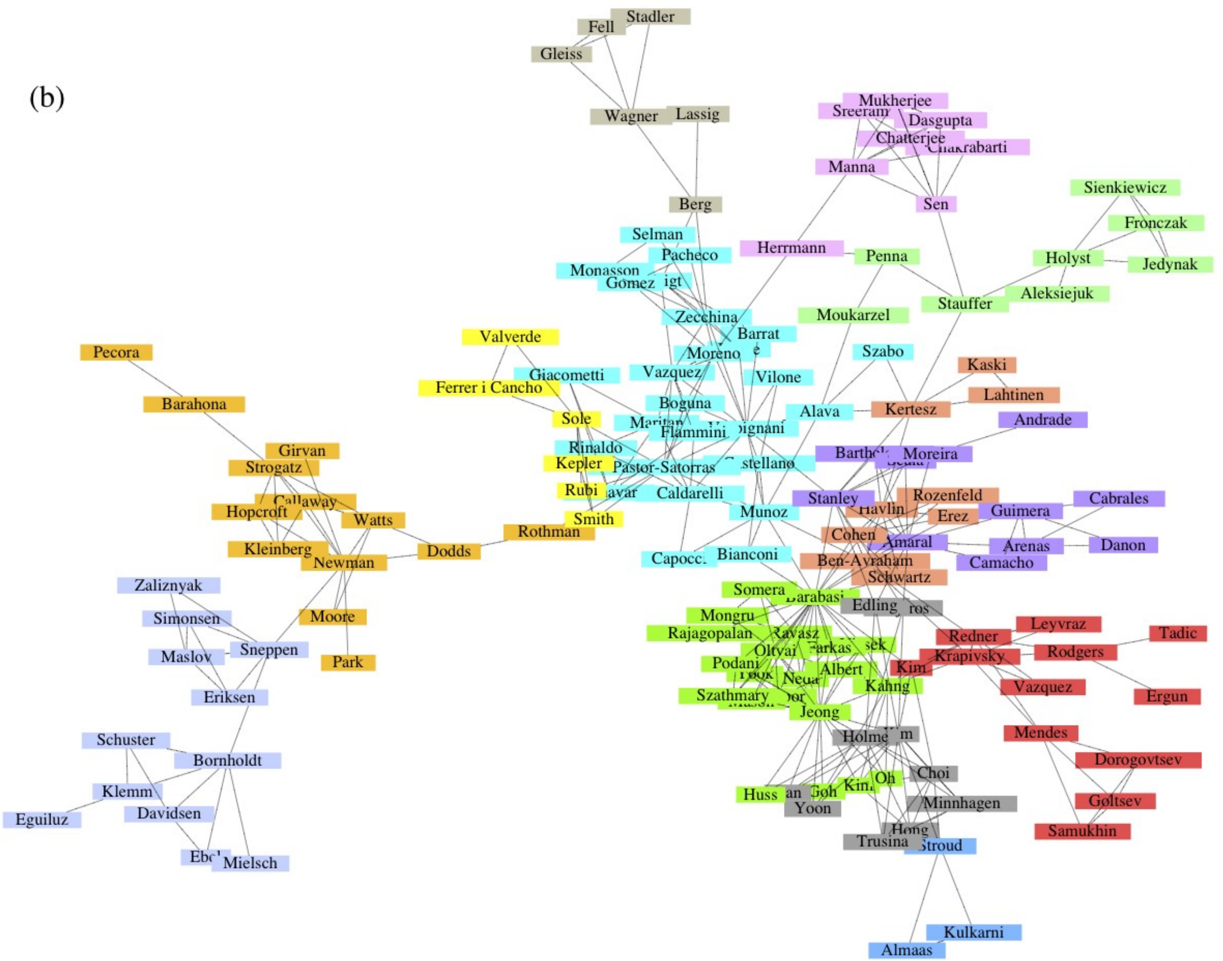
Einstein: EN=2
Paul Samuelson EN=5

….

ALB: EN: 3

NETWORK DATA: SCIENCE COLLABORATION NETWORKS

(b)

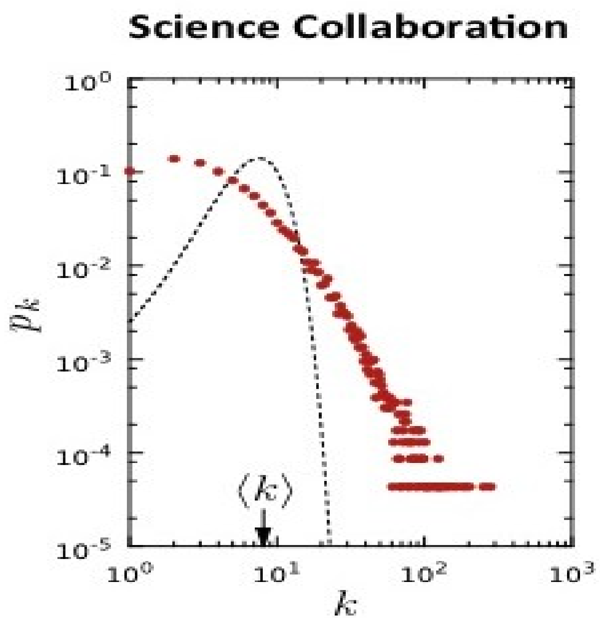


Collaboration Network:
Nodes: Scientists
Links: Joint publications

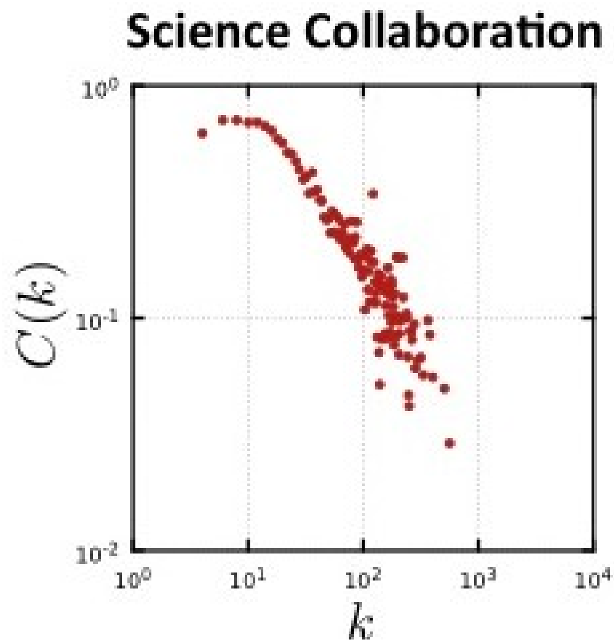
Physical Review:
1893 – 2009.

N=449,673
L=4,707,958

See also Stanford Large Network database
http://snap.stanford.edu/data/#canets.



Scale-free



Hierarchical

THE END